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The Estimation of Models of Labour Market Behaviour

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In this paper we view the labour market experience of individuals as a process of movement between the states of employment and unemployment. We note that there are three main ways of sampling members of the labour force namely sampling the members of a specific state, sampling the people entering or leaving a state and sampling the population regardless of state. The joint distribution of observable and unobservable characteristics of individuals depends on the mode of sampling adopted. We examine this dependence and its implications for the interpretation of estimates of models of labour market behaviour.

1. INTRODUCTION

Numerous studies have reported estimates of models of labour market behaviour that derive from survey data. While unemployment remains a major preoccupation of Western governments and as new survey data become available, further results can be expected to appear. In this paper we examine the interpretation to be placed on these empirical results. Since estimates of labour market models are relevant to important government policy decisions it is essential that the empirical evidence is interpreted correctly.

We argue that survey design crucially affects the interpretation of estimates of labour market models and we note that the surveys used by researchers to examine labour market behaviour differ in their designs. Some researchers use surveys of those entering or leaving unemployment—surveys of the flows between states (e.g. Kahn (1964), Kiefer and Neumann (1979), (1981), Mackay (1972), Mackay and Reid (1972), Moylan and Davies (1980), Reid (1972) and Wedderburn (1965)). Some researchers use surveys of those unemployed or employed at some sampling date—surveys of the stocks of the employed or unemployed, (e.g. Department of Employment (1974), (1977), Fowler (1968), Hill, Harrison *et al.* (1973), Lancaster (1979), McGregor (1980), Nickell (1979*a*, *b*), and Sinfield (1970)). And we find other researchers using surveys of the whole labour force, both employed and unemployed, (e.g. Disney (1979), Frank (1978), Heckman and Borjas (1980), Kay, Morris *et al.* (1980), Layard, Piachaud *et al.* (1978) and Parker, Thomas *et al.* (1972)).

In this paper we examine the comparability and the interpretation of results obtained from surveys of flows, stocks and the whole labour force.

In Section 2 of the paper we note that the probabilities of return to work and exit from employment generally depend on both unobserved and observed characteristics of individuals. We argue that the correlations between observed and unobserved characteristics can then differ in flows, stocks and the whole labour force and that these correlations can, in non-stationary or disequilibrium models, vary continuously over time. A consequence is that estimates of labour market models can be sensitive to assumptions about the conditional distribution of unobservables given observables. And different researchers making inconsistent assumptions can obtain asymptotically different estimators of apparently identical parameters.

One way to attack this problem is to obtain repeat observations and, using methods described by Chamberlain (1980), obtain estimates that are insensitive to assumptions about the conditional distribution of unobservables given observables. But often repeat observations are not available. Then it is necessary to test for the presence of the unobservable characteristics which lead to across individual heterogeneity in the probabilities of return to work and exit from employment. A test procedure is described in Chesher (1982).

Without repeat observations and with unobservable characteristics influencing the probabilities of return to or exit from employment, it is necessary to determine when the presence of unobservables leads to different correlations between unobservables and observables in flows, stocks and the whole labour force. This topic is taken up in Sections 3 and 4.

In Section 3 we examine a two state stationary equilibrium model of labour market transitions. We derive necessary conditions for observables and unobservables to be independently distributed in flow and stock, stock and whole population and so forth. The conditions require that unobservables factor out of expressions involving conditional expected employment and unemployment durations. In Section 4 we outline the extensions to disequilibrium and non stationary models. Section 5 summarizes our results and contains concluding remarks.

2. STOCK AND FLOW SAMPLING

We start by considering a world in which individuals face a stationary environment and flow into unemployment at a constant rate so as to maintain equilibrium in the stock of the unemployed.

Suppose first that individuals have identically distributed unemployment durations. Then individuals sampled from the stock of the unemployed (i.e. a sample of those unemployed at a given date) will not be typical of entrants to unemployment.

Entrants who, by good fortune, have unusually short unemployment durations will be less likely to appear in the stock sample than those who have unusually long unemployment durations. Consequently samples of the stock of the unemployed will tend to show longer complete unemployment durations than will samples of the entrants to, or flow into, unemployment. This has been noted by Salant (1977) and others and is a result of what renewal theorists call length-biased sampling (see Cox (1962)).

Now suppose, more realistically, that the individuals flowing into unemployment have different unemployment duration distributions. Let each individual have characteristics (e.g. educational attainment, gender, marital status, disability, intelligence, motivation) recorded in a vector X and let the value taken by X completely determine an individuals unemployment duration distribution.

As before, in a sample of the stock of the unemployed we will tend to observe longer unemployment durations than we will in a sample of those flowing into unemployment. But who are these individuals with long unemployment durations?

Typically they are individuals whose characteristics, X, predispose them to long durations. Suppose that X_1 and X_2 are two characteristics, uncorrelated among entrants to unemployment, and that high values of X_1 or X_2 predispose an individual to long unemployment durations. Then when we encounter an individual in the stock of the unemployed with an unusually low value of say X_1, we might expect him to have an unusually high value of X_2; sufficiently high to offset the beneficial effect of his low X_1. This suggests that we can generally expect different correlations between individuals characteristics in flows and stocks.

Now consider the researcher interested in the effect of X on some manifestation, Y, of labour market experience. The variate Y might be unemployment duration, or the wage obtained on re-employment. As long as the researcher can observe the complete vector X he need be little concerned by correlations between elements of X and their differences in flows and stocks. These differences can lead to different "regression designs" in flow and stock samples but he can control for this if X is observed.

Unfortunately economists can rarely be sure that they have observed all the characteristics in X which determine an individual's unemployment duration distribution. Let X_o be the observed sub-vector of X and X_u the remaining unobserved elements. Then the relationship between Y and X_o that the researcher sees will be affected by correlations between X_o and X_u to the extent that X_u does in fact influence Y. And if the correlations between X_o and X_u differ in flow and stock then there will apparently be different relationships between Y and the observed characteristics X_o in flows and stocks.

We have here an example of "selectivity" as discussed by Maddala (1978). Stock samples are length-biased in that long duration individuals are more likely to be selected, and observed and unobserved characteristics together influence duration.

