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AN ECONOMETRIC ANALYSIS OF RESERVATION WAGES

BY TONY LANCASTER AND ANDREW CHESHER

This paper describes an analysis of survey data on unemployed people who reported their asking wage and the wage they expected to earn. We show that if agents' behavior is described by the standard optimal job search model, one can use these data to *deduce* structural parameters rather than estimate them, and we report our deductions from two surveys[1] of British unemployed people.

1. INTRODUCTION

THIS PAPER PRESENTS an analysis of data from two surveys of unemployed people in which the foundation of the analysis is the theory of optimal job search. The principal difficulty in the econometric application of search theory is the central role in that theory of the distribution of wage offers, which is both unknown to the investigator and variable between individuals. We generally know neither the shape nor even the location of that distribution, a distribution which moreover may be personal and subjective, representing an individual's beliefs about the wage opportunities potentially available to him. Similarly we do not generally know the rate at which opportunities of employment present themselves to each individual or are believed by him to do so.

The main theoretical contribution of this paper is a method of overcoming these difficulties which does not require arbitrary statistical assumptions. We show that if we combine the restriction given by the condition for *optimal* search with the information supplied by the answers to two simple questions we can eliminate the unknown offer distribution and offer probablity from the calculation of interesting parameters. We can then *deduce* structural parameters rather than estimate them. To calculate other parameters does require an hypothesis about the shape of the offer distribution but it is not difficult to gauge the sensitivity of the results to the shape assumed. Furthermore the calculation of these elasticities requires nothing more than averages of the observations and our method should thus be robust against measurement error.

The empirical work shows that the data are consistent with optimal job search theory while the elasticities we calculate are largely, though not entirely, similar to the results of previous microeconometric studies.

In the next section we develop the model of optimal search with which we propose to interpret our observations and in the following section we describe the data and in particular the two questions whose answers are the key to the analysis.

[1] The P.E.P. survey of 1973 was described and first analyzed in [1] and the Oxford survey of 1971 was described and first analyzed in [2]. In the text we give calculations using the P.E.P. data while Appendix Table IV gives Oxford data.

2. STATIONARY OPTIMAL SEARCH

We shall apply a model in which the individual aims to maximize the expected present value of his income stream over an infinite horizon.[2] A job offer is a net weekly wage of w forever[3] and while unemployed the individual receives income at the constant rate b. Offers or opportunities of employment are realizations of the random variable w whose distribution function is $F(w)$ with complement $\bar{F} = 1 - F$. Offers arrive purely randomly in time at the average rate λ per unit time period so that the waiting time, t, between offers is an exponential (λ) variate. Future incomes are discounted continuously at rate ρ.

Consider an instant at which an offer is pending and let I denote the expected present value of income when an optimal policy is followed from such a date. If an offer of w is accepted the return is its capitalized value w/ρ. If the offer is rejected, the return, R, consists of the expected present value of b for a random waiting time, t, plus the expected discounted value of I, that is,

$$\begin{aligned} R &= E\left[\int_0^t be^{-\rho s}\,ds + Ie^{-\rho t}\right] \\ &= (b + \lambda I)/(\lambda + \rho), \end{aligned} \tag{2.1}$$

using the exponential (λ) distribution of t.[4] An acceptance policy can be characterized by the choice of a function $\phi(w)$, $0 \leq \phi \leq 1$, giving the probability with which an offer of w must be accepted, and an optimal policy must satisfy the equation

$$I = \sup_{\phi}\left[\int_0^\infty \phi(w)\frac{w}{\rho}\,dF(w) + R\int_0^\infty (1 - \phi(w))\,dF(w)\right].$$

Rearranging the term in square brackets as

$$I = \sup_{\phi}\left[\int_0^\infty \phi(w)\left[\frac{w}{\rho} - R\right]dF(w) + R\right], \tag{2.2}$$

we see the solution to be

$$\phi(w) = \begin{cases} 1, & w \geq \rho R = \xi, \\ 0, & \text{otherwise.} \end{cases} \tag{2.3}$$

[2] The model is not new of course; it derives essentially from Karlin [3]. We give it because the details, not least the notation, are essential to our econometric procedure.

[3] It is not difficult to generalize the model to allow for random anticipated employment durations at the completion of which optimal search is resumed. The same final optimality condition, (2.6) below, applies with the parameter ρ reinterpreted. In particular where job termination is purely random we replace ρ of (2.6) by $\rho + \tau$ where τ^{-1} is the anticipated mean employment duration.

[4] $E(e^{-\rho t}) = \int_0^\infty \lambda e^{-(\lambda + \rho)t}\,dt = \lambda/(\lambda + \rho)$.

