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Summary

This paper is a study of the nature of the frequency distribution of the duration of strikes in the United Kingdom as recorded by the Department of Employment. The settlement of a strike is regarded as a probabilistic process and the duration of a strike is treated as an observation on a random variable. A model for this random duration is created by supposing that at each point of time after the commencement of a strike there exists an index of the difference between the parties to the dispute. This index is itself regarded as a one-dimensional stochastic process in continuous time and space and determines the duration of the strike by the first point of time at which the difference index passes through an absorbing barrier representing "agreement". The duration of a strike then becomes the First Passage time of a stochastic process to a single absorbing barrier. As a first approximation the index process is assumed to be simple Brownian motion with drift and in consequence the duration of a strike has the Inverse Gaussian for its probability distribution. The fit of this distribution to the observations is shown to be, with few exceptions, very close.

Keywords: DURATION OF STRIKE; INVERSE GAUSSIAN

1. Introduction

A STRIKE† is defined to be a stoppage of work in connection with a dispute over terms and conditions of employment. The two most readily measurable aspects of strikes are their size—the number of men involved—and their duration and, in respect of these, strikes show great diversity. In the United Kingdom their size ranges from single figures to several hundred thousands and their duration from minutes to many weeks. This paper is concerned with one of these dimensions, namely, duration.

It appears useful to distinguish between strikes whose duration is effectively determined in advance and those whose duration is determined by the outcome of a process of bargaining and concession, although the distinction is not, perhaps, an easy one to make precise. An example of the former type is the "one day token stoppage" which appears to be in the nature of a demonstration or threat. An example of the latter type is a dispute over a pay claim in which, during the strike, bargaining continues, however spasmodically, offers and demand are revised, and the strike ends when agreement is reached, be it agreement about the claim or agreement to resort to arbitration or merely agreement to continue negotiations under normal working conditions. The model to be developed in this paper is meant to apply to strikes whose duration is not predetermined, and since we cannot, from our data, distinguish between strikes of predetermined duration and others we shall assume that the great majority of the stoppages in the statistics we analyse are not of the former type.

† The data do include a very small number of stoppages more appropriately called "lockouts" but we shall use the words "strike" and "stoppage" interchangeably.

We shall try to construct a model in which the duration of a strike is a random variable with a probability distribution characterized by a small number of parameters such that the parameters embody the systematic determinants of duration and such that these parameters can be regarded as approximately the same for some observed set of strikes.

To motivate the particular approach taken in this paper consider the following very simplified strike situation. A dispute is in progress between a body of "workers" and "management" over a pay claim. At the point of time when, for whatever reason, the men embark on strike action their demand is for 75p and the management are offering 25p. Casual observations suggest that as the strike proceeds the offer and the demand change, presumably as, at intervals of time, the workers and the management redetermine the sum which, in the light of the latest information available to them, it is optimal for them to demand or to offer respectively. These demands and offers will change as experience accumulates and expectations are revised, and work will be resumed when the management are making an offer consistent with the workers current demand, i.e. when the difference between demand and offer has fallen to zero.

This very simplified strike situation is, even though brief, descriptively inaccurate in several respects,† for example, the issue in dispute in the majority of strikes in the United Kingdom in 1965 was not claims for wage increases but, for example, questions of discipline, hours of work, sympathy, union recognition, etc. Secondly, many disputes concern not a single issue such as a pay claim but involve several points of difference. Thirdly, work appears to be resumed in some strikes when at least the stated, public, offer and demand are not in equality. Nonetheless, in spite of its inaccuracy the description serves to direct attention to the possibility of summarizing the position in a strike by a numerical measure of the difference between the parties. In the example above a natural measure of the difference between the parties would be the demand less the offer, which initially was 50p, a quantity which when the strike ended we may suppose had become zero. No such "natural" measure is available in say, a dispute about the sacking of a shop steward or in a dispute over six distinct rates of pay or bonus. However, in this paper we shall *postulate* the existence, for any dispute, of a scalar measure of the difference between the parties, and suppose that the duration of a strike is determined solely by the behaviour of this difference. This difference measure is not in general directly observable and even in the case of the simple pay dispute described above it is not necessarily to be equated with the simple difference of the demand and the offer. The difference between the parties in a dispute is a hypothetical construct defined solely by the properties we shall describe in the next section.

2. A Stochastic Model for the Difference of the Parties to a Dispute

Let time, t, varying continuously, be measured from an origin at the point of commencement of a strike and let the unit time period be a working day. For each t there is a scalar random variable $X(t)$ to be interpreted as a measure of the difference

† Though closely similar processes are often reported in the newspapers, for example, in an article entitled "Boat Rocking at Swan Hunter" a shipbuilding firm, occurs this passage. "At the beginning of last week the General and Municipal Workers Union, to which the strikers belong, were claiming £21·40 a week, the same rate as the repair yard men, and the management's offer at that time was £20·17½. Through a series of meetings this was increased first to £20·60, then to £21 and yesterday's offer turned down by a majority of only 45 reached £21·15" (*The Times*, August 9th, 1971).

of the parties to the dispute at time t. By a suitable choice of the scale and origin of $X(t)$ we shall take the difference of the parties when the strike commences to be $X(0) = 0$, and we shall suppose the strike ceases—agreement is reached—at the first point of time such that $X(t) = 1$. For each t, $X(t)$ is a random variable with the following properties:

1. For any time interval (t_1, t_2), $E(X(t_2) - X(t_1)) = \mu(t_2 - t_1)$.

This assumption means that the average progress towards settlement of the dispute in any time interval is proportional to the length of the interval. We shall suppose $\mu > 0$. Since X starts at zero and the strike ends when $X = 1$, μ is the mean *proportionate* rate of drift to settlement per working day (or non-working day!).

