Business Saving and Normal Income

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I

Mr. Hart (1) has attempted to test the significance of observations by Sargant Florence (2) and others, that the average propensity to retain profits (to save) varies with size of firm, measured by issued capital. He does this by computing the profit elasticity of savings for cross-sections of companies in the Brewing industry in each of the years 1948-51. Finding elasticities that are not significantly different from 1, he concludes that variations in the pay-out ratios are also not significant. In addition, he writes " It is tempting to compare these results with the savings-income elasticities estimated, for example, by Friedman. The present results certainly suggest that the proportionality hypothesis can be extended to companies without introducing any distinction between normal and current profits".

This paper will argue two propositions. First, that Hart's rejection of a normal income hypothesis is not warranted by his results, and that, in consequence, (a) his test is much diminished in power, and, (b) his results are, on a normal income hypothesis, confirmatory rather than contradictory of those of Sargant Florence. Secondly, it will argue that such econometric work as has been done on business saving is entirely consistent with a normal income model.

II

Firstly, what is a normal income model? Such a model would argue that a firm's dividends (D) are a function of some notion of normal or permanent or expected income (Y^*), an income figure which is not, in general, identical with current profits (Y). Suppose we found a group of firms such that it was reasonable to assume that (a) this function is linear, and (b) the firms are homogeneous with respect to the parameters of the function; then from the economic point of view the correct specification of the behavioural relationship we wish to estimate would be [2]

$$d = \beta \,.\, y^* + \mu \qquad (1)$$

If, instead of 1 we actually fit

$$d = \gamma \,.\, y + \varepsilon \qquad (2)$$

to the data, we commit a specification error. The particular form of specification error is the very familiar one of error in the variable, as is obvious by rewriting 2 as

$$d = \gamma(y^* + y - y^*) + \varepsilon \qquad (3)$$

in which the element $(y - y^*)$ can be regarded as the error in y^*.

[1] I would like, without implicating them in the mistakes, to thank Malcolm Fisher and M. J. Farrell for helpful comments.

[2] Notation: capital letters indicate variables measured from the origin, small letters variables measures from sample means. Subscripts identifying the observation are generally omitted, for simplicity. A $\hat{}$ superscript indicates a least squares estimate.

That is, in its statistical aspect, a normal income hypothesis says that by using measured income in our equation we commit a particular sort of specification error. It is a familiar result that only on very special assumptions about the errors (transitory components) will least squares estimates of γ be identical with those of β. (See e.g. Durbin [3].)

One way of looking at the relationship between $\breve{\gamma}$ and $\breve{\beta}$ would be by expanding the least squares estimator of γ as,

$$\breve{\gamma} = \frac{\Sigma d \,.\, y^* + \Sigma d \,.\, (y - y^*)}{\Sigma y^{*2} + 2\Sigma y^* \,.\, (y - y^*) + \Sigma(y - y^*)^2}$$

Another, neater, way is by using a version [1] of Theil's [4] theorem on specification errors. Denoting a matrix by a capital letter and a vector by a small one, suppose we fit by least squares,

$$y = \bar{X} \,.\, \bar{\beta} + \bar{\mu} \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

whereas the correct specification of the variables determining y is,

$$y = X \,.\, \beta + \mu \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

Then the relationship between $\breve{\bar{\beta}}$ and $\breve{\beta}$ is

$$\breve{\bar{\beta}} = \breve{P}' \,.\, \breve{\beta}$$

in which $\breve{P}'$ is the transpose of the matrix of least squares regression coefficients of each " true " determining variable, in X, on all variables in $\bar{X}$—the " false " determining variables.

In our case, y^* is the true determining variable and y the false one, therefore we have,

$$\breve{\gamma} = \breve{\alpha} \,.\, \breve{\beta} \quad \ldots\ldots\ldots\ldots\ldots \quad (6)$$

where α is defined in

$$y^* = \alpha \,.\, y + r$$

Evidently then $\breve{\gamma} < \breve{\beta}$ when $\breve{\alpha} < 1$ $(\breve{\beta} > 0)$. But $\breve{\alpha}$ is defined by,

$$\breve{\alpha} = \frac{\Sigma y^* \,.\, y}{\Sigma y^2} = 1 - \frac{\Sigma(y - y^*) \,.\, y}{\Sigma y^2}$$

Now for a cross-section of companies it is reasonable to suppose that $\Sigma(y - y^*) \,.\, y$ will be positive; the larger the measured profit the larger, in general, the error it contains. In Friedman's words, " a relatively high measured income could have been received despite unfavourable transitory effects; clearly, it is more likely to have been received because of favourable transitory effects; the winners in any particular set of races may well be better on the average than the losers but they are also more likely to have had more than their share of good luck ". ([5], p. 35.)