When we confront a selectivity problem we have to decide which population is to be the object of inference. We could take the view that, when sampling the flow into unemployment we are interested in the sorts of people who *become* unemployed and that, when sampling the stock of the unemployed, we are interested in the sorts of people likely to be *found* unemployed. Then we could rest content with different estimates of the effect of X_o on Y from flow and stock samples, accepting that the differences arise because the estimates relate to different sorts of people.[1]

We show later, in Section 4, that, in a non-stationary environment or when the labour market is out of equilibrium, correlations between X_o and X_u can vary continuously over time. In this case, to maintain the position that one is interested in the population sampled is to maintain the position that one is interested in a population (flow or stock) at a single point in time.

We have so far considered the populations that flow into unemployment and the population that is unemployed. But neither of these need be the population of interest for it is possible that we wish to make inferences about the whole labour force, both employed and unemployed. Indeed it is likely that this population is of considerable interest.

The most commonly quoted labour market statistic is the unemployment rate—the proportion of the labour force unemployed at a point in time. One may well wish to consider how the unemployment rate varies as individuals characteristics change. But this requires making inferences about the labour force, not just about the sorts of people who are likely to be *found* unemployed, nor just about the sorts of people who are likely to *become* unemployed.

Just as correlations between observables and unobservables can differ in flows and stocks, so they can differ in the labour force and in, say, the flow into unemployment. To see this we only need to apply the argument given earlier, now allowing high levels of two characteristics X_1 and X_2 to predispose an individual to frequent (rather than long) spells of unemployment. Then if in the labour force X_1 and X_2 are uncorrelated, they may be correlated in the flow into unemployment for an individual in the flow with an unusually low value of X_1 might be expected to have a rather high value of X_2.

We conclude that, if characteristics X affect the frequency of unemployment experience or durations of employment or unemployment, then the joint distribution of individual's characteristics is generally different in flows into unemployment, in the stock of the unemployed and in the labour force. If a subvector X_u of X is unobserved then this can lead to apparently different relationships between a dependent variable Y and X_o in flows, stocks and the whole labour force.

As we noted in the Introduction, when repeat observations on individuals are available the methods described by Chamberlain (1980) can be applied to obtain estimates of the effect of X_o on Y that are not influenced by correlations between X_o and X_u.[2] But repeat observations are often not available and, even when they are, we may be unable to identify interesting parameters if unobservables are estimated or conditioned out as Chamberlain suggests.

We are then led to inquire whether there are circumstances in which, even though X_o and X_u affect frequency of unemployment experience and duration of unemployment, correlations between X_o and X_u are identical in stocks, flows and the labour force.

The most interesting case to consider is that in which X_o and X_u are independent in alternative populations of interest, because the independence assumption is one commonly made by researchers, particularly when repeat observations are not available.

In the next section we examine a stationary equilibrium two state model of labour market transitions and determine conditions under which independence assumptions can hold in more than one population of interest. The succeeding section examines non-stationary and disequilibrium models.

3. A STATIONARY EQUILIBRIUM MODEL

Imagine a population of individuals—the labour force—over which a vector of characteristics X is distributed with probability density function (pdf) $f(x)$. Suppose that X is unaffected by the passage of time or labour market experience and partition X into $(X_o : X_u)$ where X_o and X_u are vectors of characteristics, X_o observed and X_u not observed by the researcher.

Each individual is, at any instant in one of two states—unemployment (state 1) or employment (state 2). The instantaneous conditional probability of exit from state $j (j = 1, 2)$ after s time periods in state j given $X = x$ and no exit prior to s is the hazard function for x-type individuals for state j. We write this[3] as $h_j(s, x)$. We assume that simultaneous exit from and re-entry to a state is not possible.

The conditional expected mean duration in state j given entry to state j and $X = x$ is $\mu_j(x)$ defined[4] in equation (1).

$$\mu_j(x) = \int_o^\infty \exp\left\{-\int_o^t h_j(s, x)ds\right\}dt. \tag{1}$$

If a hazard function decreases with s sufficiently fast then, with non-zero probability, the associated state is never left, and the associated duration distribution is defective. As time passes in models with a defective duration distribution, flows between states dry up as more and more individuals get trapped. Since we wish to study flows between states in an equilibrium model we assume non-defective duration distributions. We also assume, with little additional loss in generality, that the conditional mean durations, $\mu_j(x)$, are finite.[5]

Suppose that the process is in equilibrium and consider an arbitrary point in time, t^*. We first consider the distribution of X over those unemployed at t^* and over those employed at t^*. The results we obtain give us the distribution of X in simple random samples of the stock of the unemployed or of the employed. We then turn to consider flow samples.

At time t^* each individual is either unemployed or employed and we can define for each individual binary indicator variables D_1 and D_2 so that $D_j = 1$ if the individual occupies state j at t^*, $D_j = 0$ otherwise. The conditional probability that D_j is one given $X = x$, written $P[D_j = 1|x]$, is (see Cox (1962) p. 84):

$$P[D_j = 1 | x] = \mu_j(x)/(\mu_1(x) + \mu_2(x)). \tag{2}$$

Equation (2) tells us that individuals with long unemployment durations and short employment durations are likely to be found unemployed at an arbitrarily chosen point in time. From (2) we obtain the conditional probability mass function for D_1 and D_2 given $X = x$:

$$p(D_1, D_2 | x) = \mu_1(x)^{D_1}\mu_2(x)^{D_2}/(\mu_1(x) + \mu_2(x)). \tag{3}$$

Multiplying (3) by the pdf of X in the labour force, $f(x)$, gives the probability–probability density function of D_1, D_2 and X, (4).

$$p(D_1, D_2, x) = \mu_1(x)^{D_1}\mu_2(x)^{D_2}f(x)/(\mu_1(x)+\mu_2(x)). \tag{4}$$

Let k_j, a constant, denote the marginal probability[6] that $D_j = 1$. Then we obtain from (4) the conditional pdf of X given $D_j = 1$;

$$p(x\,|\,D_j = 1) = k_j^{-1}\mu_j(x)f(x)/(\mu_1(x)+\mu_2(x)). \tag{5}$$

Comparing this conditional distribution, the pdf of X in samples of the stock of the unemployed ($j = 1$) or the employed ($j = 2$), with the pdf of X over the labour force we see that:

(a) the whole population and stock j distributions of X are identical if and only if[7] $\mu_1(x) \propto \mu_2(x)$,

(b) the observable and unobservable characteristics X_o and X_u are independently distributed in the labour force and in stock j only if $\mu_j(x)/(\mu_1(x)+\mu_2(x))$ is a multiplicatively separable function of x_o and x_u.[8]

So if unobservables do not factor out of the conditional probability of unemployment $\mu_1(x)/(\mu_1(x)+\mu_2(x))$ then it is not possible for observable and unobservable characteristics to be independently distributed over the labour force and over the unemployed.[9]

Now consider samples of the flows into or out of unemployment. We define a short sampling interval length Δt starting at time t^*. The interval is so short that, for any individual, there is almost certainly at most one state change in $(t^*, t^*+\Delta t)$.

