An Economic Index of Riskiness*

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Abstract

Define the riskiness of a gamble as the reciprocal of the absolute risk aversion (ARA) of an individual with constant ARA who is indifferent between taking and not taking that gamble. We characterize this index by axioms, chief among them a “duality” axiom which, roughly speaking, asserts that less risk-averse individuals accept riskier gambles. The index is positively homogeneous, continuous, and subadditive, respects first and second order stochastic dominance, and for normally distributed gambles, is half of variance/mean. Examples are calculated, additional properties derived, and the index is compared with others.

JEL classification: C00, C43, D00, D80, D81, E44, G00.

Keywords: riskiness; risk aversion; expected utility; decision making under uncertainty; portfolio choice; Sharpe ratio; value at risk; coherent measures of risk.

1 Introduction

On March 21, 2004, an article on the front page of the New York Times presented a picture of allegedly questionable practices in some state-run pension funds. Among the allegations were that these funds often make unduly risky investments, recommended by consultants who are interested parties. The concept of “risky investment” is commonplace in financial discussions, and seems to have clear conceptual content. But when one thinks about it carefully and tries to

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