It is also quite reasonable to suppose that the error is generally smaller than the measured variable, implying that $\Sigma(y - y^*) \,.\, y < \Sigma y^2$. Then on these assumptions, $\breve{\alpha}$ is

[1] The theorem is usually given as a relationship between the expected value of $\breve{\bar{\beta}}$, $\breve{P}$ and the population β. For our purposes it is more convenient to give it as stating an exact relationship between L.S. estimates of $\bar{\beta}$, P, and β. Theil's theorem applies, of course, to other methods of estimation and other types of specification error.

equal to 1 minus a positive fraction, and is thus itself such a fraction. This, from 6, implies that $\breve{\gamma} < \breve{\beta}$; the estimated least squares marginal propensity to distribute dividends is biased downwards when measured income rather than normal income is employed as a determining variable.[1] Clearly also then least squares saving propensities will be biased upwards.

Supposing we had assumed a multiplicative transitory component, and a logarithmic relationship between D and Y^*, a similar argument still applies, only this time the bias is in dividend and saving elasticities.[2]

III

Before considering a third way of looking at the errors in the variables problem we can consider Hart's argument. He argues that finding a saving elasticity of unity implies that normal income = measured income; but this would only be true if there were proportionality between dividends and normal income for his sample. But clearly the proportionality assumption is not a necessary part of a normal income model at all.[3] In the same way as you can get errors in the variable bias whether or not the correct specification is one of proportionality, so you can find the normal income or Friedman effect, for they are statistically the same thing. It follows that no measured saving elasticity is evidence on the relevance of a normal income model unless one has independent evidence about the true elasticity.

We have argued that dividend elasticities using measured profits will be biased downwards, and that savings elasticities will be biased above the elasticity of saving with respect to normal income. Hart found an elasticity of 1, and so on a normal income hypothesis the true elasticity will be less than 1 implying an average propensity to save out of normal income falling with increases in firm size. Large firms save less than small ones in the Brewing industry. But this is precisely what Sargant Florence's calculations show. It follows that Hart's results are, on a normal income model, confirmatory of Florence's rather than contradictory as he supposes.

The question arises, why aren't Florence's results similarly biased? The answer, I suggest, is that he did not perform a cross-section regression analysis but simply calculated five year average ratios of dividends to profits for different size groups of firms. This sort of approach is one much more likely to escape the effects of normal income bias than is a regression analysis for three reasons. Firstly it may well be the case that if the size groups are sufficiently large the mean transitory component within each group in each year will approximate to zero. Secondly even if this is not the case it may well be that the five year mean of the mean transitory components for a size group will approximate to zero. Thirdly, even if these mean transitory components are not zero there seems no very good reason why they should such as to give rise to pay out ratios rising systematically with size of firm, i.e. between size groups.[4]

[1] One can, I suppose, quarrel with calling the normal propensity the "true" propensity and the measured income coefficient the "biased" one. But clearly it is β which measures the *propensity* to distribute, the behavioural parameter in which we are interested. γ simply represents the combined influence of y^* and a particular set of events or outcomes which, we hypothesise, are not seen by firms as relevant to tne determination of current dividends.

[2] Proof that this is the case is given in Appendix 2.

[3] See e.g. Farrell (6).

[4] This is not to say that all of Prof. Florence's results would be unaffected by the relevance of a normal income hypothesis. A comparison of the graph (2 p. 150) in which he plots a least squares regression line of D/net assets on Y/net assets, with the graph in Friedman 5 p. 34) is of some interest in this connection.

IV

An alternative way of looking at an errors in the variable model is by specifying something about the relationship between normal and measured income. One can do this at the whole sample level by making assumptions about the variances and covariances of the transitory components, as Friedman, for example, did. Or one can do it at the micro level by writing $Y^*t = g(Yt)$. The simplest way of doing this we saw on page 3 with Theil's theorem which involved a linear relationship between Y^*t and Yt. This is helpful in showing the plausible direction and extent of bias but is not much use if we wish to estimate β since substitution of 6 in 1 gives only,

$$d_t = \beta\alpha \,.\, y_t + [\beta \,.\, r + \mu] \quad \dots\dots\dots\dots \quad (7)$$

Probably the simplest function to assume which also gives a distinguishable estimate of β is the following,

$$y^*_t - y^*_{t-1} = \rho[y_t - y^*_{t-1}] \quad \dots\dots\dots\dots \quad (8)$$

in which the change in normal income is some proportion of the difference between current measured income and last periods normal income.

