This article was published in Journal of International and Comparative Economics 4, (1995), 223-241.

Decentralized Markets with Pairwise Meetings:
Recent Developments *

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Abstract

We discuss recent developments in the economics of bargaining and search in markets, and their relation to the older literature on centralized (planned) and decentralized markets. In particular, we address questions such as delay in trade, equilibrium price distributions, transmission of information, and efficiency. This new line of research may improve our understanding of how markets work, and may be of help in the design of trading institutions.

1 Planned and Non-Planned Economies.

In a planned economy the planner, sometimes referred to as the Central Planning Board (CPB), is responsible for the allocation of productive resources, the determination of the levels of pro-

*This work is an outgrowth of a paper presented at the Brown-Columbia-Johns Hopkins annual conference, Johns Hopkins University, November 1992. We thank Graciela Chichilnisky, Duncan Foley, Joe Harrington, Andreu Mas-Colell, Jerome Stein, and Shinji Yamashige for helpful comments and discussions.
duction of final goods, and the distribution of the goods among consumers. It has long been
recognized that in order to perform this function, the CPB needs to have a considerable amount
of information (endowments, technologies, tastes). Also, the computational burden on the CPB
is heavy.¹

By contrast, in a Walrasian economy in equilibrium, information is completely decentralized.
Each consumer knows his endowment and tastes, and each producer knows his technology. Com-
putations are decentralized as well. Agents take the equilibrium prices as given, and compute
their personal supplies and demands. All markets then clear, by construction of the equilibrium.²

Under well known conditions the Walrasian equilibrium is Pareto efficient. Under these
conditions, though, the CPB can induce the outcome (allocation of resources, production, dis-
tribution) achieved by the Walrasian economy. Hence, goes the argument, the performance of
a planned economy is not inferior to that of a Walrasian economy.³ Moreover, when there are
externalities, the planned economy does better. To this, of course, one can respond in defence
of the Walrasian paradigm, by invoking Coasian contracts, which under certain conditions, are
able to correct the distortions created by the externalities. Finally, as noted above, one may be
skeptical about the ability of the CPB to solve the “hundreds of thousands” of equations neces-
sary for achieving the optimal plan,⁴ and about the presumption that the CPB possesses all the
necessary information to do so. Much of the literature on economic planning is concerned with
the development of iterative processes which converge to the optimal plan, but require much less
sophistication, informational and computational, on the part of the CPB.⁵ In these processes
the CPB typically elicits information from economic agents. It should be noted that agents are
assumed to report the truth.

Work on planning processes and whether they lead to Walrasian outcomes has been carried
out, to the best of our knowledge, entirely in deterministic frameworks. The analysis can be
extended without difficulty to environments with uncertainty. If all agents and the planner
agree on the probability distributions of the random variables in the economy, then the planner
can induce the outcome that would be achieved in a Walrasian economy with a complete set of
Arrow-Debreu securities. Analogous arguments can then be made regarding Pareto efficiency,
externalities, informational requirements, and computational burden. This is true both for
aggregate uncertainty and for idiosyncratic uncertainty.⁶

¹Hayek (1935), Heal (1973), Malinvaud (1985, Chapter 8), and Vohra (1987).
²Hayek (1945)
³This argument goes back at least to Lange (1936-7).
⁴Hayek (1935, p.212).
⁵Heal (1973), Malinvaud (1985, Chapter 8), and Vohra (1987).
⁶Aggregate uncertainty is often referred to as common values uncertainty, whereas idiosyncratic uncertainty
corresponds to private values uncertainty, in the ex-ante stage, that is before agents have learned any private
When agents have private information which is not known by the planner and perhaps by other agents, the actions of the agents may reveal some of their private information. The literatures on implementation of social choice correspondences and on mechanism design have explicitly recognized the possibility of misrepresentation of private information on the part of agents.\textsuperscript{7} This literature can be viewed as dealing with planning subject to additional constraints, namely incentive compatibility requirements.

The actions taken by some agents may reveal their private information to other interested agents. The rational expectations equilibrium literature emphasizes this common values aspect of the problem.\textsuperscript{8} Agents know the equilibrium price function, which maps states of the world into prices. As in a Walrasian model, they know the market clearing price. Therefore, in equilibrium, agents can infer from this price (some of) the private information of other agents.

As long as we are not concerned with the question where do the equilibrium prices (or price functions) come from, Walrasian economies are indeed quite different from planned economies. However, once we recognize that the equilibrium prices must be computed by some central entity, the auctioneer, the resemblance of a Walrasian economy to a planned economy becomes apparent. In fact, the Walrasian auctioneer is nothing more than a CPB which uses a particular computational process. Instead of issuing commands to economic agents regarding how much to produce and to consume, the auctioneer calls the equilibrium prices to make markets clear. The similarity of Walrasian economies to planned economies is preserved “out of equilibrium” as well. Walrasian \textit{tâtonnement} closely resembles the above mentioned iterative processes used by the CPB.

Thus, Walrasian economies, often referred to as non-planned or decentralized economies, have many common features with planned economies. It is not surprising that economists have searched for other, less centralized, models of trade in large markets.

\section{A Decentralized Model: Trade in Pairwise Meetings}

In the literature on imperfect competition in general equilibrium, the central market agency plays a more limited role than in Walrasian economies.\textsuperscript{9} As an example, consider an economy where some but not all industries are perfectly competitive. The auctioneer calls prices for the goods produced in the competitive industries. Producers of goods in the imperfectly competitive industries set prices non-cooperatively. Consumers take all prices as given, and equilibrium is

\textsuperscript{7}Hurwicz (1973), Maskin (1985), Myerson (1989).
\textsuperscript{9}This literature is surveyed in Mas-Colell (1982), Hart (1985), and Bonanno (1990).
achieved when all markets clear. If all industries are imperfectly competitive, equilibrium can be achieved without an auctioneer.