ξ is the reservation wage and (2.2) thus becomes

$$I = \int_{\xi}^{\infty} \frac{(w-\xi)}{\rho}\, dF(w) + \frac{\xi}{\rho}. \tag{2.4}$$

Finally if we use (2.1) and (2.3) to put I in terms of ξ we get the equation defining the optimal reservation wage as

$$\xi = b + \frac{\lambda}{\rho} \int_{\xi}^{\infty} (w-\xi)\, dF(w). \tag{2.5}$$

Integration by parts allows us to write this in the alternative form

$$\xi = b + \frac{\lambda}{\rho} \int_{\xi}^{\infty} \bar{F}(w)\, dw. \tag{2.6}$$

Equation (2.6) is the optimality condition for reservation wage choice and it is fundamental to our analysis. We shall return to it in Section 4, after we have described more fully the observations and our proposed interpretation of them.

3. THE DATA AND THEIR INTERPRETATION

The data arise out of interviews with a random sample of the stock of unemployed people. Those individuals selected for inclusion in the sample were interviewed shortly after their selection. By this time some had found new jobs. Those who had *not* were asked the following questions which we give in detail since the precise wording is important. (a) How much take home pay would you expect to be able to earn in a new job? (b) Would you tell me the lowest amount you would be prepared to accept after stoppages?[5] Answers were given by indicating a letter from A to L coding each of twelve wage classes from "£15 a week or under" to "over £80 a week."

We need to interpret the answers to these questions in terms of the optimal search model of the last section. It seems straightforward to take the answer to (b) as the reservation wage, ξ, and this we shall do, but the interpretation of (a) is more problematic. One possibility is that it refers to the wage people expect to be earning when they have found a new job and assuming their behavior is described by the model, this must be $E(w \mid w > \xi) = x$,[6] say. The phrase "expect to be able" does, however, suggest that the question refers to the capacity to earn rather than to the person's job acceptance policy. In this case the question may have been interpreted as asking for $E(w) = \mu$, the mean wage available. What we shall do is assume that (b) provides ξ and (a) x as a working hypothesis and see whether the observations are consistent with these interpretations.

[5] That is, tax and social security deductions.
[6] "x" for expected.

There is, of course, one elementary test of this hypothesis and this is that, as a matter of arithmetic, we must have $E(w \mid w > \xi) \geq \xi$, i.e. $x \geq \xi$. The Appendix Table A-I gives the grouped joint frequency distribution of x and ξ and it can be seen that in only 3 cases out of 642 did the answer to (a) above fall in a lower class than the answer to (b). This evidence therefore is consistent with the proposed interpretations of the answer to (a) as $E(w \mid w > \xi)$. On the interpretation of the answer to (a) as μ the data suggest that almost everyone had a reservation wage in the same income class as the mean or below it. This seems implausible.

There is a second test which should be done before undertaking any more elaborate economic work and this is a joint test of both our interpretation of the data and of the optimal search model of Section 2. The model implies that no one will have a reservation wage less than his unemployment income, $\xi \geq b$. Appendix Table A-II gives the joint frequency distribution of ξ and b[7] and it can be see that this condition is violated in only 5 cases out of 653. This is consistent with both our interpretation of the answer to (b) as ξ and the optimal search model we are working with. The individuals included in the calculations reported below were male, affirmed they were actively seeking work and answered all relevant questions. It is worth noting that the proportions of all individuals asked who were coded "Don't know/Refused" on the reservation wage question was 4.2 per cent and on the expected wage question 6.4 per cent.[8]

4. THE EQUATION SYSTEM AND ITS ECONOMETRIC TREATMENT

We shall treat the reservation wage ξ, the conditional expected wage x, and the completed unemployment duration t as three jointly dependent variables satisfying a system of equations of which the first is the optimality condition of Section 2 which we repeat here for reference as

$$\xi = b + \frac{\lambda}{\rho} \int_{\xi}^{\infty} \bar{F}(w)\, dw. \tag{4.1}$$

The conditional expected wage is defined by

$$x = \int_{\xi}^{\infty} w\, dF(w) / \bar{F}(\xi),$$

[7] The benefit figure was that returned in answer to the question, "I would like to know how much you had coming in from social benefits during the main part of the time you were out of work—including any unemployment benefit, any supplementary benefit, and any family income supplement."