Substitution of 1 in 8 then gives,

$$d_t = \rho\beta \,.\, y_t + [1 - \rho] \,.\, d_{t-1} \quad \dots\dots\dots\dots \quad (9)$$

if β is constant over t and $t - 1$. Now equation 9 is of interest because, although we derived it as one possible solution to the errors in the variables problem, it is better known as Koyck's (7) suggested transformation of the geometric lag.[1] Where dividends are assumed to be equal to β times a weighted average of current and all past profits, the weights being powers of $(1 - \rho)$ declining in a geometric progression, simple manipulation gives 9 as a handier version of such a lag.

Some interesting inferences may be drawn from this approach. Firstly, and perhaps rather obviously, if 9 is found to give a reasonable fit to a time series, this need not imply that any lag exists, in any behavioural sense, but simply that Y^*t is subject to error. One way of looking at a lag is as a particular case of an errors in the variables model, a case which specifies Y^*t, as, say, some average of past incomes, and which also therefore specifies the errors, the differences between Yt and Y^*t. A lag can be regarded as a specification of a process by which errors in the variable are generated.

A second inference is that not only can solutions to an errors in the variable problem involve what look like lag equations, but techniques which are applicable to errors in the variables problems may sometimes be usefully applied to estimate lags. This may not sound like a very helpful suggestion as the estimation of linear relations in which the determining variable is subject to error has not made much progress, but equally, estimating lags can sometimes prove very difficult. For example in 9 one is quite likely to find not only multicollinearity, but also bias if, as appears to be often the case (see, e.g. 9) the disturbances are autocorrelated.[2] [3] It may then be that, say, an instrumental variables estimate of β, from 2, could be employed with the least squares estimate, $\breve{\gamma}$, to find the parameters of the lag, instead of estimating the lag directly.

[1] 8 is Nerlove and Arrow's (7) model of adaptive expectations if we assume normal = expected.

[2] In small samples least squares estimates will be biased anyway in this autoregressive equation.

[3] If 9 is specified as being derived from combining 1 and 8 and these latter are assumed stochastic, then the disturbance term in 9 will not in general be independent of $D1t - 1$, which makes for further trouble in estimating this equation.

In addition the disturbance will be specified as autocorrelated if 1 is assumed stochastic for then the disturbance in 9 will contain μt and $\mu t - 1$.

However from the point of view of this paper equation 9 is of interest because it has been fitted to dividend-profit data by several people and found to give much higher r^2 than a simple regression of dividends on profits. Lintner fitted 9 to an aggregate U.S. time series and found, " excellent correlations, random residuals, and highly significant regression coefficients over the entire period 1918-51 and all major sub-groups of years ". (10). Prais (11, 12) fitted it to cross-sectional U.K. quoted companies data and found r^2 well in excess of those from simple dividend-profit equations. Also Dobrovolsky (13) found 9 an adequate description of his U.S. data.

The success of this model suggests the relevance of the distinction between normal and measured profits. Equation 9 is consistent with a model in which $yt \neq y^*t$, in general, and in this case there will be errors in the variable or Friedman bias in both cross-section and time-series regressions of dividends on profits for such firms.[1 2]

V

The previous sections have been discussions of possible interpretations of other people's results, and it could be argued that this is a waste of time when no direct tests of these interpretations are given.[3] But there is a good reason why these points are worth making, and this is that a considerable amount of effort has been spent and is likely to continue being spent on research on company finance which quite ignores possible normal income effects. There is a quite reasonable a priori case for arguing that such research is likely to give rise to the same sort of paradoxical and contradictory results as emerged in the case of the consumption function. And indeed it is possible to regard a result of Professor Tew as a first example of such a paradox.

The technique of analysis employed by Tew and Henderson in " Studies in Company Finance " was to define " indicators " of size, growth, liquidity, etc., 16 in all, to describe the characteristics of each company in their sample of 2549 over the period 1949-53. Among these indicators were a " self-financing " indicator, the five year ratio of net saving to net investment for each firm; a " thrift " indicator the five year ratio of net saving to net income; and an " investment-income " indicator, again a five year ratio for each firm. In a chapter on self-financing, Tew gives calculations showing the correlations between the logarithms of pairs of these ratios, and interprets the results as representing behavioural relationships. For example, the correlation between $1g \,.\, S/Y$ and $1g \,.\, I/Y$ is interpreted as descriptive of the relationship between " intentions to save ", and intentions to invest ". The observed correlations are as follows:

$$r \,.\, (1g \,.\, S/I \,.\, 1g \,.\, S/Y) = .42 \qquad 1$$

$$r \,.\, (1g \,.\, S/I \,.\, 1g \,.\, I/Y) = -.86 \qquad 2$$

$$r \,.\, (1g \,.\, S/Y \,.\, 1g \,.\, I/Y) = .15 \qquad 3$$

As Tew remarks, there is nothing unreasonable in the first two correlations; they are much as theory, or common sense, would lead us to expect. But the third correlation he finds " remarkable, because it provides virtually no evidence of any connection between intentions to save and intentions to invest ".