Cooperative game theory has studied the outcomes that arise from the interaction of coalitions of agents. Notable examples are the core and the value of an exchange economy. Here the implicit role played by the planner is to enforce the agreements reached by the various coalitions. The well known core and value equivalence theorems establish the links between the cooperative approach and the Walrasian paradigm.\(^\text{10}\)

We turn to markets with pairwise meetings, which are the focus of this paper.\(^\text{11}\) In such markets prices are not centrally called. They are determined as the outcome of bilateral bargaining between traders. Traders engage in a process of search in the hope of getting the best possible deal. This, not surprisingly, entails coordination failures and delay, and hence - since delay is costly - inefficiency. A relevant exercise is to study the behavior of the equilibria as the cost of delay becomes very small, that is as the market becomes almost frictionless. The question is whether the performance of an almost frictionless decentralized market approximates the performance of a market with an auctioneer or of a market governed by a CPB.

We shall begin by studying a pairwise meetings market with symmetric information, under certainty and under uncertainty. The model is an adaptation of Wolinsky (1990) to a setting where all traders in the market are fully informed regarding the state of the world (e.g. the quality of the good being transacted) in the case of certainty, and equally uninformed in the case of uncertainty (i.e. all traders have the same beliefs regarding the state of the world). We conclude this section with a survey of earlier results in the literature on pairwise meetings markets with symmetric information. Section 3 is devoted to the presentation of results concerning markets with asymmetric information.

**A model: Symmetric information with no uncertainty.** Time runs discretely from \(-\infty\) to \(\infty\). All periods are identical. In the beginning of each period \(M\) sellers and \(M\) buyers enter the market. Let \(M\) be a large number. Sellers want to sell one unit of an indivisible good, and buyers want to buy one unit of it. The valuation of the good to the buyers is \(u\), and the cost of the good to the sellers is \(c\). We assume that there are gains from trade, namely \(c < u\).

Each period sellers and buyers are randomly matched. Each meeting results in an agreement

\(^{10}\)Aumann and Shapley (1974), Mas-Colell (1977), Mas-Colell (1985, Chapter 7). This literature has not, to the best of our knowledge, explored the implications for the equivalence principle of the presence of uncertainty or asymmetric information. Notable attempts to extend core analysis to such settings are Wilson (1978), Yannelis (1991) and Allen (1992), although the connection with market paradigms has not yet been studied.

\(^{11}\)We do not use the term “pairwise meetings economies” as, except for Gale (1986), the existing models in this field study a market for a single good.
or a disagreement. Those who agree transact and exit the market, and those who disagree stay in the market to be matched anew. Thus, the number of buyers in the market is always the same as the number of sellers.

When matched, traders make simultaneous announcements. Each trader can send one of two possible messages: $h$ or $l$. If both traders say $h$ they trade at price $p^{hh}$. If both say $l$ they trade at price $p^{ll}$. If the buyer says $h$ and the seller $l$ they trade at price $p^{hl}$. Finally, if the seller says $h$ and the buyer $l$ there is disagreement. The payoff to perpetual disagreement is zero. It is convenient to refer to messages as “tough” ($h$ for the seller and $l$ for the buyer) and “soft” ($l$ for the seller and $h$ for the buyer). It is assumed that $c < p^{ll} < p^{hl} < p^{hh} < u$.

All traders discount future payoffs by a constant factor $\delta \in (0,1)$.

Note that disagreement occurs only if both buyer and seller play “tough.” Thus, if a trader plays “soft” in some period, he will surely transact and leave the market. It follows that the only relevant decision variable for a trader is the number of periods during which he will play “tough.” Let $n_S$ and $n_B$ be these numbers for sellers and buyers.

Denote by $S$ and $B$ the proportions of sellers and buyers in the market who play “tough.” These proportions are known to all traders. Let $K$ be the total number of sellers (and therefore of buyers) in the market. The market is said to be in steady state when these three numbers are constant through time.

The market is said to be in equilibrium if each trader maximizes his expected payoff and the market is in steady state. An equilibrium is fully described by the numbers $n_S, n_B, S, B, \text{ and } K$. The equilibria of the model are determined by the following equations:

$$M = K(1 - SB),$$  \hfill (2)

$$K(1 - S) = MB^{n_S},$$  \hfill (3)

$$K(1 - B) = MS^{n_B}.$$  \hfill (4)

Equation (2) is the steady state condition for the market size - the number of entering buyers (or sellers) $M$, is equal to the number of meetings which result in trade. The other equations are the stationarity conditions for the proportions of “tough” buyers and “tough” sellers. They can be understood as follows. Consider (3). The left hand side is the number of soft sellers in the market. Any seller who has played “soft” in previous periods has already traded and left the market. Therefore these sellers must be those among the $M$ sellers who entered the market $n_S$ periods ago, and were unlucky enough to meet “tough” buyers $n_S$ times. They have now
switched to “soft,” as planned. Similarly for (4). Together with two best response conditions this is a system of five equations in five unknowns.

