Charles Jones: “US Economic Growth in a World of Ideas” and other Jones Papers

January 22, 2014
U.S. GDP per capita, log scale
Old view: therefore the US is in some kind of Solow steady state (i.e. Balanced Growth Path: BGP) with constant technological progress.

Jones critique of this:

we know that rates of investment in human capital have risen over this period. [see next 3 slides]

If technological progress were constant, this would lead to high transitional growth

Also, investment in R&D has risen in this period, which might be hard to reconcile with constant technological progress.

Therefore, it can’t be a steady state.
Factors of Production in the United States

- GDP per Hour
- Multifactor Productivity
- Capital-Output Ratio
- Human Capital per Worker
Average U.S. Educational Attainment, persons aged 25 and over
Research Intensity in the G-5 countries
Production function: \( Y = A^\sigma K^\alpha H_Y^{1-\alpha} \)

\( H_Y \) is human capital devoted to producing output ... in practice, will assume that \( H_Y/H = L_Y/Y \), even though this is probably not true

Note: since \( A \) is never observed, I don’t understand the point of having \( \sigma \) in the model.
Capital accumulation: \[ \dot{K} = s_K Y = dK \]

Human capital per worker is determined by the usual Mincer formulation:

\[ h = e^{\psi l} \]

where \( l \) (as script lowercase “L” in the paper) is years of schooling
\[ \dot{A} = \delta H_A^\lambda A^\phi \]

where \( H_A = hL_A \) is the total human capital devoted to R&D.

- This is the Jones technology production function from his earlier papers. Two key components [next two slides]
The Parameter $\phi$

$$\dot{A} = \delta H_\lambda A^\phi$$

- effect of existing knowledge on growth of new knowledge.
- The standard model assumed that $\phi = 1$. That said that new inventions are proportional in size to the existing stock of knowledge.
- Jones says that this is an arbitrary assumption. If anything, he says, $\phi = 0$ is the more natural case. This would say that inventions are of the same size.
- You could even argue for $\phi < 0$. This would say that the more knowledge there is, the smaller every new invention is – for example, if all of the good ideas had been taken. This is called the shing out effect.
- On the other hand, we might expect that $\phi > 0$ because of the “better tools” effect, which is to say that when technology is better, each worker has better tools for inventing new technologies.
The Parameter $\lambda$

$$\dot{A} = \delta H_A^\lambda A^\phi$$

- $\lambda$ captures the idea that having more people working on invention does not necessarily lead to a proportional increase in invention. The standard case is that $\lambda = 1$.

- But if there is crowding out (i.e. twice as many scientists does not lead to twice as many inventions) then $\lambda < 1$. 
\[ \dot{A} = \delta H_A^\lambda A^\phi \]

- If \( \phi < 1 \), then if \( H_A \) is constant, *growth rate* of \( A \) will asymptote to zero.
- In this case, having constant growth of \( A \) will require constant growth of \( H_A \). Call this rate of growth \( n \)

\[ \frac{\dot{A}}{A} = \delta H_A^\lambda A^{\phi-1} \text{ differentiate w.r.t. time...} \]

\[ \frac{d(\dot{A})}{dt} = \lambda \delta H_A^{\lambda-1} A^{\phi-1} \dot{H}_A + \delta(\phi - 1) H_A^\lambda A^{\phi-2} \dot{A} = \lambda n \frac{\dot{A}}{A} + (\phi - 1) \left( \frac{\dot{A}}{A} \right)^2 \]