2. For any pair of non-overlapping time intervals the changes in $X(t)$ are distributed independently.

This means, for example, that whether the parties have, by our measure, moved rapidly closer to agreement any one day is stochastically independent of whether they had done so the previous day. It has one interesting implication for the case in which bargaining has been going on prior to the commencement of the strike. The implication is that given that we know the difference of the parties when the strike commences a knowledge of their difference at some time before the strike commenced is of no help in predicting their difference at some future time and is of no help in predicting how long the strike will last. The only relevant information is their difference when the strike commenced.

Mathematically this implies that $\{X(t)\}$ is a Markov process.

3. For any time interval (t_1, t_2), $X(t_2) - X(t_1)$ is distributed normally with variance $\sigma^2(t_2 - t_1)$; $t_2 > t_1$.

This assumption has the implication that the change in the difference between the parties in any time interval may be negative—they may move further apart—although the probability of this event will be smaller the longer the time interval and the more rapid the drift towards agreement.

These three assumptions suffice to define $X(t)$ as a Wiener or Brownian motion stochastic process. The behaviour of the difference between the parties is roughly illustrated in Fig. 1.

The jagged line drawn in Fig. 1 represents a possible time path for the difference between the parties, and in this case the strike was settled after a little over 3 days.

The time taken for X to first reach the agreement barrier at $X = 1$ is known as the First Passage time through a single absorbing barrier in Brownian motion with drift. This time T is a random variable with a probability distribution which is known (Cox and Miller, 1965) to be the Inverse Gaussian with probability density function:

$$f(T) = (\sigma^2 2\pi T^3)^{-\frac{1}{2}} \exp\{-(1-\mu T)^2/2\sigma^2 T\}, \quad T > 0$$

and distribution function F such that

$$1 - F(T) = N\{(1-\mu T)/\sigma T^{\frac{1}{2}}\} - \exp\{2\mu/\sigma^2\}.N\{-(1+\mu T)/\sigma T^{\frac{1}{2}}\},$$

when $N(.)$ is the Standard Normal distribution function. The mean and variance of the distribution are:

$$E(T) = 1/\mu, \quad \text{var}(T) = \sigma^2/\mu^3.$$

For example with $\mu = 0{\cdot}20$, 20 per cent of the initial difference between the parties is eliminated per day, on average, and the expected duration of the strike is 5 days.

The density function is unimodal and positively skew—some Inverse Gaussian density functions are sketched in Fig. 2 for values of μ and σ consistent with the observations on U.K. strike durations. The hazard or age specific settlement rate defined as $\phi(t) = f/(1-F) = -d\log(1-F)/dt$ increases from zero at time zero to a

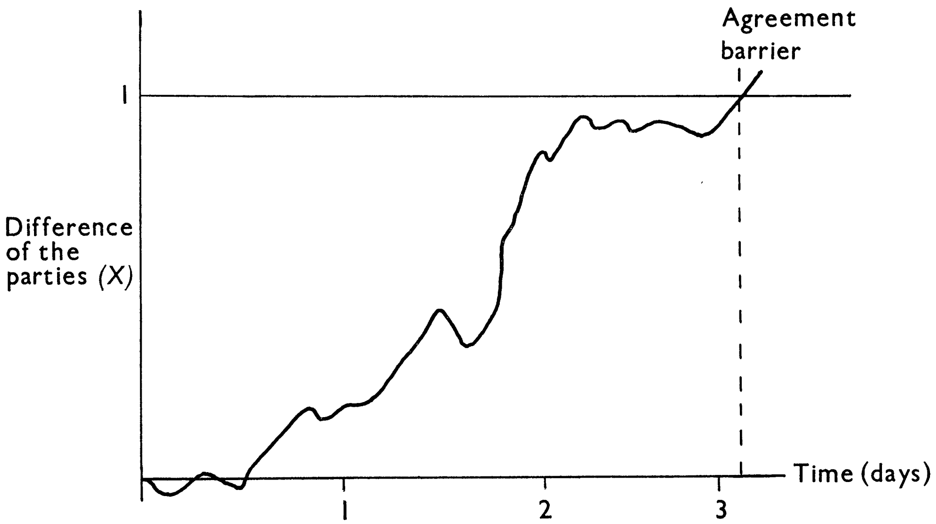

FIG. 1

single maximum located in the time interval $1/3\sigma^2 < T_m < 2/3\sigma^2$, then falls approaching the value $\mu^2/2\sigma^2$ as $t \to \infty$.† Some plots of $-\log(1-F)$, whose slope is $\phi(t)$, are also given in Fig. 2.