The (expected) payoff to a buyer is

\[ V_B = \delta^{n_B} S^{n_B} [S(u - p_{hh}) + (1 - S)(u - p_{hl})] \]

\[ + \sum_{t=0}^{n_B-1} S^t(1 - S)\delta^t(u - p^l) \]

\[ = \delta^{n_B} S^{n_B} [S(u - p_{hh}) + (1 - S)(u - p_{hl})] \]

\[ + \sum_{t=0}^{n_B-1} S^t(1 - S)\delta^t(u - p^l)(1 - \delta^{n_B} S^{n_B}). \]  

(5)

The first term is the expected gain if the buyer encounters “tough” sellers \( n_B \) times, an event which happens with probability \( S^{n_B} \). The buyer then switches to “soft” and trades with a “tough” or with a “soft” seller. The second term is the discounted expected gain from finding a “soft” seller in one of the \( n_B \) periods in which the buyer is playing “tough.”

Similarly, the payoff to a seller is

\[ V_S = \delta^{n_S} B^{n_S} [B(p^l - c) + (1 - B)(p_{hl} - c)] \]

\[ + \sum_{t=0}^{n_S-1} B^t(1 - B)\delta^t(p^l - c)(1 - \delta^{n_S} B^{n_S}). \]  

(6)

There are three equilibrium configurations:

**Configuration 1.** All sellers are “tough,” \( S = 1 \). The best response of buyers is to play “soft,” as playing “tough” for any number of periods will only induce costly delay. Thus, \( n_B = 0 \) and \( B = 0 \). From (2) it follows that \( K = M \). Sellers are indifferent between any \( n_S \in \{1, \ldots, \infty\} \).

In this equilibrium traders spend exactly one period in the market, transacting at the price \( p_{hh} \).

Therefore, sellers never switch from “tough” to “soft.”

**Configuration 2.** This configuration is symmetric to configuration 1: \( B = 1, n_S = 0, S = 0, K = M, \) and buyers are indifferent between any \( n_B \in \{1, \ldots, \infty\} \). Trade takes place at the price \( p_{hl} \), with no delay.

**Configuration 3.** Sellers are indifferent between any \( n_S \in \{0, \ldots, \infty\} \), and buyers are indifferent between any \( n_B \in \{0, \ldots, \infty\} \). Thus, \( V_S \) and \( V_B \) are flat. Equating the first differences of \( V_S \) and \( V_B \) to zero, we get the following equations:

\[ \Delta V_B = \delta(p^l - p_{hl})S^2 + [\delta(u - p_{hl}) - p^l + p_{hh} - u + p^l]S + p^l - p_{hl} = 0. \]  

(7)

\[ \Delta V_S = \delta(p^l - p_{hl})B^2 + [\delta(p^l - c) + p^l - p_{hh} + c - p^l]B + p^l - p_{hl} = 0. \]  

(8)

Denote by \( S(\delta) \) the positive root of equation (7) and by \( B(\delta) \) the positive root of equation (8).\(^{12}\)

In this configuration \( S = S(\delta), B = B(\delta), \) and therefore \( K = M/[1 - S(\delta)B(\delta)] \). The strategies \( n_B \) and \( n_S \) are determined by (3) and (4). Many transactions occur with delay.

\(^{12}\)The other roots are negative and hence not relevant for the model.
As explained above, we are interested in the asymptotic behavior of the model, as the market becomes almost frictionless, namely as the common discount factor \( \delta \) approaches 1. We shall now establish some facts which hold along any sequence of configuration 3 equilibria as \( \delta \to 1 \):

(a) \( \lim_{\delta \to 1} B(\delta) = \lim_{\delta \to 1} S(\delta) = 1 \). This follows from the continuity of \( B(\delta) \) and \( S(\delta) \), and the fact that \( B(1) = S(1) = 1 \). Most of the traders on both sides of the market are playing “tough,” searching.

(b) \( \lim_{\delta \to 1} S'(\delta) = \frac{u - c}{p_{hh} - p_{ll}} > 0; \lim_{\delta \to 1} B'(\delta) = \frac{p_{ll} - c}{p_{hh} - p_{ll}} > 0 \). This is obtained by differentiating \( \Delta V_B = 0 \) and \( \Delta V_S = 0 \) ((7) and (8)) and taking limits.

(c) \( \lim_{\delta \to 1} n_B = \lim_{\delta \to 1} n_S = \infty \). This can be seen as follows. Using (2), (4) can be written as \( \frac{1-B(\delta)}{1-S(\delta)B(\delta)} = [S(\delta)]^{\alpha} \). Taking limits, using l'Hôpital’s rule and the fact that \( \lim_{\delta \to 1} B(\delta) = \lim_{\delta \to 1} S(\delta) = 1 \), we get \( \lim_{\delta \to 1} \frac{B'(\delta)}{S'(\delta)+B'(\delta)} = 1^{\lim_{\delta \to 1} n_S} \). By fact (b) above, the left hand side is strictly less than 1, implying that \( n_B \) must approach infinity. Similarly for \( n_S \). Traders enter the market with the intention of playing “tough” (searching) for many periods. As the fraction of pairwise meetings which result in disagreement, \( SB \), approaches 1, traders spend a long time in the market.

For any \( \delta < 1 \), the equilibria in configurations 1 and 2 emulate Walrasian equilibria. In fact, any price in the interval \([c, u]\) could have arisen in a Walrasian equilibrium. By contrast, the equilibrium in configuration 3 is not Walrasian, for two reasons. First, some transactions occur with delay. Second, in any given period there is no single price at which trade takes place. A fraction \((1 - S)(1 - B)\) of trades takes place at the price \( p_{hl} \), a fraction \( S(1 - B) \) at the price \( p_{hh} \), and a fraction \((1 - S)B\) at the price \( p_{ll} \).