[1] It must be noted that 9 is also consistent with a model in which $yt = y^*t$ but in which a discrepancy arises between dt and d^*t (a " desired dividend " level). Here 9 is derived from the equilibrium equation,

$$d^*t = \beta \,.\, yt$$

and the adjustment equation, (the analogue of 8),

$$dt - dt - 1 = \rho[d^*t - dt - 1]$$

[see e.g. Nerlove [21].

[2] See over.

[3] Results of direct tests for normal income effects will be given in a forthcoming article.

Suppose we assume that,

$$Yi = Y^*i[1 + \pi_{1i}]$$
$$Di = D^*i$$
$$Si = S^*i[1 + \pi_{2i}]$$
$$Y^*i = S^*i + D^*i$$

where π_{1i} and π_{2i} express the proportionate errors in profits and savings, and the i subscript denotes the observation. Since $Di = D^*i$, we have,

$$\pi_{2i} \,.\, S_i^* = \pi_{1i} \,.\, Y_i^* = \text{the transitory component} \quad \ldots \ldots \quad (10)$$

Thus,
$$\lg\left(\frac{Si}{Yi}\right) = \lg S_i^* + \lg(1 + \pi_{2i}] - \lg Y_i^* - \lg(1 + \pi_{1i})$$
$$= \left[\lg\left(\frac{Y_i^*}{S_i^*}\right) + \lg\left(\frac{1 + \pi_{2i}}{1 + \pi_{1i}}\right)\right]$$

and,
$$\lg\left(\frac{Ii}{Yi}\right) = \lg I_i - \lg Y_i^* - \lg(1 + \pi_{1i})$$
$$= \left[\lg\left(\frac{Ii}{Y_i^*}\right) + \lg\left(\frac{1}{1 + \pi_{1i}}\right)\right]$$

where the term on the R.H.S. of each squares bracket is the error in the log of the ratio corresponding to the original multiplicative errors in Si and Yi. Suppose now that the logs of the normal ratios were positively correlated. Then each observed ratio is equal to the sum of two quantities, the first one of which is positively correlated with its opposite number, and the second one of which is plausibly negatively correlated with its equivalent in the other variable.

To see this negative correlation between $\lg\left(\frac{1 + \pi_{2i}}{1 + \pi_{2i}}\right)$ and $\lg\left(\frac{1}{1 + \pi_{1i}}\right)$ consider their covariance. Using small letters to denote variables measured from their means, and approximating $\lg(1 + \pi)$ by π, the first term in the series expansion,[1] we have,

$$\text{N. cov. } \lg\left(\frac{1 + \pi_{2i}}{1 + \pi_{1i}}\right) . \lg\left(\frac{1}{1 + \pi_{1i}}\right) = \sum_i \left[\lg(1 + \pi_{2i}) - \lg(1 + \pi_{1i})\right]\left[-\lg(1 + \pi_{1i})\right]$$

$$\triangleq \sum_i \left[\pi_{2i} - \pi_{1i}\right]\left[-\pi_{1i}\right] = \sum_i \pi_{1i}^2 - \sum_i \pi_{1i} \,.\, \pi_{2i}$$

But we know, from the assumptions of the model, that $\frac{\pi_{2i}}{\pi_{1i}} = \frac{Y_i^*}{S_i^*} = X_i$. And where π_{1i} and π_{2i} can be supposed to have zero sums,[2] this also holds for the deviations of the π's from their means, i.e. $\frac{\pi_{2i}}{\pi_{1i}} = X_i$. Thus,

$$\text{N. cov. } \lg\left(\frac{1 + \pi_{2i}}{1 + \pi_{1i}}\right) . \lg\left(\frac{1}{1 + \pi_{1i}}\right) \triangleq \sum_i \pi_{1i}^2(1 - X_i)$$

[1] Taking a term in π^2 in the expansion into account does not materially affect the following argument.