3. THE DATA

The data consist of the list of strikes recorded by the Ministry of Labour as commencing in 1965 in the United Kingdom. The Ministry attempts to record all strikes other than those lasting less than a day or involving fewer than 10 men unless one of these involves a loss of more than 100 man days. This recording rule means that, if the proportion of stoppages involving fewer than 10 men varies systematically with duration, then the proportion of stoppages of different durations that are recorded will itself vary, thus biasing the recorded duration frequency distribution away from the true one. This point was tackled by attempting to see whether there was any evidence of lack of independence of duration and number of men involved, size, to be seen in the lists of recorded strikes. The data were divided into eight industries, whose definitions are provided in the appendix, and the scatter diagrams of duration and size inspected. For seven of the eight industries the evidence was consistent with stochastic independence of duration and size. The exception was the Construction industry whose scatter diagram is shown as Fig. 3 together with the slope coefficient and standard error in the regression of log size on log duration.‡

† For an extensive account of the Inverse Gaussian distribution see Tweedie (1957).

‡ In four industries the regression coefficient of log size on log duration was positive and in four negative. Only in construction did it differ significantly from zero on a two-tailed test.

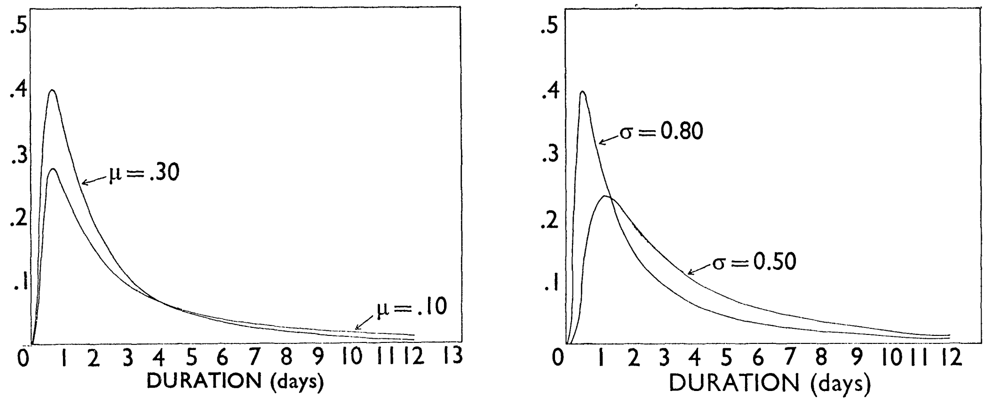

FIG. 2a. Inverse Gaussian density functions: $\sigma = 0{\cdot}70$.

FIG. 2b. Inverse Gaussian density functions: $\mu = 0{\cdot}20$.

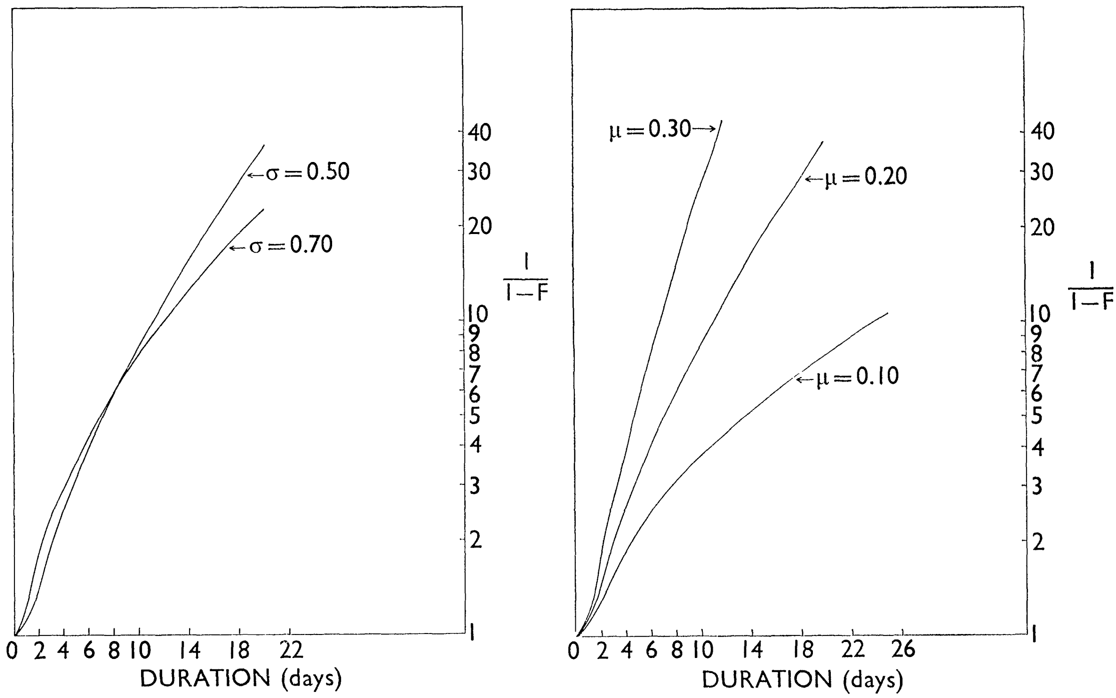

Fig. 2c. $-\log(1-F)$, where F is the Inverse Gaussian distribution function: $\mu = 0{\cdot}20$.