The allocations in equilibrium configurations 1 and 2 could have been chosen by a surplus maximizing planner. The individual payoffs \( V_B \) and \( V_S \) add up to the total (per pair) surplus \( u - c \). In configuration 3 this is not the case. Some of the social surplus is foregone as a result of the delay. This allocation would not have been selected by a surplus maximizing planner.

As was noted above, it is not surprising to find inefficiencies in a decentralized market with frictions. In this case the friction is a positive search cost driven by impatience (\( \delta < 1 \)). We want to know whether the inefficiency becomes negligible when frictions are almost entirely removed, that is when \( \delta \to 1 \). We shall show that the answer to this question is negative, saying that equilibrium configuration 3 is not asymptotically efficient. In this sense, an almost frictionless decentralized pairwise meetings economy does not approximate a centrally planned economy, nor of course a Walrasian economy. We prove that the equilibrium in configuration 3 is not
asymptotically efficient by computing the following limit:

\[
\lim_{\delta \to 1}(V_B + V_S) = \lim_{\delta \to 1}(\delta^n B^n S^n B^n)(u - p^k) + \lim_{\delta \to 1}(\delta^n B^n S^n B^n)(u - p^l)[1 - \lim_{\delta \to 1}(\delta^n B^n S^n B^n)] \\
+ \lim_{\delta \to 1}(\delta^n B^n S^n B^n)(p^h - c) + \lim_{\delta \to 1}(\delta^n B^n S^n B^n)[1 - \lim_{\delta \to 1}(\delta^n B^n S^n B^n)] \\
= \lim_{\delta \to 1}(\delta^n B^n S^n B^n)(u - p^h) + (u - p^h)[1 - \lim_{\delta \to 1}(\delta^n B^n S^n B^n)] \\
+ \lim_{\delta \to 1}(\delta^n B^n S^n B^n)(p^l - c) + (p^l - c)[1 - \lim_{\delta \to 1}(\delta^n B^n S^n B^n)] \\
= u - p^h + p^l - c \\
< u - c.
\]

(9)

It is also interesting to check whether the absence of a single trading price persists as \(\delta \to 1\). The number of pairwise meetings which result in trade in a given period is \(K[1 - SB]\). The number of meetings which result in trade at price \(p^h\) is \(K(1 - B)S\). The fraction of transactions taking place at this price is \(\frac{K(1 - B)S}{K[1 - SB]}\). We then have \(\lim_{\delta \to 1} \frac{K(1 - B)S}{K[1 - SB]} = \frac{\lim_{\delta \to 1} B'(\delta) + \lim_{\delta \to 1} S'(\delta)}{\lim_{\delta \to 1} B'(\delta) + \lim_{\delta \to 1} S'(\delta)}\), which is a constant that we can compute. Similarly, the fraction of transactions taking place at the price \(p^l\) is, in the limit, \(\frac{\lim_{\delta \to 1} S'(\delta)}{\lim_{\delta \to 1} B'(\delta) + \lim_{\delta \to 1} S'(\delta)}\), implying that the fraction of transactions taking place at the price \(p^l\) must approach zero. In the (almost) frictionless market trade at more than one price persists.

**Symmetric information with uncertainty.** Suppose there are two states of the world, \(L\) and \(H\). The state of the world is chosen by nature once and for all in the beginning of time. Traders do not know the state chosen by nature. They all believe that the probability of the state being \(H\) is \(\alpha_H \in (0, 1)\). Once trade occurs, the state of the world is revealed to them. As each trader chooses the same actions whether the state is \(L\) or \(H\), the proportions of “tough” announcements \(S\) and \(B\), and market size \(K\) are not indexed by the state of the world.

In state \(L\) the cost of the good to the seller is \(c_L\) and the valuation to the buyer is \(u_L\). In state \(H\) the cost and the valuation are \(c_H\) and \(u_H\). The following assumptions are made regarding costs, valuations, and prices:

\[
\begin{align*}
&c_L < p^l < u_L < p^h < c_H < p^h < u_H, \quad (10) \\
&(1 - \alpha_H) c_L + \alpha_H c_H < p^l, \quad (11) \\
&(1 - \alpha_H) u_L + \alpha_H u_H > p^h. \quad (12)
\end{align*}
\]

The meaning of assumptions (11) and (12) is that ex-ante there are gains from trade for buyers and sellers at any of the three possible trading prices. Assumption (10) implies that ex-post there are gains from trade to both parties in state \(L\) if the price is \(p^l\), but not otherwise, and similarly for \(p^h\) in state \(H\).
Let \( u = (1 - \alpha_H)u_L + \alpha_H u_H \). Then the payoff to a buyer is

\[
V_B = \alpha_H \delta^{n_B} S^{n_B} [S(u_H - p^{hh}) + (1 - S)(u_H - p^{hl})] \\
+ \alpha_H \frac{1 - S}{1 - \delta^{n_B}} (u_H - p^{hl})(1 - \delta^{n_B} S^{n_B}) \\
+ (1 - \alpha_H) \delta^{n_B} S^{n_B} [S(u_L - p^{hh}) + (1 - S)(u_L - p^{hl})] \\
+ (1 - \alpha_H) \frac{1 - S}{1 - \delta^{n_B}} (u_L - p^{hl})(1 - \delta^{n_B} S^{n_B}) \\
= \delta^{n_B} S^{n_B} [S(u - p^{hh}) + (1 - S)(u - p^{hl})] \\
+ \frac{1 - S}{1 - \delta^{n_B}} (u - p^{hl})(1 - \delta^{n_B} S^{n_B}),
\]

which takes exactly the same form as the payoff in (5). Similarly for the seller, with \( c = (1 - \alpha_H)c_L + \alpha_H c_H \).