[2] This assumption is not necessary in order to show that $\sum_i \pi_{1i}^2 - \sum_i \pi_{1i} \,.\, \pi_{2i}$ is plausibly negative but it gives a neater argument without being very unreasonable.

which, since X_i, the reciprocal of the normal savings ratio, may be expected to be generally > 1, is very plausibly negative.

This suggests, but does not, of course, prove, that the correlation between the observed ratios will be smaller than the correlation between the normal ratios. We might expect to find, on a normal income model, that the observed correlation understated the true, behavioural, relationship in the sample.

Appendix I contains a statement of conditions under which errors in ratios will reduce r^2, together with an application of these results to those of Tew. It is shown there that of his three correlations, the third, remarkable, one is very likely to be subject to downwards bias. And it is stated that his other two, more acceptable, results appear rather less likely to be affected in this way.

VI

There has been one previous attempt to apply a normal income model to business dividends, by M. R. Fisher (15), although it has attracted little attention. Fisher's data consisted of aggregate data for 10 U.S. industries over the period 1939-50. He employed the following formula from Friedman (5) to determine the coefficient of variation of normal profits over time in each industry. (He made, of course, the assumption that observed aggregate profits equals aggregate normal profits plus an error.)

$$v_{Y*} = \sqrt{v_Y^2 \, . \, Py} \qquad (11)$$

where v_Y is the coefficient of variation of measured profits and Py is the ratio of the variance of normal profits to the variance of measured profits.

He finds a correlation of $-.725$ between the coefficient of variation of normal profits and the mean pay-out ratio over time in each industry. That is, where i denotes the industry,

$$r \, . \, (v^*_{Yi} \, . \, (\sum_t Dt \, / \sum_t Yt)_i) = -0.725$$

He then argues that industries with high variation of normal profits had high or optimistic profit expectations, and that his correlation shows that the higher the profit expectations the more profits were retained.

Suppose however we assume that the geometric lag of equation 9 describes the relationship between profits and dividends over time in each industry.[1] (Lintner's and Dobrovolsky's work render this assumption very reasonable.) Then if this is so, and if the parameters of 9 are stable over the period, it is not difficult to show that the aggregate pay-out ratio in each industry and in each year, will depend on the growth rate of profits, even where β, the true, or normal pay-out ratio is the same in each industry.

We assume that aggregate profits in each industry grew at the rate i throughout the period, calling the initial period $t = \alpha$. Thus 9 becomes,

$$D_1 = \rho\beta \, . \, Y_1 + (1 - \rho) \, . \, D_0$$

and in the next period we have

$$D_2 = \rho\beta \, . \, Y_1((1 + i) + (1 - \rho)) + (1 - \rho)^2 \, . \, D_0$$

and so on. It is then straightforward to sum these dividend and profit series and find an expression for the aggregate pay-out ratio over the period. The manipulation is clearly

[1] Assuming the constant term is zero, for simplicity.

tedious and will not be reproduced here but it is in fact the case that for constant, β, ρ and i this ratio approaches the asymptotic limit,[1]

$$\sum_t Dt \,/ \sum_t Yt \rightarrow \frac{\rho\beta}{1 - \dfrac{1-\rho}{1+i}} \quad (t \rightarrow \infty \qquad (12)$$

So, from 12, the faster the growth of measured profits the smaller the observed pay-out ratio.

So far we have not said anything to contradict Fisher, for he found the pay-out ratio varying with the variation (growth) of *normal* profits, and we have only shown a mechanism by which this ratio will vary with the growth of *measured* profits. However Fisher's formula for $v^*_{\bar{Y}}$ (11) involves Py the variance of normal dividend by the variance of measured profits, which is found as the ratio of the marginal propensity to distribute to the average, or pay-out ratio. But for Py to be given by the ratio $\frac{mpd}{apd}$ requires (a) a zero covariance between the normal and transitory components of profits, and (b) a zero mean transitory component. Neither of these assumptions will be satisfied under geometric profit growth[2] and the model 9.[3] In particular the mean transitory component will be plausibly positive (necessarily positive if $Do = \beta \,.\, Yo$), and it will be an increasing function of i. In fact employing 12, the mean transitory component in the limit is given as

$$\frac{\bar{D}}{\bar{Y}} = \frac{\beta \bar{Y}^*}{\bar{Y}} = \frac{\rho\beta}{1 - \dfrac{1-\rho}{1+i}} \text{ so } \bar{Y} - \bar{Y}^* = \bar{Y}\left(1 - \frac{\rho}{1 - \dfrac{1-\rho}{1+i}}\right)$$

This means that Fisher's $v^*_{\bar{Y}}$ will not, in general, equal the coefficient of variation of normal profits. What it actually measures depends on the ratio $\frac{mpd}{apd}$ represented by Py in equation 11. In fact calculations from Fisher's data tell us that whatever v_{Y^*} is, it is correlated to the extent of .888 with v_Y, the coefficient of variation of measured profits.