Fig. 2d. $-\log(1-F)$, where F is the Inverse Gaussian distribution function: $\sigma = 0{\cdot}70$.

For this industry the data suggest that the longer a strike the fewer men involved and we might thus expect to find some under-recording of the longer strikes relative to the shorter ones. For the other industries, however, it seems as though we are reasonably safe in supposing that the recorded proportions of strikes of different durations are not systematically different from the corresponding proportions of all strikes.

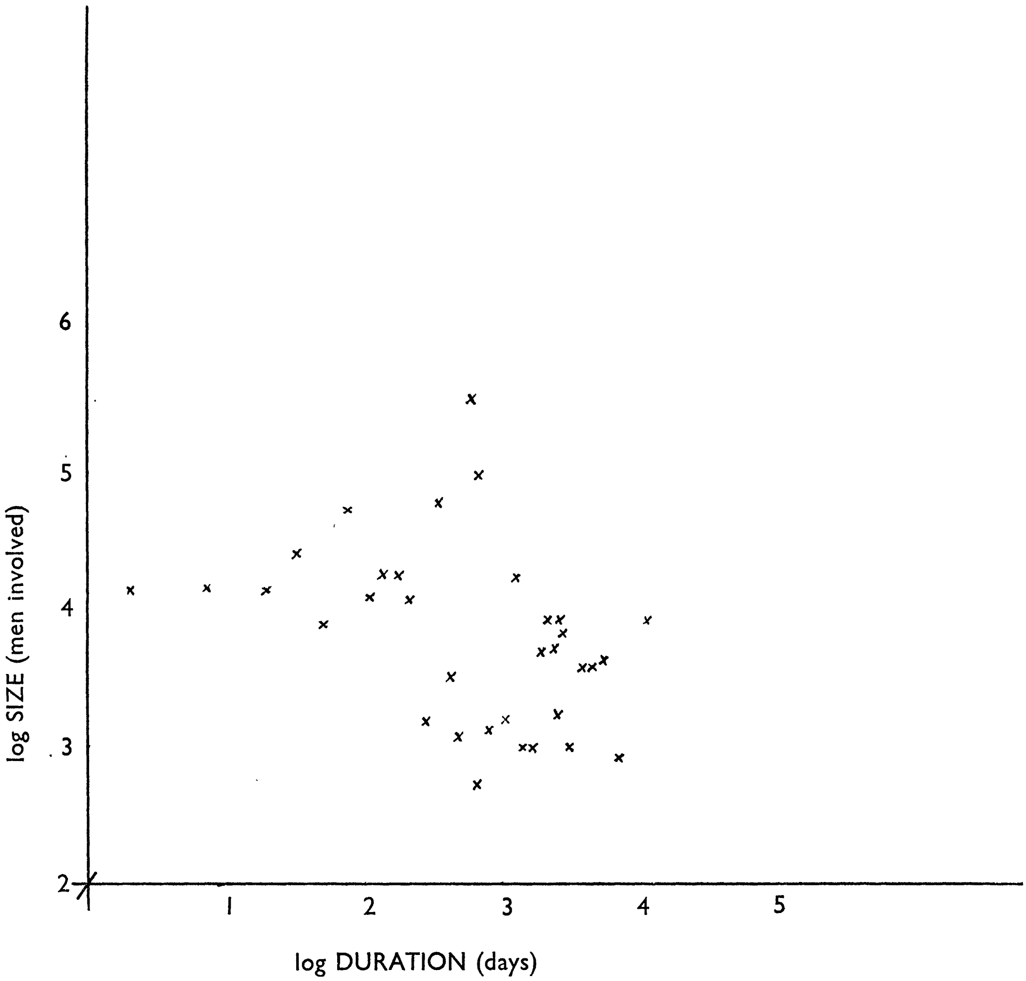

Fig. 3. Size and duration in the Construction Industry. Slope coefficient: $-0{\cdot}154$. Standard error: $0{\cdot}069$.

A second consequence of this recording rule is that we must work with a duration frequency distribution truncated on the left at the point $T = 1$ day. A complete count of the stoppages lasting less than a day does not appear to be available and indeed when considering very short stoppages one comes upon a problem of definition, a point that is taken up in Section 6 of this paper.

4. TESTS OF THE MODEL

The problem is to compare the predicted theoretical duration distribution with the observations. The observations consist of the cumulative distribution subject to a presumed under-recording of strikes lasting less than a day.

We cope with this difficulty by working with a duration distribution truncated at the point $T = 1$ day. The observations arise in grouped form so that all stoppages lasting more than 1 day but not more than 2 days form the first group, those lasting more than 2 days but not more than 3 days form the second group, and so on. In addition there are in the ministry list stoppages recorded as lasting *exactly*† 1 day—these were discarded in forming the truncated sample cumulative distribution. That is, they were treated as forming part of an incompletely observed first group, namely the stoppages lasting not more than 1 day.