The stationarity equations are the same as in the case with no uncertainty. Therefore the analysis is identical. The only difference is that individual and social welfare are measured in ex-ante terms. Unlike the certainty case, there will be a non-negligible fraction of the population trading at a price which is not ex-post individually rational. This feature of the model will play a key role when we introduce asymmetric information.

**Earlier work.** In early models of decentralized markets with pairwise meetings the outcome of the bargaining stage was determined by the Nash bargaining solution. The classic references are Diamond and Maskin (1979), Diamond (1981), and Mortensen (1982). The next wave of research incorporated strategic bargaining between buyers and sellers. The major results for markets with symmetric information and no uncertainty are due to Rubinstein and Wolinsky (1985) and to Gale (1986, 1987). See Osborne and Rubinstein (1990), where other important work in this area is surveyed, as well as Peters (1991, 1992) and Muthoo (1993).

Rubinstein and Wolinsky model the bargaining process as an alternating-offer procedure with random proposer. Offers take the form of any split of the gains from trade. Agents bargain with their trading partner. Agreement entails trade and exit. In the case of disagreement, with some probability the traders will remain matched and will bargain for one more round, and with some probability they will leave the match and return to the pool of agents in the market. Some of these traders will find another match, whereas others will remain unmatched, and so on in every period. The initial measure of buyers in the market is larger than that of sellers, whereas the measures of entering buyers and sellers are equal. As attention is restricted to steady states (the number of entering traders equals the number of exiting traders), the initial abundance of buyers relative to sellers is preserved through time.

Gale (1986) studies a non-stationary market with pairwise meetings, where traders bargain over bundles of commodities. The analysis is carried out “in the limit” \( (\delta = 1) \). The equilibria are shown to be Walrasian. Gale (1987) is more similar to Rubinstein and Wolinsky (1985),
but departs from their model in several significant ways. There are many types of buyers and
sellers in the market. Buyers have graduated valuations and sellers have graduated costs. When
valuations and costs are finely graduated, a market with continuous demand and supply curves
with a unique Walrasian equilibrium, is approximated. For a market with a finite number of
traders Gale generates a sequence of equilibria which converges, as frictions are removed, to the
Walrasian equilibrium.

Then Gale turns to the analysis of a stationary market. He points out that Rubinstein and
Wolinsky focus on the stock of traders in the market. As in steady state the number of buyers
in the market exceeds the number of sellers, the resulting Walrasian price in their model is (in
our notation) $u$, whereas the sequence of equilibria of the pairwise meetings market converges,
as $\delta \to 1$, to a price which is strictly lower than $u$. Gale proposes to focus on the flow of traders
to the market. In the Rubinstein and Wolinsky model there is an equal number of buyers and
sellers, all with the same valuations and costs, entering the market each period. Any price
between the valuation and the cost is Walrasian according to this definition. In Gale’s model,
though, due to the graduated costs and valuations, the Walrasian flow equilibrium price is well
defined. Gale shows that the equilibria of the pairwise meetings market converge to this price as
frictions are removed. The Wolinsky (1990) version of the model presented above takes a neutral
position on this issue, as any price in $[c, u]$ is Walrasian, both in the stock and the flow sense.\textsuperscript{13}

### 3 Symmetric Information

Let there be two states of the world as above. Suppose that a fraction $x_B \in [0, 1]$ of the buyers
and a fraction $x_S \in [0, 1]$ of the sellers know the state of the world upon entering the market. The
rest of the traders are uninformed, and have a common prior $\alpha_H$. If $x_B \in [0, 1]$ and $x_S \in [0, 1)$
the information structure is two sided. If $x_B = 1$ or $x_S = 1$ it is one sided. Note that the cases
$x_B = x_S = 1$ and $x_B = x_S = 0$ are precisely the cases studied in the previous section. In this
section we rule them out.

The appropriate Walrasian notion of equilibrium for such a market is a rational expectations
equilibrium, where prices convey information to uninformed traders and, at the same time, clear
markets. The following is a fully revealing rational expectations equilibrium (FRREE) for this
market. Consider the price function: $p^L$ if the state of the world is $L$, $p^{hh}$ if the state of the
world is $H$. The price clearly reveals the state of the world to the uninformed. Once the state is
revealed all agents are willing to trade in the respective states of the world at the above prices,
so the market clears.

\textsuperscript{13}If costs and valuations were graduated, Gale’s point would, of course, become relevant.
We turn to the relevant notion of a planned economy. Suppose the planner does not know the state of the world, and cannot distinguish between informed and uninformed traders. Due to his limited information, the planner cannot directly allocate resources in the economy. The best he can do is to design a mechanism (a game form) in which the agents in the economy can choose to participate. Those who participate play according to the rules of the mechanism non-cooperatively. The resulting equilibrium determines the allocation of resources in the economy.

From the Revelation Principle\textsuperscript{14} we know that the planner can achieve any equilibrium allocation of an arbitrary mechanism, by the truth-telling equilibrium of some direct mechanism. We can therefore restrict attention to direct incentive-compatible mechanisms. We say that such a mechanism is interim incentive efficient if there is no other direct incentive-compatible mechanism which Pareto dominates it in the interim sense, that is when agents’ payoffs are evaluated given their private information.\textsuperscript{15} In general, FRREE are not necessarily interim incentive efficient.\textsuperscript{16} In our particular case, the FRREE of (the centralized version of) the Wolinsky model is interim incentive efficient. The proof can be found in Serrano and Yosha (1993b).