[8] The people included in the present sample include most of the 'still unemployed' portion of the sample analyzed in [5]. Though the sets of individuals overlap, the present analysis largely concerns variables—expected and reservation wages—not studied in the earlier paper. There is in fact more data in this survey which is, on the face of it, relevant to a search theoretic interpretation of behavior.

which, after integration by parts, may be written as

$$x = \xi + \int_{\xi}^{\infty} \bar{F}(w)\,dw / \bar{F}(\xi). \tag{4.2}$$

For the third equation we note that for an individual to resume work in any short interval of length dt he needs both to receive an offer, an event of probability $\lambda\, dt$, and to have that offer exceed his asking price, an event of probability $\bar{F}(\xi)$. Hence the hazard or re-employment probability function, θ,[9] satisfies

$$\theta\, dt = \lambda \bar{F}(\xi)\, dt. \tag{4.3}$$

Equation (4.3) completes the system since a specification of the hazard function is equivalent to a specification of the distribution of unemployment duration (see [5]) so (4.3) can be regarded as the duration equation.

We now wish to use this system to help us understand the data. In particular we wish to know what the data tell us about the numerical values of parameters that are of importance for policy purposes. The interesting numbers would measure the response of the reservation wage and the probability of re-employment to changes in the level of unemployment benefit, b, and the rate of arrival of job offers, λ. These elasticities can be deduced by differentiating the optimality condition (4.1), partially with respect to b and λ, and then differentiating θ with respect to λ and ξ. For example, differentiating (4.1) with respect to b and solving[10] gives

$$\frac{\partial \xi}{\partial b} = \frac{1}{1 + \lambda \bar{F}(\xi)/\rho} = \frac{1}{1 + \theta/\rho}$$

and hence

$$\frac{\partial \log \xi}{\partial \log b} = \frac{b}{\xi}\,\frac{1}{1 + \theta/\rho}\,.$$

To find the effect of b on θ,

$$\frac{\partial \log \theta}{\partial \log b} = \frac{\partial \log \bar{F}(\xi)}{\partial \log \xi} \cdot \frac{\partial \log \xi}{\partial \log b}$$

$$= -\frac{f(\xi)}{\bar{F}(\xi)}\,\frac{b}{1 + \theta/\rho}\,.$$

[9] Note that θ is independent of time in this stationary model.

[10] Note that we are assuming λ independent of b which effectively rules out endogenous choice of search intensity. We have examined the effect of allowing λ to be optimally chosen and report our conclusions in Section 7.

TABLE I

ELASTICITIES

Variables	Formula and Bounds	Solution
Reservation Wage with Respect to Benefit Level	$0 \leq \dfrac{\partial \log \xi}{\partial \log b} = \dfrac{b}{\xi} \dfrac{1}{1+\theta/\rho} \leq 1$	$\dfrac{b}{\xi} \cdot \dfrac{x-\xi}{x-b}$
Reservation Wage with Respect to Offer Probability	$0 \leq \dfrac{\partial \log \xi}{\partial \log \lambda} = \dfrac{1}{\xi} \dfrac{\xi - b}{1+\theta/\rho} \leq 1$	$\dfrac{\xi - b}{\xi} \cdot \dfrac{x-\xi}{x-b}$
Re-employment Probability with Respect to Benefit Level	$\dfrac{\partial \log \theta}{\partial \log b} = -\dfrac{f(\xi)}{\bar{F}(\xi)} \cdot \dfrac{b}{1+\theta/\rho} \leq 0$	
Re-employment Probability with Respect to Offer Probability	$\dfrac{\partial \log \theta}{\partial \log \lambda} = 1 - \dfrac{f(\xi)}{\bar{F}(\xi)} \dfrac{\xi - b}{1+\theta/\rho}$	

The complete list of elasticities together with any restrictions they necessarily satisfy is given in Table I. The interpretation of the final column will be given subsequently.

It is noteworthy that we not only can sign three out of the four elasticities but that both reservation wage elasticities must necessarily not exceed one. Indeed we can go further since their sum is equal to $(1 + \theta/\rho)^{-1}$ which is no greater than one, i.e. they cannot both exceed a half.[11] The assumption of optimal job search restricts the parameter space unusually severely. On the other hand, as has been noted before, we cannot sign, on theoretical grounds, the effect of an increase in the probability of an offer on the probability of return to work. An increase in λ per se raises θ but it also causes an increased asking price which reduces θ. It is plausible that the more offers there are the more rapidly individuals return to work but it is not necessarily so.