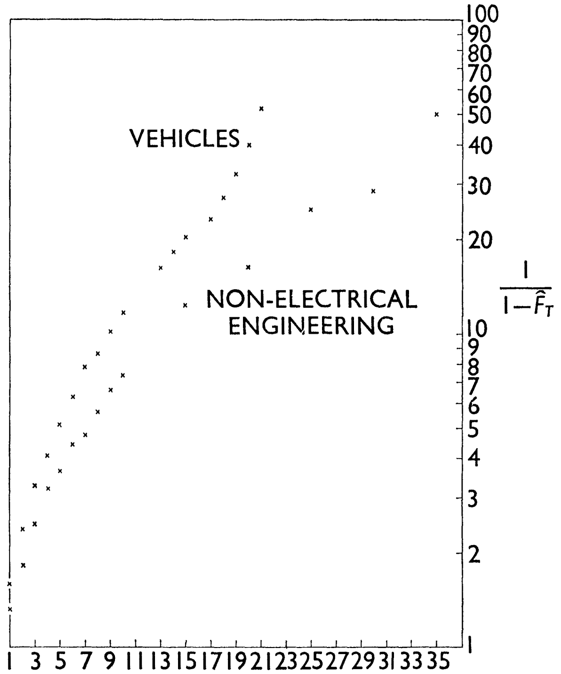

Fig. 4. $-\log(1-\hat{F}_T)$, where $\hat{F}_T$ is the observed truncated cumulative distribution.

Thus if F is the Inverse Gaussian distribution function we shall work with an assumed population model having distribution function, say, $F_T = \{F - F(1)\}/\{1 - F(1)\}$. This truncated distribution has the same hazard function as F itself and so a preliminary graphical test of the model is to inspect the sample truncated cumulative distribution, say $\hat{F}_T$. A smooth curve drawn through a plot of $-\log(1-\hat{F}_T)$ against T should have a slope which provides an estimate of the hazard function of F_T, equivalently of F. That is, it should have the typical shape of the Inverse Gaussian hazard described earlier. Two examples of such plots are given in Fig. 4. The general impression is not inconsistent with the theoretical distributions in that the slope tends to diminish, at least from somewhere in the neighbourhood of 1 day duration.

† Recall that the ministry rule is to disregard stoppages lasting *less than* 1 day.

It was therefore felt worth while to attempt a numerical measure of the goodness of fit of the model. Accordingly a programme was written to calculate Maximum Likelihood estimates of μ and σ from the observed grouped duration distribution truncated at 1 day. The programme is an iterative one which requires starting values for μ and σ. An aid in the choice of these values is the observed mean duration which, in view of the truncation, can be reasonably presumed to exceed the population mean $1/\mu$. Secondly the observed slope of the plots of $\ln(1/(1-\hat{F}_T))$ for large T provides a guide to the ratio μ/σ since on our theory this slope is tending to the limit $\mu^2/2\sigma^2$ from above.

The results of these Maximum Likelihood calculations in the form of point estimates of the parameters μ and σ and a comparison of observed and predicted frequencies in the grouped truncated duration distributions for the eight broad industry groups are presented in Table 1.

The fit of the Inverse Gaussian appears, overall, to be good, and in some industries such as Vehicles and Shipbuilding is remarkably close. In only one case, that of Electrical Machinery, is the value of χ^2 significant at the 10 per cent level. The major contribution to χ^2 here comes from the peculiar numbers of stoppages of 3 and 4 days' duration. The number of strikes observed in this group is relatively small. The prediction errors do not appear to have a systematic character in the sense that the predicted number of strikes of some particular duration is almost always too small, for example, and in addition the Inverse Gaussian form appears to cope well with the long tail of the sample distributions.

The estimates of the drift parameter μ varies between about 0·13 and 0·25 apart from Transport with its value of 0·991. The implicit mean durations range from about 4 to 8 days again apart from Transport with a mean duration of about 1 day.

The estimates of the parameter σ vary between 0·61 and 0·81 apart from much higher values in two of the eight groups.† The relative similarity of the estimates of μ and σ in groups 1 to 6 suggests that the observations from these six groups could be usefully pooled. The results of calculations made on this pooled data are listed in Table 2.

The fit of the Inverse Gaussian to these pooled data is still very close. The evidence suggests that the strikes of these six industries could reasonably be thought of as a random sample from the same Inverse Gaussian distribution. In terms of our model these results suggest that these industries are very similar in regard to the systematic factors which, we suppose, determine the mean rate of progress towards the settlement of a strike and the variation around this trend.

Another feature of the result is the association between the estimates of μ and σ. Over all eight sectors the Spearman rank correlation of $\hat{\mu}$ and $\hat{\sigma}$ is 0·917 which differs significantly from zero at the 1 per cent level on a two-tailed test. A graphical examination of the plot of $\hat{\mu}$ against $\hat{\sigma}$ is not strongly inconsistent with the hypothesis $\sigma/\mu = \text{constant}$. Were this to be so it would indicate that the difference index processes in the eight sectors differ only by a multiplicative constant. This in turn could be explained by the scaling by which we placed the absorbing-agreement-barrier at unity in each industry. Constancy of the ratio of σ to μ after scaling to place the

† These values of σ imply a hazard decreasing from some T no greater than about 1·5 days. Thus in view of the truncation at the point 1 day and the grouping into 1-day intervals we could not expect to observe the initially increasing portion of the hazard in our initial graphical inspection of the data! (See Fig. 4.)