Next, we need to characterize the set of equilibria of the pairwise meetings markets, one sided and two sided, and study their asymptotic properties. There are two conceptually distinct questions that need to be addressed in such an analysis. First, one wants to know whether the pairwise meetings market is, in the limit, “Walrasian,” that is whether the set of equilibria converges as $\delta \to 1$ to the FRREE. A different manner to put the question is to ask whether, in the limit, uninformed traders learn from informed traders through the process of trade. There are three forces which are relevant for the learning by the uninformed. Force CL (Cost of Learning): As $\delta \to 1$ it is less costly for the uninformed to remain in the market and learn from meetings with other traders who may be informed. Force MI (Misrepresentation of Information): As $\delta \to 1$ the informative content of the pairwise meeting decreases because it costs less for informed traders to try and extract surplus from the uninformed (e.g. a seller who knows that the valuation and cost of the good are low, will ask for a high price in the hope that the buyer he faces is uninformed and will agree.) Forces CL and MI are present when information is one sided or two sided. When information is two sided, there is an additional force at work. Force N (Noise): As $\delta \to 1$ the informative content of the pairwise meetings decreases because there are more uninformed traders on both sides of the market trying to learn. Force CL works in favor of learning, while forces N and MI work against it. The answer to our question, regarding the learning by the uninformed, depends on the interplay between these forces as $\delta \to 1$.

\textsuperscript{14}See Myerson (1989) and the references therein.  
\textsuperscript{15}Holmström and Myerson (1983).  
\textsuperscript{16}Laffont (1985).
The second question of interest is whether the equilibria of the pairwise meetings market converge in payoffs, as $\delta \to 1$, to interim incentive efficient equilibria of the planned economy. It is conceivable, in principle, that the uninformed traders will learn from informed traders when frictions are made very small, but that in discounted present (expected) value terms they will be worse off than they would be in a FRREE (as a result of the long stay in the market). Similarly, failure by the uninformed to learn, does not necessarily mean that a social inefficiency is involved. In principle, this could simply mean that surplus is distributed to informed traders at the expense of the uninformed.

Wolinsky (1990) and Serrano and Yoshia (1993a) characterize the equilibria of the model for two sided and one sided information, respectively, and address the first question, namely the convergence to a FRREE. Serrano and Yoshia (1993b) address the question of interim incentive efficiency when information is one sided and two sided. We summarize the major findings below.

Let $S^h_L$ be the proportion of informed sellers in the market who play “tough” in state $L$. Similarly for $S^h_H$, $B^L_L$, and $B^H_H$. Denote by $K_L$ and $K_H$ the market size in the two states. Let $n_{SL}$, $n_{SH}$, $n_{BL}$, and $n_{BH}$ denote the strategies of the informed traders, and let $n_S$ and $n_B$ denote the strategies of the uninformed traders. The equilibria of the model are determined by the following equations:

$$M = K_H(1 - S^h_H B^H_H),$$  
Eq. (14)

$$M = K_L(1 - S^h_L B^L_L),$$  
Eq. (15)

$$K_H(1 - S^h_H) = M[x_S(B^H_H)^n_{SH} + (1 - x_S)(B^H_H)^n_{S}],$$  
Eq. (16)

$$K_H(1 - B^H_H) = M[x_B(S^h_H)^n_{BH} + (1 - x_B)(S^h_H)^n_{B}],$$  
Eq. (17)

$$K_L(1 - S^h_L) = M[x_S(B^L_L)^n_{SL} + (1 - x_S)(B^L_L)^n_{S}],$$  
Eq. (18)

$$K_L(1 - B^L_L) = M[x_B(S^h_L)^n_{BL} + (1 - x_B)(S^h_L)^n_{B}].$$  
Eq. (19)

Together with six best response conditions this is a system of twelve equations in twelve unknowns.

When information is one sided, say when all sellers are informed, the variable $n_S$ and the corresponding best response condition are dropped. From now on, whenever we refer to one sided information we have in mind this case. The case where all buyers are informed is symmetric. Also, when information is one sided equations (14) and (17) turn out to be redundant. In order to complete the system equation (17) is replaced by

$$B^H_H = \frac{(1 - x_B)n_B}{(1 - x_B)(n_B + 1) + x_B}.$$  
Eq. (20)
A detailed derivation of the equilibrium conditions is provided in Wolinsky (1990) and in Serrano and Yoshia (1993a).

The analysis is performed in terms of the fraction of transacting uninformed buyers who in state $L$ end up trading at a price which is not ex-post individually rational,

$$f_B = \frac{K_L(1 - B_L)}{M(1 - x_B)}$$

and the fraction of transacting uninformed sellers who in state $H$ end up trading at such a price,

$$f_S = \frac{K_H(1 - S_H)}{M(1 - x_S)}$$

If along a sequence of equilibria, as $\delta \to 1$, we have $\lim_{\delta \to 1} f_B = \lim_{\delta \to 1} f_S = 0$ (or $\lim_{\delta \to 1} f_B = 0$ when information is one sided), we say that the sequence is asymptotically ex-post individually rational, in other words learning occurs in the limit so that almost all traders transact at the FRREE prices.

Suppose information is one sided. For sellers in state $H$ and for informed buyers in state $L$ it is a dominant strategy to play “tough.” Switching to “soft” entails trade at a sure loss. Informed buyers in state $H$ realize that playing “tough” for any number of periods will entail disagreement and delay, and therefore enter the market playing soft. The true tension is between the sellers in state $L$ and the uninformed buyers. Although the cost of the good to the sellers is $c_L$, they have an incentive to play “tough,” in the hope of extracting surplus from uninformed buyers. This, though, entails a loss in the form of potential delay. The uninformed buyers try to gauge market conditions by playing “tough” for a while, in the attempt to learn whether the population of sellers they are facing is “unequivocally tough” (state $H$), or only “partially tough” (state $L$). As $\delta \to 1$ the cost of misrepresentation to sellers and the cost of learning to the uninformed buyers decrease (forces MI and CL). Force N is not present when information is one sided, as there are uninformed traders only on the side of the market where learning occurs. Meetings with traders from the other side of the market are always informative.