In other words both v_Y and v_{Y^*} are indicators of the growth rate of measured profits and because of this we would expect them to be inversely related to the pay-out ratio, as in fact they are. More generally, in any situation in which a model like 9 is thought relevant it is necessary to separate out the effects of the growth rate before conclusions about variations in behavioural parameters can be validly drawn.

VII

To conclude. This paper has done three things. It has pointed out what appears to be a logical error in Hart's supposing that his results allow him to reject a normal income hypothesis. Secondly it has discussed a normal income model in its *statistical* aspect, pointing

[1] This limit is of some interest in that it implies the dependence, cet. par., of the observed business savings ratio on the rate of growth of profits. It thus suggests the relevance of a " rate of growth " hypothesis (see Farrell (6) in the business saving function, as well as in the consumption function, though for rather different reasons. A version of this limit was also found by Stone and Brown (16) when they employed a geometric lag model to represent a Friedman type expenditure function.

[2] Or, more generally, when there is any marked upwards bend in profits.

[3] Friedman (5) employed the assumptions of independence of normal (permanent) and transitory components and zero mean transitory component; he also, at one point, employed a geometric lag model to describe the generation of normal income, as 9 does. To the extent that any of his income series could be approximated by a geometric growth curve, the possibility of inconsistency between his assumptions and his practice arises.

out the relationship of such a model to (a) the well-known errors in the variables problem, and (b) lag models: and suggesting that the success of Koyck-type lag equations in the context of business saving indicates the relevance of normal income effects. Thirdly it has pointed out that such effects could well account for a curious result [1] of Professor Tew and probably render his other results imperfect measures of " propensities ".

The evidence presented here can certainly not be regarded as sufficient to establish beyond doubt the relevance of a normal income hypothesis, statistically analogous to Friedman's consumption function hypothesis. It is, I think, sufficient to establish a doubt about the validity of results derived from manipulating quantities involving measured aggregate profits and measured retained profits.

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APPENDIX 1

Conditions under which errors in the variables will lower r^2

As in the text, small letters denote variables measured from their means and capitals variables measured from the origin. A * superscript denotes the true variable and Greek letters the (variable) error component. Subscripts identifying the observation are omitted for simplicity.

1. We define,

$$u = u^* + \gamma_1 \qquad v = v^* + \gamma_2$$

2. The true r^2 is given by,

$$\frac{(\Sigma u^* v^*)^2}{\Sigma u^{*2} \,.\, \Sigma v^{*2}} = r_1^2$$

3. The false r^2 is given by,

$$\frac{(\Sigma uv)^2}{\Sigma u^2 \,.\, \Sigma v^2} = r_2^2$$

4. To find:

conditions under which,

$$i \quad r_2^2 \quad - \quad r_1^2 \quad < \quad 0$$

5. Substitution of the definitions, 1, into 2 gives,

$$\gamma_1^2 = \frac{[\Sigma ur - \Sigma\gamma_2 u^* - \Sigma\gamma_1 v^* - \Sigma\gamma_1\gamma_2]^2}{[\Sigma u^2 - 2\Sigma\gamma_1 u^* - \Sigma\gamma_1^2][\Sigma v^2 - 2\Sigma\gamma_2 v^* - \Sigma\gamma_2^2]}$$

Some sets of conditions under which i will hold can be seen, using 5, to be:

A. If the errors, γ_1, γ_2, are independent of each other and of each true component, i becomes,

$$\frac{[\Sigma uv]^2}{\Sigma u^2 \,.\, \Sigma v^2} - \frac{[\Sigma uv]^2}{[\Sigma u^2 - \Sigma\gamma_1^2][\Sigma v^2 - \Sigma\gamma_2^2]} < 0 \quad \text{TRUE}$$

[1] A result which has already found its way into at least one undergraduate text.

B. If the errors are independent of each true component, but negatively correlated, i becomes,

$$\frac{[\Sigma uv]^2}{\Sigma u^2 . \Sigma v^2} - \frac{[\Sigma uv - \Sigma \gamma_1 \gamma_2]^2}{[\Sigma u^2 - \Sigma \gamma_1^2][\Sigma v^2 - \Sigma \gamma_2^2]} < 0 \quad \text{TRUE}$$

C. A more general set of sufficient conditions for i to hold is given by,

$$\Sigma \gamma_2 u^*, \ \Sigma \gamma_1 v^*, \ \Sigma \gamma_1 \gamma_2 \leqslant 0$$

$$\Sigma \gamma_1 u^*, \ \Sigma \gamma_2 v^* \geqslant 0$$

as can be seen by inspection of 5.