Let us now turn to the final column of the table which is labelled 'solution' because it provides the answer to the problem of calculating these elasticities. The only unknown variable in the first two elasticities of the table is $\theta/\rho = \lambda\bar{F}(\xi)/\rho$. But substituting (4.2) into (4.1) gives immediately

$$\frac{\theta}{\rho} = \frac{\xi - b}{x - \xi},$$

enabling us to express the first two elasticities as exact functions of the data for each individual. These expressions are given in the solutions column of the table. Hence without any theory whatsoever about the offer distribution we can

[11] Indeed since the expected duration of unemployment for someone behaving according to the model is $d = 1/\theta$ the sum of the reservation wage elasticities may be written as $(1 + 1/d\rho)^{-1}$ which for sensible values of d and ρ is much closer to zero than to one. For example with a year as the unit time, $d = 1/2$ and $\rho = 0.20$ give an elasticity of $1/(1 + 10) = .09$. This suggests numerically very small reservation wage elasticities.

calculate, using the observed ξ, x and b, the benefit and offer probability elasticities of the reservation wage. Moreover we can do this for every individual, evaluating the elasticity at his particular, optimally selected, variable values. We do not require a regression analysis let alone the attempted maximization of some awkward and quite possibly nonconcave likelihood function. This seems a remarkable example of the informational value of economic theory.

The remaining spaces in the solutions column are blank since we cannot deduce these elasticities merely by substituting out the unknowns. We require to know $f(\xi)/\bar{F}(\xi)$ which is the hazard function of the offer distribution at the chosen reservation wage. Thus to calculate the remaining elasticities requires a further assumption and we return to this question in Section 6 after we have reported values for $\partial \log \xi / \partial \log b$ and $\partial \log \xi / \partial \log \lambda$.

Before doing so we comment on the question of goodness of fit of the model. One of the most obvious ways in which the model can fail is where individuals adopt a policy of varying their reservation wage in response to the failure to find a job. Indeed the time dependence of the reservation wage and more generally of the hazard function has been a major focus of previous studies. One solution would be to work with a less restrictive optimal search model in which the solution involves a generally time dependent reservation wage. Such models appear however to be an order of magnitude more complicated from the applied econometric point of view than the stationary case. Another solution would be to drop the assumption of *optimal* search and simply write the reservation wage as a function of time with coefficients to be estimated. But as we have seen the equation of optimal reservation wage choice is enormously helpful in solving the estimation problem so this solution appears to throw out the baby with the bath water.

We therefore choose in this paper a third solution. Two of the most plausible reasons why job seekers might alter their reservation wage are, firstly, learning about the offer distribution and the availability of jobs and, secondly, fluctuations in the level of their prospective income while unemployed. The effect of learning presumably diminishes over time while the main variation in the benefit stream in the British system occurs at 26 weeks. Thus we would argue that a constant reservation wage model—which we shall also take to imply a constant θ —seems most appropriate a priori for the longer term unemployed. It seems to us rather implausible for people who have been out of work six months or more still to be significantly varying their real asking price. We shall therefore calculate average elasticities for individuals classified by their elapsed duration of unemployment, and place more confidence in the appropriateness of our method for the longer duration groups.[12]

[12] It is tempting to argue that if we found that the calculated elasticities do not vary systematically with elapsed duration this is evidence in favor of stationarity. But this is not strictly so since selection by duration is not random with respect to λ, F, and b. The longer term unemployed must tend to have a higher proportion of people with fewer offers and/or inferior offer distributions relative to their benefit levels. Since the elasticities depend upon λ, F, and b we might expect, even in the stationary case, to find some systematic variation in elasticities between duration groups.

5. RESERVATION WAGE ELASTICITIES

In principle the formulae of Table I enable us to calculate elasticities for every sample individual, but in practice the ξ, x, and b data are rather coarsely coded. This means that if we replaced each class by its midpoint we would get zero denominators in our elasticity formulae for a number of individuals. To avoid this and to minimize the measurement error that the use of class midpoints introduces we replace the x, ξ, and b in the formulae of the solution column by their averages over all individuals in each duration group.[13] The results are 'average' elasticities for each elapsed duration group and these are given in Tables II and III.

An alternative way to look at the figures is in money terms rather than as elasticities. From the data given in the Appendix we can calculate that for those unemployed more than a year an extra £1 in unemployment benefit would, on average, raise the asking price by about 25p, whereas for those unemployed less

TABLE II

ELASTICITY OF THE RESERVATION WAGE WITH RESPECT TO UNEMPLOYMENT BENEFIT

Duration Group (Weeks)	Elasticity
$T \geq 52$	0.14
$35 \leq T < 52$	0.11
$26 \leq T < 35$	0.12
$13 \leq T < 26$	0.17
$T < 13$	0.11
All T	0.135

TABLE III

ELASTICITY OF THE RESERVATION WAGE WITH RESPECT TO THE OFFER PROBABILITY

Duration Group (Weeks)	Elasticity
$T \geq 52$	0.12
$35 \leq T < 52$	0.10
$26 \leq T < 35$	0.10
$13 \leq T < 26$	0.12
$T < 13$	0.09
All T	0.107

[13] It is important to note that these 'average' elasticities are not estimates of some parameter common to all members of the group. $\partial \log \xi / \partial \log b$, for example, is not independent of ξ or b so each individual will have his own elasticity at his b and optimally selected ξ point. Our averages are to be interpreted as typical values of these varying numbers for the group in question.

than 13 weeks such a rise would increase the asking price by about 21p. For the second elasticity we can deduce that a 10 percent rise in the arrival rate of offers or equivalently a 10 per cent fall in the mean waiting time between job offers would increase the asking price by about 27p for those unemployed more than a year and by about 23p for those unemployed less than 13 weeks.