In Serrano and Yoshia (1993a) it is shown that when information is one sided, for large enough $\delta$ there are two equilibrium configurations. There is a corner equilibrium where the sellers in state $L$ enter the market playing “soft” ($n_{SL} = 0$). As the behavior of sellers is fully revealing (“tough” in state $H$ and “soft” in state $L$), the best response for an uninformed buyer is to sample the population of sellers exactly once ($n_B = 1$). If the buyer meets a “tough” seller, he learns that the state must be $H$, and switches. Trade occurs at the FRREE prices: $p^H$ in state $L$ with no delay, and $p^{hh}$ in state $H$ with a delay of one period.

The second equilibrium configuration is interior. Sellers in state $L$ are indifferent between any $n_{SL} \in \{0, \ldots, \infty\}$ (the cost of delay just offsets the incentive to play “tough”), and buyers
respond by choosing \( n_B \) optimally. As \( \delta \to 1 \), both \( n_B \) and \( n_{SL} \) approach infinity. In this struggle between forces MI and CL, the latter typically has the upper hand.

More formally, the following results were shown: (a) For any cost, valuation, and price configuration satisfying (10), there is an open region of the parameters \( \alpha_H \) and \( x_B \) for which, along any sequence of equilibria such that limits exist, \( \lim_{\delta \to 1} f_B = 0 \). (b) For an open region of the parameters \( \alpha_H \) and \( x_B \), sequences such that \( \lim_{\delta \to 1} f_B > 0 \) could not be ruled out (although no example of such a sequence was discovered). (c) For any cost, valuation, and price configuration satisfying (10), and any \( \alpha_H \in (0,1) \) and \( x_B \in (0,1) \), there exist a sequence of corner equilibria, and a sequence of interior equilibria such that \( \lim_{\delta \to 1} f_B = 0 \).

We turn to two sided information. Wolinsky (1990) shows that the following configurations are possible in equilibrium: (a) \( n_{SH} = \infty, n_{BH} = 0, n_{BL} = \infty, n_{SL} = 0, n_S < \infty, n_B < \infty, \) (b) \( n_{SH} = \infty, n_{BH} \in \{0, \ldots, \infty\}, n_{BL} = \infty, n_{SL} = 0, n_S < \infty, n_B = \infty, (c)n_{SH} = \infty, n_{BH} = 0, n_{BL} = \infty, n_{SL} \in \{0, \ldots, \infty\}, n_S = \infty, n_B < \infty. \) There are no corner equilibria. In all equilibrium configurations, as \( \delta \to 1 \), \( n_B \) and \( n_S \) approach infinity. Again, force CL overcomes force MI. Wolinsky shows that as \( \delta \to 1 \), the traders behind force MI become a negligible fraction of the market. They realize that extracting surplus from increasingly patient uninformed trading partners is becoming harder and harder. Understanding that they can switch to “soft” and still gain from trade, they do so and leave the market. Yet, this does not result in complete learning on the part of the uninformed, as force N (Noise) comes into play. As \( \delta \to 1 \), the overwhelming majority of the population, on both sides of the market, is becoming uninformed. Thus, although uninformed traders are very patient, and hence very inclined to stay in the market and learn, the informative content of the pairwise meetings is becoming smaller and smaller. Force N overcomes force CL. Wolinsky shows that along any sequence of equilibria where \( \delta \to 1 \), and such that \( \lim_{\delta \to 1} f_S \) and \( \lim_{\delta \to 1} f_B \) exist, at least one of these limits is positive. A non-negligible fraction of the uninformed traders end up not learning, that is they trade at a price which is not ex-post individually rational.

Serrano and Yosha (1993b) establish that in this particular model, convergence to FRREE prices is synonymous with convergence to FRREE payoffs, and hence to asymptotic interim incentive efficiency. More formally, the main result is that a sequence of equilibria is asymptotically interim incentive efficient if and only if it is asymptotically ex-post individually rational.

Therefore, when there is a one sided information asymmetry, the pairwise meetings model is efficient in the limit. When the information asymmetry is two sided, it is not. It is interesting

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17Assumptions (11) and (12) were made by Wolinsky (1990) in order to prove existence of equilibrium, but were not needed for one sided information.
to note that when there is no informational asymmetry, the limiting model is not efficient.\footnote{This includes the cases studied in the previous section, and (roughly speaking) the two sided information case when $\delta$ is close enough to 1, as explained above.}

It seems that one sided information is somewhat special. When both sides of the market are informed, or when both are uninformed there are inefficiencies caused by delay (and by noise in the two sided information case). When only one side of the market is uninformed, the informed side prefers to give up at some point. This results in complete learning without loss of surplus in the form of delay.\footnote{In all the models described in this section uncertainty is of the common values type. Samuelson (1992) studies a model with pairwise meetings, asymmetric information, and private values uncertainty, finding delay in equilibrium. He does not explore, though, the asymptotic properties of the model.}

\textbf{Remark.} Related experimental results are reported in Harrison (1992).\footnote{See also the many references therein.} In one of the experiments, traders, endowed with dividend paying shares and with money, engage in trade on a stock market which meets three times. The amount of dividend paid by the shares that each individual owns is private information. The object of interest is the sequence of market clearing prices for these shares. It is possible to compute, by fully rational backward induction, the rational expectations equilibrium (REE) price of this market. Harrison reports that although the sequence of spot prices converges to the REE price, convergence is slow. The gradual convergence is interpreted as myopia on the part of traders.\footnote{When futures markets are introduced, convergence becomes faster, but not instantaneous.} Similar results are obtained when uncertainty about a common values state of nature is introduced, as well as when some traders are informed regarding the state of nature ("insiders.")