Define for each individual the indicator variables E_{12} and E_{21} so that $E_{ij} = 1$ if the individual moves from state i to state j in the sampling interval and $E_{ij} = 0$ otherwise. Since the process is in equilibrium $P[E_{12} = 1\,|\,x] = P[E_{21} = 1\,|\,x]$ for all x and, since the sampling interval is short, $P[E_{12} = 1 \cap E_{21} = 1\,|\,x] \doteq 0$ for all x. We first develop an expression for $P[E_{ij} = 1\,|\,x]$.

On average an individual with characteristics $X = x$ spends $\mu_1(x)$ time periods unemployed and $\mu_2(x)$ employed and therefore re-enters a given state on average once every $\mu_1(x)+\mu_2(x)$ time periods. So, in equilibrium, $P[E_{ij} = 1\,|\,x]$, the probability of transition into a given state in a sampling interval Δt time periods long is approximately $\Delta t/(\mu_1(x)+\mu_2(x))$. To obtain this expression formally, note that in the equilibrium one state renewal process, for which the associated point process has events defined by re-entry to state j, the renewal density is constant and the probability of an event (i.e. re-entry to the chosen state j) in the sampling interval is (see Cox (1962) p. 55) approximately $\Delta t/(\mu_1(x)+\mu_2(x))$. In the context of our two state model this is $P[E_{ij} = 1\,|\,x]$. Multiplying $P[E_{ij} = 1\,|\,x]$ by the pdf of X over the labour force we obtain (6), the approximate probability–probability density function for $E_{ij} = 1$ and X.

$$p(E_{ij} = 1, x) \doteq \Delta t f(x)/(\mu_1(x)+\mu_2(x)). \tag{6}$$

Dividing (6) by the marginal probability that $E_{ij} = 1$ we obtain the conditional distribution (7) of X given $E_{ij} = 1$. For small Δt this is the distribution of X in simple random samples of the flow into or out of unemployment. The constant k is introduced so that (7) integrates to one.

$$p(x\,|\,E_{ij} = 1) \doteq k^{-1}f(x)/(\mu_1(x)+\mu_2(x)). \tag{7}$$

Comparing this distribution with the pdf of X over the labour force, $f(x)$, we see that:

(a) the labour force and flow distributions of X are identical if and only if $\mu_1(x)+\mu_2(x)$ does not depend on x.

(b) observable and unobservable characteristics are independent in the labour force and in the flows into or out of unemployment only if $\mu_1(x)+\mu_2(x)$ is a multiplicatively separable function of x_o and x_u.[10]

The four distributions of individuals' characteristics are shown in Table I.

TABLE I

Distributions of individual's characteristics in flows, stocks and the labour force

Population	Distribution of characteristics (pdf)
Labour force	$f(x)$
Individuals unemployed at t^*	$k_1^{-1}\mu_1(x)f(x)/(\mu_1(x)+\mu_2(x))$
Individuals employed at t^*	$k_2^{-1}\mu_2(x)f(x)/(\mu_1(x)+\mu_2(x))$
Individuals becoming unemployed or employed in $(t^*, t^*+\Delta t)$	$k^{-1}f(x)/(\mu_1(x)+\mu_2(x))$

Comparing the flow and stock distributions in Table I we see that:

(a), the flow and stock j distributions of X are identical if and only if $\mu_j(x)$ does not depend on x,

(b), unobservable and observable characteristics are independent in the flow into (or out of) unemployment and in the stock of the unemployed only if conditional mean duration of unemployment, $\mu_1(x)$, is a multiplicatively separable function of observables and unobservables.

We conclude that it *is* possible to maintain independence of observables and unobservables over say the flow into unemployment aand the stock of the unemployed *as long as* unobservables factor out of conditional (on X) mean unemployment duration. In the empirical literature we find that the model used by Kiefer and Neumann (1981) does not satisfy the factoring condition whereas the model used by Lancaster (1979) does.

What happens when these factoring conditions fail to hold? To demonstrate the difficulties that can arise, suppose that mean unemployment duration is the linear function of scalar x_o and x_u, $\mu_1(x)=\alpha+\beta x_o+x_u$. Suppose that, over entrants to unemployment, X_o and X_u are independently distributed and that X_u has mean zero, variance σ^2 and marginal pdf $m(x_u)$. Then, using the results in Table I we find that the conditional pdf of X_u given X_o in the stock of the unemployed is $g(x_u\,|\,x_o)$ shown in (8).

$$g(x_u\,|\,x_o)=m(x_u)\left\{1+\frac{x_u}{\alpha+\beta x_o}\right\}. \tag{8}$$

We see that the regression of X_u on X_o in the stock of the unemployed is the non linear function of x_o:

$$E(X_u\,|\,x_o)=\sigma^2/(\alpha+\beta x_o).$$

Now imagine two researchers, one sampling the flow into unemployment, the other the stock of the unemployed, both obtaining realisations of a variate Y with $E(Y\,|\,x_o, x_u)$ equal to $\gamma_o+\gamma_1 x_o+\gamma_2 x_u$. For the flow sample, $E(Y\,|\,x_o)=\gamma_o+\gamma_1 x_o$ but for the stock sample, $E(Y\,|\,x_o)=\gamma_o+\gamma_1 x_o+\gamma_2\sigma^2/(\alpha+\beta x_o)$. The stock and flow regressions of Y on x_o are drawn in Figure 1 where we asume $\beta, \gamma_1, \gamma_2>0$. The regression of Y on x_o is linear in the flow into unemployment but non-linear in the stock of the unemployed.