6. THE PARETO TAIL

For policy purposes it is important to know how our extra £1 of unemployment benefit would alter not the asking price but rather the probability of re-employment and hence the duration of unemployment and the size of the stock of unemployed. To get at this response we need to add an hypothesis. What we propose to do is to adopt the simplest hypothesis that will enable us to complete the solutions column of Table I and to explore the consequences of this hypothesis in this section. Our hypothesis is that the relevant portion of the wage offer distribution for any individual—that exceeding his benefit level—is a member of the Pareto family. In the next section we shall ask how sensitive our conclusions are to this assumption.

The hypothesis is thus that, at least for $w \geq b$,

$$\bar{F}(w) = (w_0/w)^{1/\sigma}, \qquad \sigma < 1/2, \quad w \geq \max\{w_0, b\}, \tag{6.1}$$

where w_0 and σ are positive parameters of the distribution which may vary from one individual to another. To interpret these parameters we note that, if $w_0 \geq b$,

$$E(w) = w_0/(1-\sigma),$$

$$\operatorname{var} w = w_0^2\sigma^2/(1-\sigma)^2(1-2\sigma),$$

$$\text{coefficient of variation} = \sigma/\sqrt{1-2\sigma}\,,$$

$$\operatorname{var}\log w = \sigma^2.$$

Thus σ is the standard deviation of log wage offers and approximately (as $\sigma \to 0$) the coefficient of variation of offers. The constant w_0 is proportional to the mean offer which we thus allow to vary between people. This model allows a variety of slopes for the offer density function though all must be decreasing to the right of the benefit level.

This assumption, though restrictive, is certainly fruitful. The first application we can make is to estimate σ, the coefficient of variation of offers, since an easy calculation shows that

$$E(w \mid w > \xi) = x = \xi/(1-\sigma). \tag{6.2}$$

Since we know x and ξ we could in principle calculate σ for each individual but in view of the grouping of our data we report instead the value got by replacing x and ξ in (6.2) by averages for each elapsed duration group. We note the absence of systematic variation in σ with elapsed duration and a typical coefficient of variation of the order of 10–15 per cent.[14] This value, while small, is certainly sufficient to allow substantial income gains to an individual who holds out for a better offer.

The interpretation of σ is worth remarking on at this stage. There is nothing in our argument so far which requires F (and λ) to correspond to some objectively existing random mechanism which generates wage offers of various sizes and which each individual happens to know for sure. The x and ξ numbers returned by each person are in fact telling us about the F and λ which he believes, implicitly or explicitly, to face him. Similarly θ would be the consequential re-employment probability and need have no necessary objective accuracy as a description of what his chances of re-employment 'really are.' There is, however, an obvious incentive for individuals' beliefs to be accurate and we need in fact to assume they are when, as we are about to do, we report elasticities of θ with respect to the benefit level. We want to know the response of individuals' actual chances of work to an extra £1 of benefit and not merely the response of his implicit belief in his chance of re-employment.

To complete the solution column of Table I is now straightforward since the hazard of the Pareto distribution at the point ξ is readily seen to be

$$\frac{f(\xi)}{\bar{F}(\xi)} = \frac{1}{\sigma\xi}. \tag{6.3}$$

We therefore use the σ values of Table IV in calculating

$$\begin{aligned} \frac{\partial \log \theta}{\partial \log b} &= -\frac{b}{\sigma\xi}\,\frac{x-\xi}{x-b}, \\ \frac{\partial \log \theta}{\partial \log \lambda} &= 1 - \frac{1}{\sigma}\,\frac{\xi - b}{\xi}\,\frac{x-\xi}{x-b}. \end{aligned} \tag{6.4}$$

If an individual moves through time with constant hazard, θ, then his completed unemployment duration is exponentially distributed with mean, say d, equal to $1/\theta$. The benefit and λ elasticities of mean duration are then the negative of (6.4) and these are given in Tables V and VI.

The benefit elasticities of mean unemployment duration are about unity and show no great systematic variation with elapsed duration. An elasticity of one is consistent with the results of Nickell [7] though somewhat higher than that reported in [5]. Table VI shows that 10 per cent fall in the mean time between offers is associated with a 2 per cent rise in the reservation wage. It should be noted that the sign of this effect is entirely a consequence of the Pareto tail assumption.