Application to Tew's Results

Take the correlation between S/Y and I/Y and define

$$S = S^*(1 + \pi_2) \qquad Y = Y^*(1 + \pi_1)$$

in which π_1 and π_2 are variables and represent the proportionate errors. Assuming that the error in retained profits, S, arises from the error in measured profits—that the transitory component of profits goes into savings and does not affect dividends, we have,

$$\pi_2 . S^* = \pi_1 Y^* = \text{the transitory component}$$

Using the notation, $\lg(A/B) - \lg(A/B) = \lg(a/b)$, and, $\lg(1 + \pi) - \lg(1 + \pi) = \lg(1 + \pi)$; and putting,

$$u = \lg(s/y) \qquad u^* = \lg(s^*/y^*)$$

$$v = \lg(i/y) \qquad v^* = \lg(i/y^*)$$

we find that,

$$\gamma_1 = \lg\left(\frac{1 + \pi_2}{1 + \pi_1}\right) \qquad \gamma_2 = \lg\left(\frac{1}{1 + \pi_1}\right)$$

Consider the conditions, C. It seems reasonable to assume that the proportionate error in the variable is independent of their size, and this implies that the two terms, $\Sigma \gamma_2 u^* = \Sigma \gamma_2 v^* = 0$. Further, we argued in the text that $\Sigma \gamma_1 \gamma_2 < 0$. The two remaining conditions then are,

$$\text{(i)} \quad \Sigma \lg\left(\frac{1 + \pi_2}{1 + \pi_1}\right) . \lg(s^*/y^*) \geqslant 0$$

$$\text{(ii)} \quad \Sigma \lg\left(\frac{1 + \pi_2}{1 + \pi_1}\right) . \lg(i/y^*) \leqslant 0$$

Using the first term in the series expansion of $\lg(1 + \pi)$, as in the text, we find that,

$$\text{(i)} \quad \triangleq \Sigma(\pi_2 - \pi_1) . \lg(s^*/y^*) = \Sigma(\pi_2 - \pi_1) . \lg\left(\frac{S^*}{Y^*}\right)$$

Since the normal savings ratio can be assumed to fall almost uniformly in the interval $0 - 1$, $\lg\left(\frac{S^*}{Y^*}\right)$ can be assumed almost uniformly negative. But the difference $(\pi_2 - \pi_1)$

is positive or negative according as the transitory component of profits is positive or negative. Thus the sum (i) may be assumed small relative to the variances and covariance of the errors which are sums of uniformly positive or negative terms.

For (ii) write,

$$\lg(i/y^*) = b \, . \lg(s^*/y^*) + \mu$$

and assume that,

$$\Sigma(\pi_2 - \pi_1) \, . \, \mu = 0$$

Then proceeding analogously to (i) we find that,

$$\text{(ii)} \triangleq b \, . \, \Sigma(\pi_2 - \pi_1) \, . \lg\left(\frac{S^*}{Y^*}\right)$$

which, since b is unlikely to be much over unity, may again be assumed small.

We had to consider seven terms, the five specified in the conditions C, plus the variances of the errors. Of these seven, two may reasonably be assumed zero; three, the variances and covariances of the errors, certainly have the required sign; and the remaining two may plausibly be assumed small relative to the latter terms. Taking into consideration the fact that weaker conditions than C for downwards bias in r^2 can certainly be formulated, it is clear that multiplicative errors in S and Y of the type specified in this model would certainly cause downwards bias in the observed r^2 between the savings—income and investment—income ratios.

It is possible to proceed along similar lines with Tew's other correlations and to show that conditions C are somewhat less likely to be satisfied in these cases. That is, it is precisely his most remarkable correlation that is most likely to biased by errors in the variables.

APPENDIX 2

Conditions under which multiplicative errors in S^* and Y^* raise the least squares measured income elasticity of savings.