We see that, depending on the distribution of x_o in the stock sample, the researcher fitting the linear equation $Y=\delta_o+\delta_1 x_o+U$ to the stock sample can obtain negative, zero or positive estimates of δ_1 in large samples. And the estimated coefficient on x_o will, in large samples, be smaller than γ_1, the coefficient that, under the usual conditions, the flow sample analyst will estimate consistently. These stock and flow differences are important if x_o is a government policy instrument, for instance some measure of unemployment benefit receipts. Of course if the unobservable X_u and the observable X_o are independent in the stock of the unemployed a similar argument applies to the flow sample.

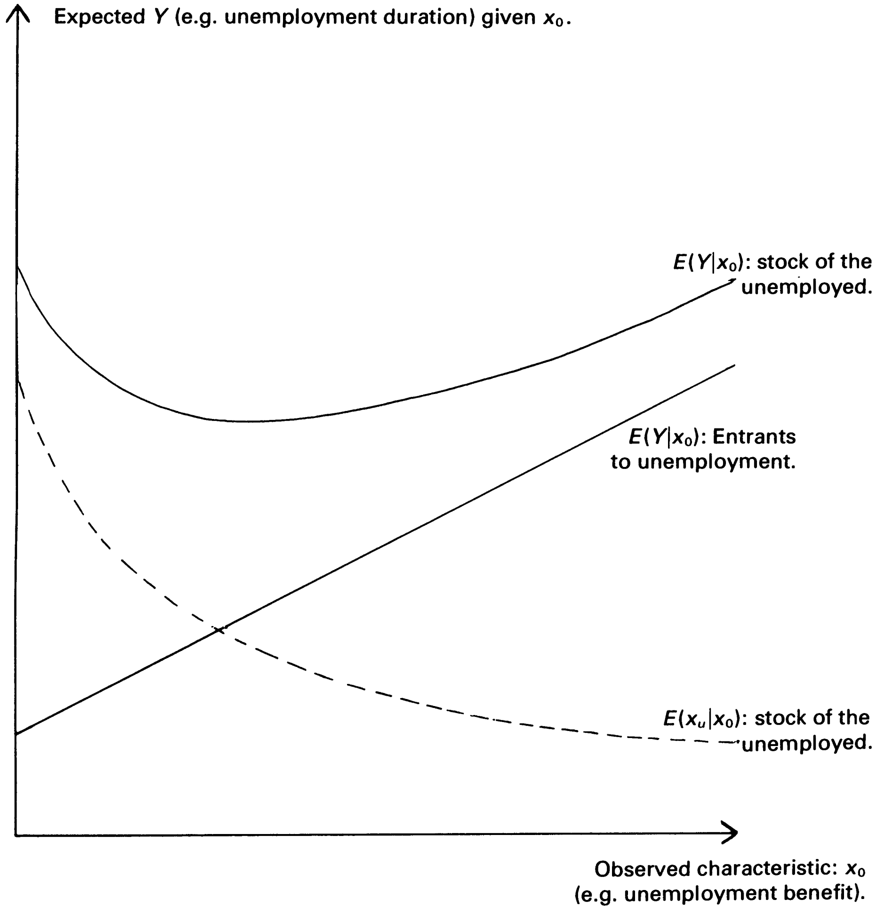


FIGURE 1

Expected Y (e.g. unemployment duration) given x_o (e.g. unemployment benefits) for entrants to unemployment and the unemployed assuming independent x_o and unobserved x_u over entrants to unemployment.

Even when observables and unobservables are distributed independently in stocks and flows their joint distributions in stocks and flows generally differ. To illustrate this suppose X is k-variate normal with mean μ and variance matrix Σ in the flow into unemployment. Suppose the hazard functions are $h_j(s, x)$ given in (9). We note

$$\begin{aligned} h_j(s,x) &= s^{\alpha_j-1}\exp(y_j) \\ y_j &= x'\beta_j \end{aligned} \tag{9}$$

that the hazard functions (9) are examples of proportional hazard functions as described by Cox (1972). In this case conditional expected duration in state j given entry to state j and $X=x$ is:

$$\mu_j(x) \propto \exp\{-x'\beta_j/\alpha_j\}. \tag{10}$$

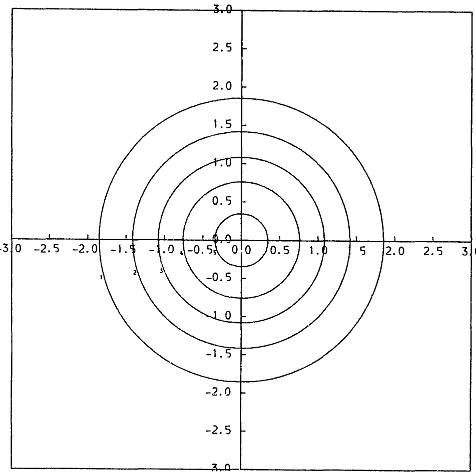
Applying the results in Table I we find that for those in state j, X has a k-variate normal distribution with mean $\mu - \Sigma\beta_j/\alpha_j$ and covariance matrix Σ. The factoring condition holds for $\mu_j(x)$ shown in (10) whichever elements of X make up X_u. Indeed in this special case the complete variance matrix is identical in stock and flow.

Stock, flow and whole labour force pdf's of bivariate X for this normal distribution example are contoured in Figure 2. Here we have set $\alpha_1=\alpha_2=1$, set X as bivariate $N(O, I_2)$ in the flow into unemployment and chosen $\beta_1'=(-1-1)$ and $\beta_2'=(1\ 1)$. We see that X is circular normal in flow and stocks but that in the stocks X has non zero mean. Distributions differ in flows and stocks but independence is maintained.