TABLE 1

Observed and predicted numbers of strikes by duration in eight industry groups

1. *Metal Manufacture*

Duration	*Observed*	*Predicted*
2	43	47·1
3	37	30·1
4	21	21·0
5	19	15·3
6	11	11·6
7	8	9·2
8	8	7·4
9	9	6·1
10	3	5·1
11–15	16	16·7
16–20	4	9·1
21–25	4	5·6
26–30	3	3·7
31–40	3	4·3
41–50	5	2·3
>50	4	3·7
	198	

$\hat{\mu} = 0{\cdot}137$, $\hat{\sigma} = 0{\cdot}612$
$\hat{\sigma}/\hat{\mu} = 4{\cdot}47$, $\chi^2_{13} = 12{\cdot}4$

2. *Non-electrical Engineering*

Duration	*Observed*	*Predicted*
2	41	43·3
3	28	24·9
4	18	16·1
5	8	11·3
6	9	8·3
7	3	6·4
8	7	5·1
9	5	4·1
10	3	3·4
11–15	11	10·6
16–20	4	5·4
21–25	4	3·1
>25	8	6·0
	149	

$\hat{\mu} = 0{\cdot}197$, $\hat{\sigma} = 0{\cdot}721$
$\hat{\sigma}/\hat{\mu} = 3{\cdot}66$, $\chi^2_{10} = 5{\cdot}8$

3. *Distributive Trades*

Duration	*Observed*	*Predicted*
2	14	16·7
3	13	9·6
4	4	6·2
5	6	4·3
>5	17	17·3
	54	

$\hat{\mu} = 0{\cdot}231$, $\hat{\sigma} = 0{\cdot}697$
$\hat{\sigma}/\hat{\mu} = 3{\cdot}02$, $\chi^2_2 = 3{\cdot}1$

4. *Vehicles*

Duration	*Observed*	*Predicted*
2	34	34·0
3	19	18·2
4	10	11·3
5	8	7·7
6	6	5·6
7	5	4·2
8	2	3·3
9	3	2·6
10	2	2·1
11–15	6	6·5
16–20	4	3·0
>20	4	4·3
	103	

$\hat{\mu} = 0{\cdot}255$, $\hat{\sigma} = 0{\cdot}811$
$\hat{\sigma}/\hat{\mu} = 3{\cdot}18$, $\chi^2_9 = 1{\cdot}3$

TABLE 1 (*cont.*)

Duration	5. *Construction* *Observed*	*Predicted*	*Duration*	6. *Shipbuilding* *Observed*	*Predicted*
2	44	44·3	2	27	28·2
3	33	34·6	3	19	18·4
4	28	25·3	4	19	12·5
5	23	19·0	5	7	9·0
6	11	14·7	6	4	6·8
7	12	11·7	7	5	5·3
8	9	9·4	8	3	4·2
9	6	7·8	9	1	3·5
10	13	6·5	10	2	2·8
11–15	16	21·0	11–15	11	9·0
16–20	7	11·0	16–20	6	4·6
21–25	6	6·4	>20	8	7·8
26–30	6	4·0			
>30	11	9·2		112	
	225				

$\hat{\mu} = 0{\cdot}134$, $\hat{\sigma} = 0{\cdot}502$, $\hat{\sigma}/\hat{\mu} = 3{\cdot}75$, $\chi^2_{11} = 13{\cdot}1$

$\hat{\mu} = 0{\cdot}165$, $\hat{\sigma} = 0{\cdot}605$, $\hat{\sigma}/\hat{\mu} = 3{\cdot}67$, $\chi^2_9 = 8{\cdot}3$

Duration	7. *Transport* *Observed*	*Predicted*	*Duration*	8. *Electrical Machinery* *Observed*	*Predicted*
2	50	43·0	2	24	23·5
3	19	18·3	3	5	11·6
4	10	10·3	4	16	7·1
5	5	6·7	5	6	4·9
6	2	4·7	6–7	5	6·3
7–20	13	16·5	8–10	6	5·5
>20	3	2·6	>10	10	13·1
	102			72	

$\hat{\mu} = 0{\cdot}988$, $\hat{\sigma} = 2{\cdot}895$, $\hat{\sigma}/\hat{\mu} = 2{\cdot}93$, $\chi^2_4 = 4{\cdot}0$

$\hat{\mu} = 0{\cdot}233$, $\hat{\sigma} = 1{\cdot}093$, $\hat{\sigma}/\hat{\mu} = 4{\cdot}69$, $\chi^2_4 = 16{\cdot}2$

Notes: The duration x days refers to strikes lasting more than $x-1$ days but not more than x days. The subscript to χ^2 refers to the appropriate degrees of freedom. The precise definitions of each industry group are given in the Appendix. Standard errors for $\hat{\mu}$ and $\hat{\sigma}$ have not been computed.

barrier at unity is consistent with a model in which over all sectors the two parameters of the normal distribution of $\Delta X(t)$ are identical but that sectors differ in the position of the agreement barrier.†