Employing the model and notation of appendix 1 we have,

$$\lg s = \lg s^* + \lg(1 + \pi_2)$$

$$\lg y = \lg y^* + \lg(1 + \pi_1)$$

The least squares normal elasticity is given by,

$$e^* = \frac{\Sigma \lg s^* \, . \lg y^*}{\Sigma\,[\lg y^*]^2}$$

The least squares measured elasticity is given by,

$$e = \frac{\Sigma \lg s^* \, . \lg y^* + \Sigma \lg(1 + \pi_1) \, . \lg(1 + \pi_2) + \Sigma \lg s^* \, . \lg(1 + \pi_1) + \Sigma \lg y^* \, . \lg(1 + \pi_2)}{\Sigma\,[\lg y^*]^2 + 2\Sigma \lg y^* \, . \lg(1 + \pi_1) + \Sigma\,[\lg(1 + \pi_1)]^2}$$

which reduces to,

$$e = \frac{\Sigma \lg s^* \, . \lg y^* + \Sigma \lg(1 + \pi_1) \, . \lg(1 + \pi_2)}{\Sigma\,[\lg y^*]^2 + \Sigma\,[\lg(1 + \pi_1)]^2}$$

if the error proportions, π_1 and π_2, are independent of their own true components.

Thus $e > e^*$ when,

$$\frac{\Sigma \lg (1 + \pi_1) \, . \, \lg (1 + \pi_2)}{\Sigma \, [\lg (1 + \pi_1)]^2} > \frac{\Sigma \lg s^* \, . \, \lg y^*}{\Sigma \, [\lg y^*]^2}$$

That is, when the elasticity of $(1 + \pi_2)$ with respect to $(1 + \pi_1)$ or, roughly, $d\pi_2/d\pi_1$ exceeds the normal saving elasticity. Approximating $\lg (1 + \pi)$ by π we have,

$$\frac{\Sigma \lg (1 + \pi_1) \, . \, \lg (1 + \pi_2)}{\Sigma \, [\lg (1 + \pi_1)]^2} \triangleq \frac{\Sigma \pi_1 \, . \, \pi_2}{\Sigma \, \pi^2}$$

But we know that the error in savings arises from the error in profits and their relationship is given by $\dfrac{\pi_2}{\pi_1} = \dfrac{Y^*}{S^*}$. If we make the assumption that the error proportions have zero sums the same relationship holds for the deviations of the error proportions from their means, i.e. $\dfrac{\pi_2}{\pi_1} = \dfrac{Y^*}{S^*}$. Thus,

$$\frac{\Sigma \, \pi_1 \, . \, \pi_2}{\Sigma \, \pi^2} = \frac{\Sigma \, \pi^2 \, . \, Y^*/S^*}{\Sigma \, \pi^2}$$

This means that the elasticity of $(1 + \pi_2)$ with respect to $(1 + \pi_1)$ can be approximated by a weighted mean of the reciprocals of the normal savings ratios.

On the further not unreasonable assumption that the covariance of π_1^2 and $\dfrac{Y^*}{S^*}$ is zero this weighted mean is equal to the arithmetic mean of the reciprocals of the normal savings ratios.

$$\frac{\Sigma \lg (1 + \pi_1) \, . \, \lg (1 + \pi_2)}{\Sigma \, [\lg (1 + \pi_1)]^2} \triangleq \frac{1}{N} \, . \, \Sigma \, \frac{Y^*}{S^*}$$

Thus the condition for upwards bias in the measured elasticity can be approximated as,

$$\frac{1}{N} \, . \, \Sigma \, \frac{Y^*}{S^*} > e^*$$

In practice it should usually be possible to infer the plausible direction of bias from this formulation and prior information about the normal elasticity and savings ratios. As a particular example we can take the Brewing industry to which Hart's data refers. Tew and Henderson's data for quoted companies over 1949-53 indicate an average measured savings ratio of .38. Taking this as an approximation to the normal ratio we see that the true elasticity would have to be as great as $1/.38 = 2.6$ before the observed least squares elasticity would not be biased upwards. That is, given the existence of errors of the sort specified in this model, it is very likely that his calculated figure does exceed the normal elasticity.[1]

[1] It must be noted that the model requires that we take logs of $(1 + \pi_2)$ which implies that π_2 should always > -1, i.e. positive normal savings should not be associated with observed dis-savings. [And generally of course we require that normal and measured profits and savings exceed zero.] Making use of

$$\pi_1/\pi_2 = S^*/Y^*$$

we find that $-S^*/Y^*$ gives the lower limit to π_1 the proportionate error in normal profits, if the model is to apply. For a savings ratio of .1 the proportionate error should not be $\leqq -10$ per cent of normal profits, while for a savings ratio of .5 the transitory component can be as low as -50 per cent of normal profits. Since transitory components can fairly plausibly be expected to fall in the interval ± 25 per cent of normal profits, the model only applies to data with normal savings ratios generally in excess of .25.

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