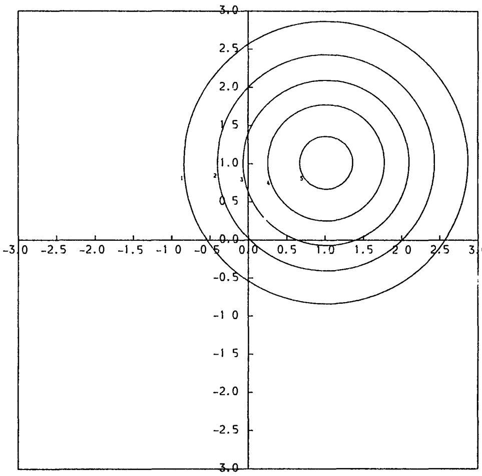
In the whole labour force X has a bimodal distribution and X_1 and X_2 are positively correlated. The bimodality of X's distribution in the labour force follows directly from the assumption of independent normal X_i in the flow into unemployment.[11] That assumption requires us to believe that individuals in the labour force are likely to be either "employment prone" or "unemployment prone".

FIGURE 2

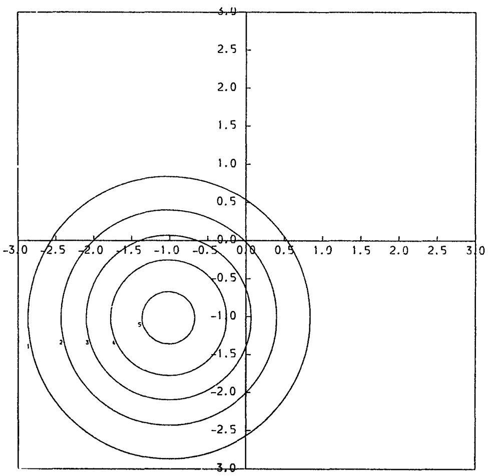
Contours of pdf's of two characteristics X_1 and X_2 assuming X_1 and X_2 independently normally distributed for entrants to unemployment.



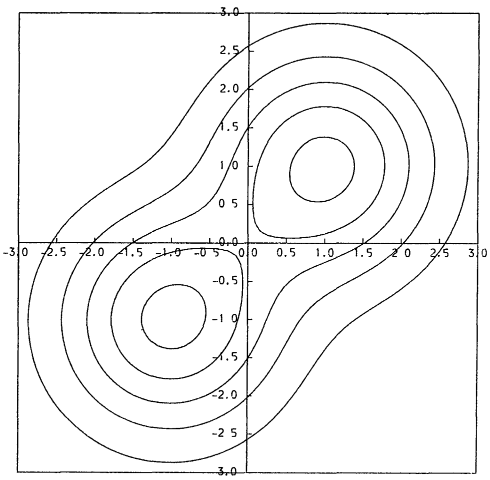
(a) pdf of X_1 and X_2 for entrants to unemployment.



(b) pdf of X_1 and X_2 for the unemployed.



(c) pdf of X_1 and X_2 for the employed.



(d) pdf of X_1 and X_2 for the whole labour force

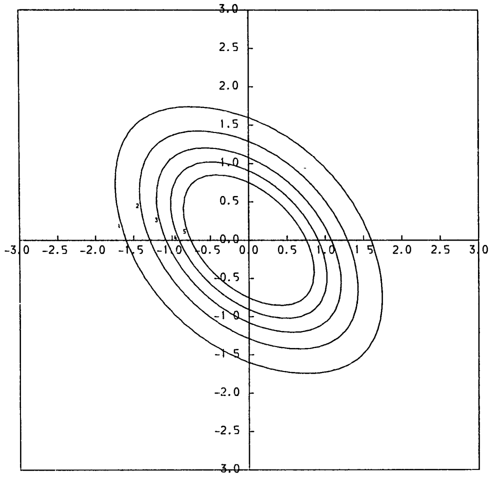
Notes: 1. Contours equally spaced. Contour levels in (d) are one half of contour levels in (a)–(c).
2. Horizontal axis, X_1, vertical axis, X_2.

In Figure 3 we show contours of X's pdf's in flows, stocks and the whole labour force when X is assumed $N(O, I_2)$ in the *whole* labour force rather than in the flows. Then we obtain non-normal distributions for X in flows and stocks and X_1 and X_2 are negatively correlated except in the whole labour force, where they are independent by assumption.[12]

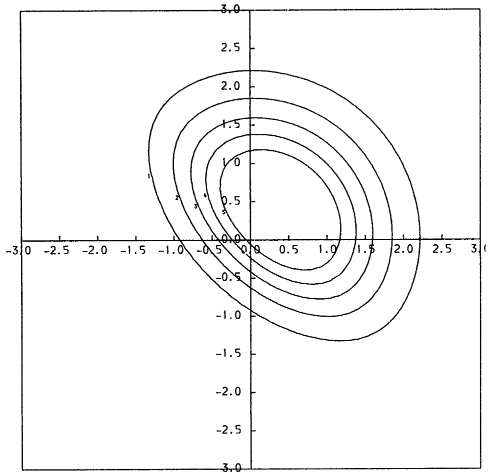
In this example we have used a proportional hazard model $h_j(s, x) = s^{\alpha_j - 1} \exp(x'\beta_j)$ in which the hazard is a multiplicatively separable function of the elements of x. We find that expected duration given x is then multiplicatively separable in x. Generally proportional hazards that are multiplicatively separable in x_o and x_u do not lead to separable expected durations. The hazard $h(s, x) = (1+s)g(x)$ provides a simple example.

FIGURE 3

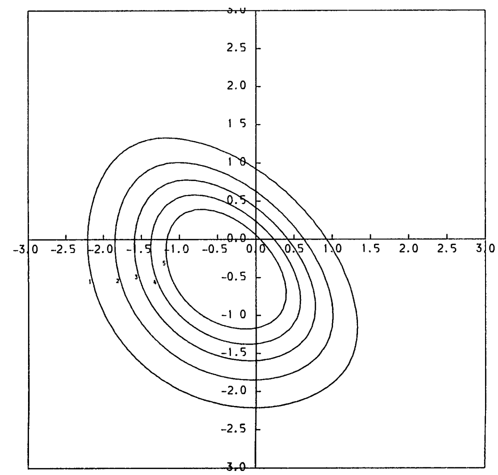
Contours of pdf's of two characteristics X_1 and X_2 assuming X_1 and X_2 independently normally distributed over the whole labour force.