[14] Kiefer and Neumann [4] estimate the coefficient of variation of the offer distribution for their U.S. data at 0.14.

TABLE IV

ESTIMATES OF σ BY DURATION GROUP

Duration Group	σ	N
$T \leq 13$	0.11	131
$13 < T \leq 26$	0.15	104
$26 < T \leq 35$	0.12	67
$35 < T \leq 52$	0.11	56
$52 < T$	0.12	281
All T	0.131	639

TABLE V

BENEFIT ELASTICITY OF UNEMPLOYMENT DURATIONS

Elapsed Duration Class	Benefit Elasticity
$T \leq 13$	1.0
$13 < T \leq 26$	1.1
$26 < T \leq 35$	1.0
$35 < T \leq 52$	1.0
$52 < T$	1.2
All T	1.03

TABLE VI

ELASTICITY OF UNEMPLOYMENT DURATIONS WITH RESPECT TO THE MEAN RATE OF ARRIVAL OF OFFERS BY ELAPSED DURATION CATEGORIES

Duration Group (Weeks)	Elasticity
$T \geq 52$	-0.16
$35 \leq T < 52$	-0.09
$26 \leq T < 35$	-0.15
$13 \leq T < 26$	-0.19
$T < 13$	-0.10
All T	-0.190

7. SOME SENSITIVITY CHECKS

An analysis that depends, as ours does, on some rather strong hypotheses, needs to be complemented by a study of the sensitivity of its conclusions to deviations from these hypotheses. We have been looking at the effect of the level of unemployment income, b, on the reservation wage, ξ, and on the probability of re-employment, θ, and we calculate the former elasticity, ϵ_R, by

$$\bar{\epsilon}_R = \frac{b}{\xi} \cdot \frac{x - \xi}{x - b}, \tag{7.1}$$

and minus the latter, ϵ_D, by

$$(7.2) \quad \bar{\epsilon}_D = -\bar{\epsilon}_R \partial \log \bar{F}/(\xi)/\partial \log \xi,$$

$$= \bar{\epsilon}_R/\sigma, \qquad \text{for the Pareto tail,}$$

$$= \bar{\epsilon}_R x/(x-\xi).^{15}$$

The question of sensitivity is that of the direction and extent of the inaccuracy of the formulae (7.1) and (7.2) when the assumptions are false.

A full account of the sensitivity analysis would be lengthy and we confine ourselves to reporting our conclusions briefly, with one exception. We can allow for risk aversion by changing the maximand to the expected present value of $u(w)$ where u is an increasing concave function. In the case of $u(w) = \ln w$ we find $\bar{\epsilon}_R$ and $\bar{\epsilon}_D$ somewhat understate the true elasticities. It is not unreasonable that in general a risk averse person should be more sensitive to variations in his 'fallback income,' b, than a risk netural one.

We can allow for choice of search intensity, in which λ is endogenous, by introducing an offer production function $\lambda = \lambda(z)$ in which expenditure of z generates offers at the rate λ, $\lambda' > 0$; $\lambda'' < 0$. The individual then chooses simultaneously z and ξ to maximize the expected present value of his income stream. This modification carries the implication that $\bar{\epsilon}_R$ overstates ϵ_R but it appears that it cannot be said whether $\bar{\epsilon}_D$ is too low or too high, although some calculations with specific forms for λ and $\bar{F}$ suggest the error in $\bar{\epsilon}_D$ might be small.

We can study the effect of omitting leisure,[16] l, from the model by taking the maximand to be the expected present value of $u(w, l)$ where l takes the value 1 when the individual is unemployed and searching and $1 - h$ when he is employed. Here h is interpreted as the weekly hours of work associated with a 'full-time job.' That is, we take it to be the weekly hours implicitly assumed by the respondent when he replies to questions about weekly wages. The particular utility function $u(w, l) = wv(l)$ amounts to altering b to rb, $r = v(1)/v(1-h)$, in the optimality condition (2.6) and implies that $\bar{\epsilon}_R$ will underestimate ϵ_R, possibly severely, to the extent that unemployment and search is preferred to work at the same income level.

So far as the benefit elasticity of the reservation wage is concerned, therefore, our studies suggest that, if anything, $\bar{\epsilon}_R$ will understate the true response and therefore the duration elasticity. However this latter also depends upon our form for $f(\xi)/\bar{F}(\xi)$ which in the Pareto model was the decreasing function $1/\sigma\xi$. To study the effect of this assumption we consider the alternative model in which

[15] The bars on $\bar{\epsilon}_R$ and $\bar{\epsilon}_D$ distinguish the specific computational formulae from the true but unknown elasticities.