Along the Balanced Growth Path, technology growth is constant, so the above is zero, so

\[ \frac{\dot{A}}{A} = \frac{\lambda}{1-\phi} n \]
Measuring $H_A$

Jones allows for technology to be advanced by R&D in all of the cutting edge countries. So, while the other variables above were all supposed to have country subscripts (but I was too lazy to put them in), the $H_A$ was not (nor was the $A$). Specifically:

$$H_A = \sum_{i=1}^{M} h_i^\theta L_{A,i}$$

where $\theta > 0$ (I don’t really understand this part. I would just make $\theta = 1$).
$N(1 - s_h) = L_Y + L_A$

where $s_h$ is the fraction of the adult population that is in school

define

$s_R = L_A/N$
Rewriting the Production Function

Original Production Function: \( Y = A^\sigma K^\alpha H_Y^{1-\alpha} \)

define \( y = Y/L \) and \( k = K/L \)

so we can rewrite production function as

\[
y = \left( \frac{K}{Y} \right)^{\frac{\alpha}{1-\alpha}} \frac{L_Y}{L} hA^{\frac{\sigma}{1-\alpha}}
\]

- This way of re-writing is similar to Hall and Jones development accounting. The logic is that along a BGP where \( s_R \) and \( s_h \) are constant, the first three terms on the RHS will also be constant.

- Slight simplifying cheat: Jones defines output per worker as \( y = Y/L_Y \) instead of putting \( L \) in the numerator. This doesn’t matter much because share of labor force doing R&D is very small.
Rewritten production function: \[ y = \left( \frac{K}{Y} \right)^{1-\alpha} \frac{L}{L} h A^{\frac{\sigma}{1-\alpha}} \]

Along the BGP, everything on the RHS except \( A \) is constant, so
\[ \hat{y} = \frac{\sigma}{1-\alpha} \hat{A} = \gamma n \]

where \( \gamma = \frac{\lambda}{1-\phi} \frac{\sigma}{1-\alpha} \)

- but we know that we are not on a balanced growth path because \( h \) is in fact rising.
human capital production function (Mincer): $h = e^{\psi l}$ ($l$ is years of schooling)

- take logs and differentiate w.r.t. time: $\hat{h} = \psi \frac{dl}{dt}$
- $\psi$ is the (Mincerian) return to education. He uses 7%.
- $\frac{dl}{dt}$ averages .090 years per year
- so $\hat{h}$ will be assumed to be $.090 \times .07 = .0063$
Average Annual Growth Rates, 1950 - 1993

Growth of $A$ calculated as a residual (top half of table)

<table>
<thead>
<tr>
<th>Growth Rate of</th>
<th>Variable</th>
<th>Sample Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output per hour</td>
<td>$\hat{y}$</td>
<td>0.0200</td>
</tr>
<tr>
<td>Capital-output ratio</td>
<td>$\hat{K} - \hat{Y}$</td>
<td>-0.0015</td>
</tr>
<tr>
<td>Share of labor in goods</td>
<td>$\hat{\ell}_y$</td>
<td>-0.0001</td>
</tr>
<tr>
<td>Human capital</td>
<td>$\hat{h}$</td>
<td>0.0063</td>
</tr>
<tr>
<td>Multifactor productivity</td>
<td>$\hat{A}$</td>
<td>0.0146</td>
</tr>
<tr>
<td>R&amp;D labor</td>
<td>$\hat{H}_A$</td>
<td>0.0483</td>
</tr>
<tr>
<td>G-5 labor force</td>
<td>$\hat{n}$</td>
<td>0.0120</td>
</tr>
<tr>
<td>Share of labor in R&amp;D</td>
<td>$\hat{\ell}_A$</td>
<td>0.0363</td>
</tr>
<tr>
<td>Annual change in $\ell_h$</td>
<td>$\Delta \ell_h$</td>
<td>0.0902</td>
</tr>
</tbody>
</table>

Note: this is really $\sigma/(1 - \alpha)\hat{A}$ and not $\hat{A}$ that is being calculated, but we will deal with that in a minute.
Derivation of $\gamma$