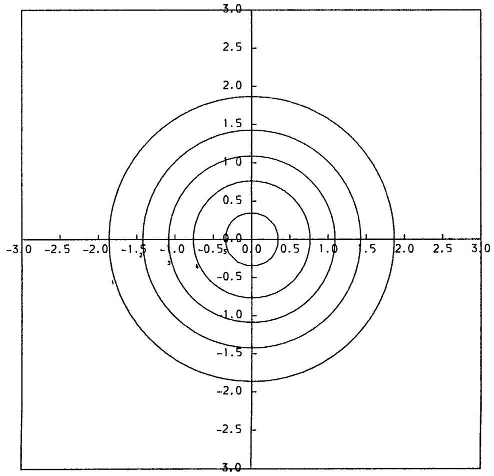
(*a*) pdf of X_1 and X_2 for entrants to unemployment.



(*b*) pdf of X_1 and X_2 for the unemployed.



(*c*) pdf of X_1 and X_1 for the employed.



(*d*) pdf of X_1 and X_2 for the whole labour force.

Notes: 1. Contours equally spaced. Contour levels identical in (*a*)–(*d*).
2. See note 2 for Figure 2.

Here expected duration is proportional to $\exp(g(x)/2)g(x)^{-1/2}\Phi(-g(x)^{1/2})$ where Φ is the standard normal distribution function, and, when $g(x)$ is multiplicatively separable, expected duration is not.

In this section we have examined a stationary equilibrium two state model of labour market transitions. We find that if we can maintain factoring conditions on expressions involving expected mean durations of employment and unemployment then assumptions concerning the independence of unobservable and observable characteristics valid for one population (e.g. the flow into unemployment) are valid for another (e.g. the whole labour force).

In the absence of such conditions independence assumptions valid for one population are not valid for another and researchers analysing flow, stock and labour force samples will generally obtain different answers to the same questions unless the correlations between observable and unobservable characteristics are taken into account.

In the next section we briefly consider the extension to more realistic disequilibrium and non-stationary models. We note now that the extension to multi-state models is straightforward, merely requiring us to develop new expressions for $P[D_j=1|x]$ and $P[E_{ij}=1|x]$.

4. DISEQUILIBRIUM AND NON-STATIONARY MODELS

We now assume that the hazard functions $h_j(\ )$ depend on calendar time, t, and write the hazard functions as $h_j(t, x)$. To obtain tractable results we assume that the hazards do not depend on elapsed duration in a state given t. As before we assume that the individuals in the labour force have characteristics, X, with pdf $f(x)$, and that they alternate between state 1—unemployment, and state 2—employment.

Define for each individual two random functions of time $D_1(t)$ and $D_2(t)$ to indicate the state occupied at time t. Let $D_j(t)=1$ if state j is occupied at time t, $D_j(t)=0$ otherwise. At instants of transition $D_j(t)$ records the state left.

Suppose we can obtain an expression for $P(D_j(t)=1|x)$, the conditional probability that state j is occupied at time t given $X=x$. Then, arguing as in Section 3, the pdf of X over those in state j at time t is:

$$p(x|D_j(t)=1)\propto P[D_j(t)=1|x]f(x). \tag{11}$$

For unobservable and observable characteristics to be independently distributed in the labour force and in state j at time t we require $P[D_j(t)=1|x]$ to be a multiplicatively separable function of observables and unobservables. We develop expressions for $P[D_j(t)=1|x]$ later. First we consider the flow from, say, state 1 to state 2 in the short sampling interval $(t, t+\Delta t)$.

For each individual define $E_{12}(t)=1$ if a transition from state 1 to 2 occurs in the sampling interval, $E_{12}(t)=0$ otherwise. From the definition of the state 1 hazard function we have:

$$P[E_{12}(t)=1|x, D_1(t)=1]\doteqdot h_1(t, x)\Delta t. \tag{12}$$

Multiplying by $P[D_1(t)=1|x]$ and noting that $E_{12}(t)=1$ implies $D_1(t)=1$ we obtain:

$$P[E_{12}(t)=1|x]\doteqdot h_1(t, x)P[D_1(t)=1|x]\Delta t. \tag{13}$$

Arguing as before we find that the pdf of X in the population flowing from state 1 to 2 at time t is, as Δt passes to zero:

$$\begin{aligned} p(x|E_{12}(t)=1) &\propto h_1(t, x)P[D_1(t)=1|x]f(x) \\ &\propto h_1(t, x)p(x|D_1(t)=1). \end{aligned} \tag{14}$$

Note that (14) does *not* hold if hazard functions depend on duration and calendar time.

Examining (14) we see that observable and unobservable characteristics can be independently distributed in state j at time t^* and in the outflow from state j at the *same* time t^* if the hazard for leaving state j is a multiplicatively separable function of observables and unobservables.

We also see that the ratio of the pdf's of X in state 1 and in the outflow from state 1 at the same date t^* is proportional to the hazard function for exit from state 1 at time t^*. This suggests that the hazard functions can be estimated up to a factor of proportionality given estimators of the stock and flow pdf's of X. This procedure is similar in spirit to the choice based sampling methods used in discrete choice modelling, (see e.g. Manski and Lerman (1977) and Cosslett (1981). We can apply this approach in the model of Section 3 to estimate elasticities of mean durations in states, for there the ratio of stock and flow pdf's of X is proportional to mean duration in the selected stock.

To conclude this section we briefly examine the form of $P[D_j(t)=1\,|\,x]$, the probability that appears in the expressions (11) and (14) for the stock and flow pdf's of X.

We suppose that at time $t=0$ the conditional probability that an individual is in state j given $X=x$ is $P[D_j(0)=1\,|\,x]=P_j[0,x]$. In the Appendix we show that $P[D_1(t)=1\,|\,x]$ has the time path given in (15). Note that $P[D_2(t)=1\,|\,x]=1-P[D_1(t)=1\,|\,x]$.

$$P[D_1(t)=1\,|\,x]=\exp\left\{-\int_o^t h_1(s,x)+h_2(s,x)ds\right\}$$
$$\times\left[P_1[0,x]+\int_0^t h_2(s,x)\exp\left\{\int_o^s h_1(u,x)+h_2(u,x)du\right\}ds\right]. \tag{15}$$

We appear to require very strict conditions on the hazard functions and on initial allocations to states for (15) to be a multiplicatively separable function of observable and unobservable characteristics. If (15) is not multiplicatively separable then correlations between observable and unobservable characteristics in the stock of the unemployed vary continuously as time passes.