[16] In formulating the approach described in this paragraph we have benefited from reading some unpublished work by Steve Nickell.

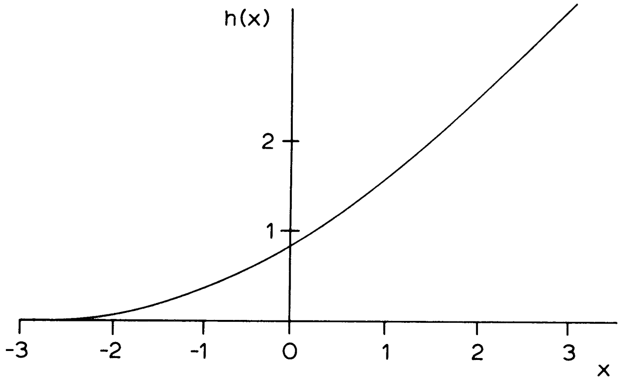

FIGURE 1—The standard normal hazard function: $h = \phi/\overline{\Phi}$.

the wage offer distribution facing an individual, at least to the right of b, is $N(\mu, \sigma^2)$. This is a case opposite to the Pareto in the sense that $f/\overline{F}$, the hazard, is a monotonically *increasing* function as can be seen from Figure 1 which sketches the hazard of the $N(0, 1)$ distribution, which we denote by h. For this Normal model $\overline{F}(w) = \overline{\Phi}((w-\mu)/\sigma)$ where Φ is the standard Normal distribution function with ϕ the density function and $\overline{\Phi} = 1 - \Phi$. The hazard at ξ is

$$\begin{aligned} f(\xi)/\overline{F}(\xi) &= \phi(\xi^*)/\sigma\overline{\Phi}(\xi^*) \\ &= h(\xi^*)/\sigma \end{aligned}$$

where

$$\xi^* = (\xi - \mu)/\sigma. \tag{7.3}$$

Thus from Table I we see that the benefit elasticity of unemployment duration is

$$\epsilon_D = \frac{h(\xi^*)}{\sigma} \frac{b(x-\xi)}{(x-b)}. \tag{7.4}$$

To evaluate (7.4) we use the result that for a Normal distribution $x = \mu + \sigma h(\xi^*)$ which, using (7.3) to eliminate μ, gives

$$x - \xi = \sigma\big(h(\xi^*) - \xi^*\big).$$

Solving for σ and substituting in (7.4) gives

$$\begin{aligned} \epsilon_D &= h(\xi^*)(h(\xi^*) - \xi^*)\frac{b}{x-b} \\ &= h'(\xi^*)\frac{b}{x-b} \end{aligned} \tag{7.5}$$

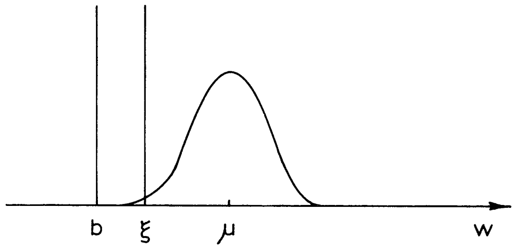

FIGURE 2—A wage offer distribution in which $f(\xi)/\bar{F}(\xi) \sim 0$.

since $h'(y) = h(h - y)$. As the figure suggests, $h'(y)$ increases from zero at $y = -\infty$ approaching unity as $y \to \infty$; $h'(0)$ is 0.65. Thus on the Normal model we deduce only that

$$0 \leq \epsilon_D \leq \frac{b}{x - b} \sim 1.0.$$

If the typical sample individual has ξ near μ, i.e. ξ^* near zero, then we have $\epsilon_D \sim 0.65$ a figure calculated in [5] for an overlapping sample but using radically different methods.[17] On the other hand it is possible to argue that $\epsilon_D \sim 0$ on the Normal model. This will happen when ξ is several standard deviations below μ as in Figure 2. Note, however, that in order for such a low ξ to be optimal, λ, the offer arrival rate, must be low and note also that, since $\xi > b$, there can be very few jobs that pay b or less. Thus if one is to believe that ϵ_D is near to zero one must also believe that very few jobs open to our sample pay the benefit income or less and that our sample individuals are, on the average, unemployed because they receive few, if any, job offers.

8. CONCLUDING REMARKS

Our main conclusions are that the people in our sample can answer questions suggested by optimal job search theory and that their answers are both numerically consistent with that theory and can be used to yield elasticities not dissimilar to those found in previous microeconometric studies. Our inferences about the important policy parameter, the benefit elasticity of the hazard, are however sensitive to the assumptions made, especially to the treatment of leisure in the maximand of the optimal job search model and to the precise functional form relating the hazard, θ, to the reservation wage.