TABLE 2

All industry groups apart from transport and electrical machinery

Duration	*Observed*	*Predicted*
2	203	212
3	149	136
4	100	92
5	71	66
6	49	50
7	33	39
8	29	31
9	26	26
10	23	21
11	14	18
12	12	15
13	9	13
14	11	11
15	15	10
16	6	9
17	7	7·7
18	6	6·9
19	4	6·1
20	4	5·5
21–25	17	20·5
26–30	16	12·9
31–35	8	8·6
36–40	8	5·8
41–50	12	7·0
>50	8	8·8
	840	

$\hat{\mu} = 0{\cdot}160, \quad \hat{\sigma}/\hat{\mu} = 3{\cdot}86$

$\hat{\sigma} = 0{\cdot}617, \quad \chi^2_{22} = 17{\cdot}3$

5. STRIKES CLASSIFIED BY BOTH INDUSTRY AND "CAUSE"

The Department classifies stoppages by the "principal issue in dispute" which we shall refer to for brevity as "cause". Where sufficient observations were available the calculations were done with groups of stoppages classified by both industry and "cause". One point of studying more finely classified groups of stoppages, apart from its intrinsic interest, is that such groups may, perhaps, be thought more homogeneous with respect to the systematic determinants of duration. The falling hazard characteristic of the whole industry groups—see Fig. 4—is consistent with a model in which the observations come from several different populations none of which

† It is tempting to develop this point further but to do so would probably place more weight on our hypothetical "difference index" than it can bear, at least for the present.

individually exhibit a falling hazard.† Thus it is of relevance to enquire whether the Inverse Gaussian continues to describe more finely classified data.

Generally the fit of the Inverse Gaussian was slightly better than on the coarser grouping and the evidence suggested that the observations from Industries 1 to 6 could be pooled. We report in Table 3 only the results of the fitting on this pooled data for three separate "causes".

A number of questions naturally occur at this point about the variation in the estimated parameters of the model between different industries and causes. The study of the variation in the fitted model between different types of stoppage and the reasons for such variations as are significant appears, however, to be a lengthy task involving considerable computation and rather than start on it now it was decided to devote a separate paper to the topic reserving the present work simply to an investigation of the fit of the model to duration data.

In the next section we shall attempt to examine the implications of the model for the frequencies of stoppages lasting a day or less.

6. The Short Stoppages

The frequency distributions studied thus far relate to stoppages lasting over a day and we have ignored such data as we have on stoppages lasting a day or less on the hypothesis that, in view of the Department's recording rules, these latter observations are presumably incomplete. However, when we estimate the parameters of a truncated distribution we can use the estimated distribution to give a prediction of the number of stoppages that occurred and lasted a day or less. In fact, if $\hat{F}$ is the estimated distribution function and n is the number of stoppages recorded as lasting over a day then we would naturally predict the total of stoppages of all durations as $n/(1-\hat{F}(1))$ and thus by subtraction we would have a prediction of the number of short strikes. In this section we shall list the predicted numbers of short stoppages for each set of observations to which we have fitted the model and compare these predictions with such information as we possess about the actual frequency.

The principal item of information about the short stoppages is the recorded number of stoppages lasting exactly 1 day—recalling that the Department's rule is to ignore stoppages lasting *less than* a day except where the number of men involved is large. This number can then be assumed to be a lower bound to the total number of stoppages lasting a day or less and we should reasonably expect, if our model does in fact apply to the short stoppages, to find the predicted number of short stoppages generally exceeding the number recorded as lasting exactly a day. In Table 4 we list the predicted number of short stoppages on the basis of the estimates of $\hat{\mu}$ and $\hat{\sigma}$ from the truncated data and, in the column headed "recorded", the number of stoppages recorded as lasting exactly 1 day.

It will be observed that in many cases the predicted number of short stoppages is less than the recorded number of 1-day disputes and in only one instance, the Transport group, does the predicted number substantially exceed the recorded total. The evidence then suggests that the model which fits the distribution of stoppages lasting over a day underestimates the frequency of stoppages lasting a day or less.

† In fact experiments were made with a mixture of two exponential distributions using the grouped truncated duration data. In some cases the fit was relatively good but in no case was it as close as the Inverse Gaussian which has only two parameters compared to the three of the mixed exponential. Experiments were also made with the two and three parameter Weibull distribution—found by Horvath (1968) to describe some U.S. duration data—and with the lognormal distribution, but the fit was generally poor.