Equation (15) describes the time path of the probability of being unemployed in a rather general non stationary disequilibrium model. It is worthwhile investigating whether this probability can be reasonably regarded as multiplicatively separable in observables and unobservables in the simpler special cases in which (a) the process is stationary but out of equilibrium and (b) the process is non-stationary but in a dynamic equilibrium in the sense that the effect of the initial allocation to employment and unemployment has dissipated.

First suppose that the model is stationary ($h_j(t,x)$ does not depend on t) but out of equilibrium. Then, after some rearrangement (15) simplifies to:

$$P[D_1(t)=1\,|\,x]=\frac{h_2(x)}{h_1(x)+h_2(x)}$$
$$+\left\{P_1(0,x)-\frac{h_2(x)}{h_1(x)+h_2(x)}\right\}\exp\{-(h_1(x)+h_2(x))t\}. \tag{16}$$

Even in this stationary model we require strict conditions on the hazards and initial allocations to avoid changing correlations over time between observables and unobservables in stocks and flows if we have a static distribution for X in the whole labour force. Of course as t becomes large and equilibrium is approached the second term in equation (16) becomes negligible and, since there is no duration dependence so that $\mu_j(x)=h_j(x)^{-1}$, we obtain the same results as in Section 3.

Finally suppose that the process is non-stationary but that t is large enough for the effect of the initial allocation to states to have dissipated. Then, as we show in the

Appendix, equation (15) simplifies to:

$$P[D_1(t)=1\,|\,x]=$$

$$\left(1+\left[\int_o^t h_1(s,x)\exp\left\{\int_o^s h_1(u,x)+h_2(u,x)du\right\}ds\right]\Big/\left[\int_o^t h_2(s,x)\exp\left\{\int_o^s h_1(u,x)+h_2(u,x)du\right\}ds\right]\right)^{-1}. \tag{17}$$

Only under restrictive conditions on the hazard functions is (17) multiplicatively separable in observables and unobservables. We conclude that, in non stationary or disequilibrium models, unless hazard functions and initial allocations to employment and unemployment satisfy very restrictive conditions, a static distribution of characteristics in the whole labour force generally implies continuously changing distributions of characteristics in the flows into and out of unemployment and over the employed and unemployed. This generally implies continuously changing correlations between observables and unobservables in these flows and stocks. We are then led to the conclusion that researchers sampling flows or stocks who do not allow for these correlations will generally obtain parameter estimates that relate only to the population (flow or stock) sampled and only to the experience of that population at the date of sampling.

5. CONCLUDING REMARKS

Economists are unlikely to have data on all the characteristics of individuals that affect individual's labour market experience. Consequently we can expect unobservable as well as observable characteristics to influence the probabilities of entry to and exit from unemployment.

We have shown that, in this situation, the joint distributions of observable and unobservable characteristics are generally different in the populations flowing into or out of unemployment, in the populations unemployed or employed and in the whole labour force. In particular unobservables and observables can be independently distributed in one population (e.g. the whole labour force) and dependent in another population (e.g. the flow into unemployment). In non-stationary or disequilibrium models the distribution of observable and unobservable characteristics differs in flows and stocks and generally varies continuously as time passes if the distribution of characteristics in the whole labour force is static.

This suggests that estimates of labour market models should be interpreted carefully. For, if unobservables do influence the probabilities of entry to or exit from unemployment then the results reported may only apply to the population sampled (flow, stock or labour force) and, in non-stationary environments, they may only apply to the period of time studied and therefore give misleading predictions.

In Section 3 of the paper we showed that, in stationary equilibrium models, simple factoring conditions on expressions involving conditional expected employment and unemployment durations enable independence assumptions to hold over more than one population of interest. However, in non-stationary or disequilibrium models the required conditions appear to be unrealistically restrictive.

Under these circumstances it is desirable to produce estimates that are robust to mis-specification of the conditional distribution of unobservables given observables. One way to achieve this is to obtain repeat observations on individuals. Then, following Chamberlain (1980), we can either estimate unobservables or make inferences conditional on sufficient statistics for the unobservables. But often we may be unable to identify interesting parameters.

In this case, or when repeat observations are unavailable, we might proceed by constructing and estimating models that embody the factoring conditions developed here. If this route is taken then it is essential to conduct rigorous specification tests on the fitted model. For, as far as we are aware, economic theory provides no justification for these factoring conditions.

APPENDIX

The time path of the probability of state occupancy in the non-stationary disequilibrium model

Here we develop expressions for the probability, $P[D_1(t)=1|x]$, that state 1 is occupied by an individual at time t given that he has characteristics $X=x$. All probabilities below are to be regarded as conditional on $X=x$. We suppress the argument x in the hazard functions which are now written $h_1(t)$ and $h_2(t)$.

Consider a point in time, $t-\Delta t$, shortly before the sampling date t. $D_1(t)=1$ if state 2 was occupied at $t-\Delta t$ and a transition occurred in $(t-\Delta t, t)$ or if state 1 was occupied at $t-\Delta t$ and no transition occurred in $(t-\Delta t, t)$. So, for small Δt, we have:

$$P[D_1(t)=1]=P[D_1(t-\Delta t)=1](1-h_1(t)\Delta t)+P[D_2(t-\Delta t)=1]h_2(t)\Delta t. \qquad (A1)$$