- BGP equations: $\hat{y} = \frac{\sigma}{1-\alpha} \hat{A} = \gamma n$ where $\gamma = \frac{\lambda}{1-\phi} \frac{\sigma}{1-\alpha}$
- Jones makes normalization that $\sigma = 1 - \alpha$, which makes me wonder why we have been carrying around $\sigma$ all this time....
- so now we have: $\hat{y} = \hat{A} = \gamma n$ where $\gamma = \frac{\lambda}{1-\phi}$
- that holds when R&D labor force has been growing at a constant rate for a long time
- that is not true – but for lack of anything better, say it were true. Then we could derive $\gamma$ by dividing the growth of $A$ by $n$
  [Note: on the BGP, $\hat{y} = \hat{A}$, even though that is not true in our data because $h$ is growing. But we assume $A$ is on a BGP, so we use its growth rate.]
- This yields $\gamma = .0146/.0483 = .30$
- Jones has a fancier econometric section
Rewritten production function: \( y = \left( \frac{K}{Y} \right)^{\frac{\alpha}{1-\alpha}} \frac{L_Y}{L} hA^{\frac{\sigma}{1-\alpha}} \)

Take logs and differentiate w.r.t. time:

\[
\hat{y} = \left( \frac{\alpha}{1-\alpha} \right) (\hat{k} - \hat{y}) + \hat{h} + \left( \frac{L_Y}{L} \right) + \left( \frac{\sigma}{1-\alpha} \hat{A} - \gamma n \right) + \gamma n
\]

Note we have subtracted and added the term \( \gamma n \) on the right side. This is the only term that is non-zero on the BGP. The second to last term is technology growth in excess of what will obtain on the BGP.

See Table [next slide]: output growth decomposed into transitional and non-transitional parts.
### Accounting for U.S. Growth, 1950 - 1993

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\hat{y}$</th>
<th>$\frac{\alpha}{1 - \alpha} (\hat{K} - \hat{Y})$</th>
<th>$\ell_y$</th>
<th>$\hat{h}$</th>
<th>$\hat{A} - \gamma n$</th>
<th>$\gamma n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.050</td>
<td>0.0200</td>
<td>-0.0007</td>
<td>-0.0001</td>
<td>0.0063</td>
<td>0.0140</td>
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</tr>
<tr>
<td>0.200</td>
<td>0.0200</td>
<td>-0.0007</td>
<td>-0.0001</td>
<td>0.0063</td>
<td>0.0122</td>
<td>0.0024</td>
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<tr>
<td>0.333</td>
<td>0.0200</td>
<td>-0.0007</td>
<td>-0.0001</td>
<td>0.0063</td>
<td>0.0106</td>
<td>0.0040</td>
</tr>
</tbody>
</table>

Transition Dynamics:

- Output per Hour
- Capital Intensity
- Labor Reallocation
- Educational Attainment
- Excess Idea Growth
- Steady-State Growth
What are We to Make of Growth Being Constant?

- remember our initial motivating fact
- Jones has shown that we are not on a BGP. Both $\hat{h}$ and “excess idea growth” are important (and K/Y growth before 1950, maybe)
- Jones introduces “Constant Growth Path” as an alternative to BGP
BGP arises when rates of accumulation (saving rate, level of education, fraction of labor force doing R&D) are constant (and population is growing).

CGPs arise when all these accumulation rates are themsevles *growing at a constant rate* (except for years of education, which has to have a constant time derivative).

Of course, that can’t go on forever like a BGP can, but it could last a while.

Observed output could be a CGP

My view: yes, it could be, but it could also be a result of any of a zillion other patterns of varying accumulation rates (ACGP - Arbitrary Constant Growth Path).

Deeper question: why would either CGP or ACGP happen?
Jones Conclusions

- Jones says: constantcy of growth of output (key motivating fact) is an illusion
- US has experienced “grand traverses” in accumulation rates:
  - investment rate rose in 19th century
  - schooling rose in 20th; R&D share still rising (temporarily)
- Once all this traversing is over, growth will slow down.
- Brown University critique: Hello? Why take population growth as fixed and permanent while allowing all these other accumulation rates to change?