There is some other evidence on the frequency of short stoppages in the motor industry and which is referred to in Turner *et al.* (1967). These authors cite figures showing that at one plant in the industry there were 234 stoppages between August 1955 and March 1957 of which only 13 were included in the Department of Employment list; at another plant it is claimed there were 57 stoppages of which 10 were

TABLE 3

Duration of stoppages in connection with disputes over claims for wage increases

Industries 1–6 *pooled*

Duration	*Observed*	*Predicted*
2	88	92·3
3	65	60·5
4	52	51·2
5	26	29·7
6	21	22·4
7	16	17·5
8	15	14·0
9–10	16	21·0
11–15	31	30·4
16–20	13	15·7
21–30	14	14·9
> 30	16	13·3
	373	

$\hat{\mu} = 0{\cdot}159, \quad \hat{\sigma} = 0{\cdot}603$

$\hat{\sigma}/\hat{\mu} = 3{\cdot}79, \quad \chi^2_9 = 6{\cdot}4$

Duration of stoppages in connection with disputes over the employment of particular persons or classes

Industries 1–6 *pooled*

Duration	*Observed*	*Predicted*
2	53	56·0
3	42	37·0
4	22	25·4
5	29	18·5
6	13	14·1
7	10	11·1
8	6	9·0
9	9	7·4
10	4	6·3
11–15	16	20·5
16–20	9	11·2
21–30	11	11·4
> 30	17	13·0
	241	

$\hat{\mu} = 0{\cdot}133, \quad \hat{\sigma} = 0{\cdot}604$

$\hat{\sigma}/\hat{\mu} = 4{\cdot}54, \quad \chi^2_{10} = 12{\cdot}3$

Duration of stoppages in connection with disputes over working arrangements, rules, discipline

Industries 1–6 *pooled*

Duration	*Observed*	*Predicted*
2	43	43·9
3	24	21·6
4	12	13·2
5	8	9·0
6	9	6·6
7	3	5·1
8	5	4·0
9	4	3·3
10	4	2·7
11–15	6	8·6
16–20	4	4·6
21–30	5	4·5
> 30	5	4·9
	132	

$\hat{\mu} = 0{\cdot}249, \quad \hat{\sigma} = 1{\cdot}093$

$\hat{\sigma}/\hat{\mu} = 4{\cdot}39, \quad \chi^2_{10} = 4{\cdot}2$

recorded by the Department and at a third there were some 60 stoppages of which "about a dozen" appear to have been recorded by the Department. It appears likely that the great majority of these unrecorded stoppages were either too small or too short to justify their inclusion in the records. Again, evidence from the motor industry

TABLE 4

Predicted and recorded numbers of short strikes

Data		*Predicted*	*Recorded*
Metal Manufacturing:	Cause I	18	17
	All causes	34	68
Non-electrical Engineering:	Cause I	13	22
	All causes	46	49
Distributive Trades:	All causes	17	38
Vehicles:	Cause I	20	16
	All causes	46	61
Construction:	Cause I	5	9
	Cause II	8	5
	All causes	19	31
Shipbuilding:	Cause I	7	9
	All causes	20	17
Electrical Machinery	All causes	55	31
Transport:	All causes	435	78
Industries 1–6 pooled:	Cause I	64	84
	Cause II	39	54
	Cause III	103	83

Note: Causes I, II and III refer to the "causes" described in the headings of Table 3.

to the Royal Commission on Trades Unions and Employers Associations gave figures for the numbers of stoppages and Turner and colleagues compared these with the number of these stoppages which would be recordable on the Department's criterion. For instance for the first 10 months of 1965 the manufacturers reported 863 stoppages while in the whole year only 96 were recorded by the Department of Employment. Again it would appear that the vast majority of the unrecorded stoppages were both short and small which suggests that our model does in fact seriously underestimate the frequency of short stoppages.

Supposing that there are many more short stoppages than our model predicts why should this be so? The rationale of our model is that the duration of a strike is determined by a process of bargaining and negotiation, of revision of demands and offers until a point of agreement is found. We remarked earlier that there appear to be stoppages whose duration is not determined in this way but which are such that the duration is effectively determined in advance. The obvious hypothesis to advance is that the great majority of the stoppages which last less than a day are of this latter type and in this connection it is relevant to quote Turner on stoppages in the motor industry. "It is known that there are many stoppages which last less than a day or shift. An embracing definition of a strike would include lunch-break shop-floor meetings which fortuitously overrun into working hours, as well as full-scale walkouts of several hundred men who go home an hour or so before the end of a normal shift (but who return the next day). In between there are the many 'downers', or mid-shift stoppages, which may involve a department or section of several scores of men and last for several hours. These are a normal feature of the day-to-day conduct of industrial relations at most of the car plants, deliberately undertaken for aggressive or defensive reasons, to express protests, back up claims, achieve action, demonstrate the genuineness of a grievance, and so on. The call in these cases is still 'all out'—but not out of the factory gates, only out into the yard or nearest open space to await immediate developments. It is 'downers' in particular which are least likely to become public knowledge." These remarks would appear to support the view that many of the short stoppages have duration determined in a different way from stoppages that last over a day—that in fact they are often in the nature of protests, demonstrations or threats whose duration is effectively decided in advance.

But the view that many stoppages are different in character from those that last over a day is mere guesswork at the moment. To test this view and indeed to make it more precise would appear to demand a detailed study of all the stoppages in some plant or industry over a period of time and we shall leave the question at this point.

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APPENDIX

Definition of Industry Groups in terms of Minimum List Heading for Standard Industrial Classifications of 1965.

Group	*M L H*
1. Metal Manufacturing	311–322, 383–399
2. Non-electrical Engineering	331–352
3. Distributive Trades and Services	810–906
4. Vehicles	381–382
5. Construction	500
6. Shipbuilding and Marine Engineering	371–372
7. Transport	700–709
8. Electrical Machinery	361–369