Rearranging and letting Δt pass to zero we obtain:

$$\frac{d}{dt}P[D_1(t)=1]=-(h_1(t)+h_2(t))P[D_1(t)=1]+h_2(t). \qquad (A2)$$

At time $t=0$ individuals are allocated to states 1 and 2 so that:

$$P[D_1(0)=1]=P_1[0]. \qquad (A3)$$

We seek the solution to the non linear differential equation (A2) satisfying the initial condition (A3). Define the integrating factor (see Piaggio (1952)) $J(t)$:

$$J(t)=\exp\left\{\int h_1(t)+h_2(t)dt\right\}=\exp(I(t)). \qquad (A4)$$

The constant of integration in $I(t)$ is set to zero, without loss of generality. Multiplying (A2) by $J(t)$ and integrating with respect to t we obtain:

$$P[D_1(t)=1]=\exp\{-I(t)\}\left[c+\int h_2(t)\exp\{I(t)\}dt\right]. \qquad (A5)$$

The constant c in (A5) is to be determined by the initial condition (A3). After some rearrangement we obtain the solution for $P[D_1(t)=1]$:

$$P[D_1(t)=1]=P_1[0]\exp\{-(I(t)-I(0))\}+\exp(-I(t))\int_0^t h_2(s)\exp\{I(s)\}ds \qquad (A6)$$

For large t with $h_1(t)$, $h_2(t)$ bounded away from zero the first term in (A6) is negligible. Then:

$$\begin{aligned}P[D_1(t)=1]&\doteqdot\exp(-I(t))\int_o^t h_2(s)\exp(I(s))ds\\&=\exp(-(I(t)-I(0)))\int_o^t h_2(s)\exp(I(s)-I(0))ds\\&=\exp\left(-\int_o^t h_1(s)+h_2(s)ds\right)\int_o^t h_2(s)\exp\left(\int_o^s h_1(u)+h_2(u)du\right)ds \qquad (A7)\end{aligned}$$

Consider the integral:

$$\int_o^t (h_1(s)+h_2(s)) \exp\left\{\int_o^s h_1(u)+h_2(u)du\right\}ds.$$

Performing the substitution $g(s)=\int_o^s h_1(u)+h_2(u)du$ we obtain:

$$\int_o^t (h_1(s)+h_2(s)) \exp\left\{\int_o^s h_1(u)+h_2(u)du\right\}ds$$

$$=\exp\left\{\int_o^t h_1(s)+h_2(s)ds\right\}-1 \quad \text{(A8)}$$

which we note is approximately equal to $\exp\{\int_o^t h_1(s)+h_2(s)ds\}$ for large t with $h_1(s)+h_2(s)$ bounded away from zero. Substituting into (A7) we obtain the large t time path of $P[D_1(t)=1]$,

$$P[D_1(t)=1] \doteqdot \frac{\int_o^t h_2(s)\exp\left\{\int_o^s h_1(u)+h_2(u)du\right\}ds}{\int_o^t [h_1(s)+h_2(s)]\exp\left\{\int_o^s h_1(u)+h_2(u)du\right\}ds}. \quad \text{(A9)}$$

We can rearrange (A9) to give:
$P[D_1(t)=1] \doteqdot$

$$\frac{1}{1+\left[\int_o^t h_1(s)\exp\left\{\int_o^s h_1(u)+h_2(u)du\right\}ds\right]\Big/\left[\int_o^t h_2(s)\exp\left\{\int_o^s h_1(u)+h_2(u)du\right\}ds\right]}.$$

Finally note that $P[D_2(t)=1]=1-P[D_1(t)=1]$.

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NOTES

1. If we do maintain this position then, if samples are stratified by X_o, or variables correlated with X_o, we must allow for stratification in estimation. For if we do not then the estimates obtained will generally apply to neither stock nor flow.

2. Chamberlain (1980) suggests either estimating the unobservable x_u's associated with each individual or basing inference on conditional likelihood functions conditioned on sufficient statistics for the unobservables.

3. For small Δs, $h_j(s, x)\Delta s$ is approximately the conditional probability of exit from state j in the interval $(t+s, t+s+\Delta s)$ given entry to state j at time t, no exit prior to $t+s$ and $X=x$.

4. The conditional distribution function of duration in state j given $X=x$ is

$$F_j(t|x)=1-\exp\left\{-\int_o^t h_j(s, x)ds\right\}$$

—see Cox (1962) p. 5. Equation (1) follows because for a non-negative variate Z with distribution function $F(z)$, $E(Z)=\int_o^\infty 1-F(z)dz$.

5. A sufficient condition for a non-defective duration distribution is that for all x there exists finite $s^*(x)$ and $\alpha(x)>0$ such that for all $s>s^*(x)$, $h_j(s, x)\geqslant \alpha(x)/s$. Expected duration is then certainly finite if for all $s>s^*(x)$, $h_j(s, x)>\alpha(x)/s$, or, if only the weak inequality holds, then $\alpha(x)>1$. Essentially the hazard should decline less fast than one over duration as duration lengthens.

6. k_1 is the analogue of the unemployment rate.

7. $p(x \mid D_j = 1) = f(x)$ if and only if $\mu_j(x)/(\mu_1(x)+\mu_2(x))$ is constant which occurs if and only if $\mu_1(x) \propto \mu_2(x)$ since $\mu_j(x)/(\mu_1(x)+\mu_2(x))$ equals $[1+\mu_2(x)/\mu_1(x)]^{-1}$ for $j=1$ and, for $j=2$, $[1+\mu_1(x)/\mu_2(x)]^{-1}$.

8. Independence occurs in both distributions only if both pdf's are multiplicatively separable function of x_o and x_u. For this we require the ratio of the pdf's to be multiplicatively separable but the ratio is proportional to $\mu_j(x)/(\mu_1(x)+\mu_2(x))$.

9. If X_o and X_u are independent over the labour force and the unemployed then they are independent over the employed only if the probability of being unemployed depends just on x_o or just on x_u. For if $\mu_1(x)/(\mu_1(x)+\mu_2(x))$ is multiplicatively separable then $1-\mu_1(x)/(\mu_1(x)+\mu_2(x))$ is only multiplicatively separable if $\mu_1(x)/(\mu_1(x)+\mu_2(x))$ if a function of either x_o or x_u.

10. The arguments of notes (7) and (8) apply here.

11. The whole labour force distribution of X is a mixture of the two stock distributions with weights independent of X and equal to the marginal probabilities of unemployment and employment. This follows because at any instant each member of the labour force is either employed or unemployed.

12. Further results using this example are given in Lancaster and Chesher (1981).

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