We also think the methods used in the paper, which involve inference about structural parameters using statistical data and economic theory but without

[17] The coefficient of variation is given by $\sigma/\mu = (x - \xi)/(\xi h(\xi^*) - x\xi^*)$ which at $\xi^* = 0$ is about 0.18.

maintaining a statistical model in the usual sense,[18] are of interest. Our companion paper [6] analyzes an overlapping data set with more conventional methods and provides additional information about the parameters of the job search model.

University of Hull, England
and
University of Birmingham, England

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[18] To aggregate our individual specific elasticities rigorously *would* require use of sampling theory. In the text we have used a rather rough 'averaging.'

APPENDIX

TABLE A-I

JOINT FREQUENCY DISTRIBUTION OF EXPECTED (x) AND RESERVATION (ξ) WAGES (P.E.P. DATA)

		x											
		14	17.5	22.5	27.5	32.5	37.5	42.5	47.5	55	65	75	
	14	24	11	6	1								42
	17.5	1	60	79	5	2							147
	22.5	1		78	71	11	2	2					165
	27.5				81	69	11	8	2	1			172
ξ	32.5		1			30	27	5	2	4			69
	37.5						14	7	2	1	3		27
	42.5							4	6	2	1		13
	47.5								2	2	1	1	6
	55									1			1
													642

NOTE: 642 is our basic sample of 653 less the 11 who failed to answer the expected wage question.

TABLE A-II

JOINT FREQUENCY DISTRIBUTION OF RESERVATION (ξ) WAGES AND REPORTED BENEFIT LEVELS (b) (P.E.P. DATA)

		ξ									
		14	17.5	22.5	27.5	32.5	37.5	42.5	47.5	55	
	4	4	3	3	2						12
	7.5	26	71	45	21	6	2	1	1		173
	12.5	10	58	49	33	5	5	5	1		166
b	17.5	3	15	54	66	27	8	3	3	1	180
	22.5		1	15	48	29	13	4			110
	27.5			1	4	3	1		1		10
	32.5					2					2
											653

TABLE A-III

GEOMETRIC MEAN RESERVATION AND EXPECTED WAGES (£/WEEK) AND REPORTED BENEFIT LEVELS WITHIN ELAPSED DURATION GROUPS (P.E.P. DATA)

Duration Group	ξ	$\bar{x}$	$\bar{b}$
$52 < t$	22.49	25.71	13.11
$35 < t \leq 52$	22.97	25.89	12.13
$26 < t \leq 35$	24.78	28.04	13.20
$13 < t \leq 26$	24.64	28.85	14.35
$t \leq 13$	24.92	28.01	12.85
All t	23.59	26.92	13.17

NOTE: $N = 639$—the original 653 less the 11 who failed to answer the expected wage question and the 3 who reported $x < \xi$.

TABLE A-IV

GEOMETRIC MEAN RESERVATION AND EXPECTED WAGES (£/WEEK) AND REPORTED BENEFIT LEVELS WITHIN ELAPSED DURATION GROUPS (OXFORD DATA)

Duration Group	ξ	x	$\bar{b}$
$52 < t$	17.74	20.32	10.81
$35 < t \leq 52$	19.45	21.88	10.84
$26 < t \leq 35$	20.18	22.39	10.33
$13 < t \leq 26$	21.23	23.82	9.84
$t \leq 13$	21.28	25.23	6.90
All t	19.47	22.09	10.22

NOTE: $N = 627$.

REFERENCES

[1] DANIEL, W. W.: "A National Survey of the Unemployed," P.E.P. London, Broadsheet No. 546, 1974.

[2] HILL, M. J., R. M. HARRISON, A. V. SARGENT, AND V. TALBOT: *Men Out of Work*. Cambridge: Cambridge University Press, 1973.

[3] KARLIN, S.: *Studies in Applied Probability and Management Science*, ed. by K. J. Arrow, S. Karlin, and H. Scarf. Stanford: Stanford University Press, 1962.

[4] KIEFER, N. M., AND G. R. NEUMANN: "Individual Effects in a Non-Linear Model: Explicit Treatment of Heterogeneity in the Empirical Job-Search Model," *Econometrica*, 49(1981), 965–980.

[5] LANCASTER, T.: "Econometric Methods for the Duration of Unemployment," *Econometrica*, 47(1979), 939–956.

[6] LANCASTER, T., AND A. D. CHESHER: "Simultaneous Equations with Endogenous Hazards," in *Studies in Contemporary Economics*, ed. by G. R. Neumann and N. Westergaard-Nielsen. Heidelberg: Springer-Verlag, forthcoming 1984.

[7] NICKELL, S. J.: "Estimating the Probability of Leaving Unemployment," *Econometrica*, 47(1979), 1249–1266.