We would like to point out an effect that may be at work in the above experiments, and which may contribute to the slow convergence. It may be that (as in the Wolinsky model) traders are behaving strategically, in the sense that they are trying to extract surplus from other traders. This results in "toughness," and hence in a slower convergence to REE. When private (inside) information regarding the common values uncertainty is introduced, convergence may be slowed down further as a result of the attempt by uninformed traders to learn from informed traders through the market clearing price.

\section{Concluding Remarks}

In this section we shall briefly address two issues. First, we shall try to assess to what extent the Wolinsky (1990) model and its symmetric information variant we developed can indeed be regarded as models of decentralized trade. Then we shall discuss some possible implications of
the theory for the design of markets and trading institutions.

**How decentralized is the Wolinsky model?** There are (at least) four kinds of arguments which can be formulated as criticisms of the above model as a model of decentralized trade:

(a) In the event of disagreement in a particular round of bargaining, traders are not free to keep negotiating with their trading partner, and are forced back into the pool of unmatched traders to be matched anew, almost surely with a new trading partner. The other models surveyed above have a similar feature. Muthoo (1993) addresses this issue, and finds that allowing traders to negotiate as long as they wish results in non-convergence to a Walrasian outcome. See also Osborne and Rubinstein (1990, Chapter 10).

(b) We have seen that there is more than one equilibrium, both in the symmetric and asymmetric information versions of the model. As was kindly pointed out by a referee, multiplicity of equilibria requires some coordination between players in order for equilibrium to be attained. In this sense the model is not completely decentralized, as in the background there is a coordinating invisible hand. In most of the other models surveyed above, equilibrium is unique.

(c) In the Wolinsky model with asymmetric information, traders are assumed to know, upon entering the market, the proportions of “tough” trading partners in every state of the world. These proportions are endogenous variables, so it cannot be argued that their knowledge is a consequence of the knowledge of the model. Nor are these proportions directly chosen by traders in the market, so the knowledge of the proportions cannot be attributed to the knowledge of other traders’ equilibrium strategies. The proportions can be interpreted as beliefs of traders regarding the probabilities that trading partners will play “tough” in each state of the world. But then, why assume that all traders have the same beliefs? In any event, there is an implicit assumption regarding the presence of some sort of coordination device.

**Implications for the design of markets and trading institutions.** For some markets, decentralized models of trade in pairwise meetings are not very plausible. Organized stock and commodity exchanges are obvious examples. Another example are food markets. Consider, for instance, the market for potatoes in the EEC. It is hard to conceive of buyers searching for sellers across Europe, or of potato growers searching for buyers. The farmers sell “to the market” and buyers buy “from the market.” The price is usually centrally determined by a commission in Brussels.

Pairwise meetings models are suitable for the analysis of markets such as labor markets, housing markets, and markets for bank loans. In such markets there is a strong element of search and of bargaining, and the prices at which transactions take place are affected by the relative
“toughness” of buyers and sellers. Naturally, such markets involve inefficiencies related to the costs of search. These costs include the foregone time, transportation costs, the opportunity cost of holding money (buyers) and inventories (sellers) during the process of search, and so forth.

Governments can contribute to the reduction of these costs. For example, the availability of efficient systems of communication and transportation reduces search costs considerably. A developed consumer credit system permits buyers to hold smaller amounts of money in liquid form. The question is whether a substantial reduction in search related costs will result in approximately efficient outcomes. The above analysis suggests that quite often this will not be the case. It was shown that when information is symmetric and when information is asymmetric and two sided, there are sequences of equilibria such that the inefficiency persists even when search related costs tend to zero. At the basis of this phenomenon lies a coordination failure. When a trader enters the market he knows that some of his trading partners are “tough” whereas others are not. This induces him to become “tougher,” in the hope of meeting a “soft” trading partner. This “strategic toughness” is present on both sides of the market. It becomes stronger as the cost of staying in the market decreases, and results in delay which increases very fast, fast enough to generate a welfare loss.

A familiar example of a market with symmetric information where such delays are observed is the market for senior academic faculty. Information regarding the quality of the academic institutions and of the senior faculty is relatively well known. Yet, negotiations take a terribly long time. An example of a market with two sided asymmetric information is the market for corporate executives. Let the state of the world be the compatibility between a firm and an executive (communication skills, style of management, etc.). It may be that the firm is (relatively) informed regarding the compatibility whereas the executive is not, it may be the other way around, or it may be that both or none are informed. Again, this market exhibits long searches, on the part of firms and unemployed executives alike, which is consistent with the theory.

Curiously, when information is asymmetric and one sided, we have seen that almost frictionless markets are typically efficient. A possible intuition for this would run as follows. When it is common knowledge between the trading parties that one party, say the seller, is fully informed, then both parties understand that when the cost of production is low, the seller’s back is not against the wall. The seller, who is not desperate to sell at the high price and who knows that the buyer is aware of this fact, finally throws the towel and plays “soft.”

An example of a market with one sided asymmetric information is the market for private placements of securities. Firms who issue securities typically know more about the true value of the securities than the buyers. The above analysis suggests that in order to induce an ap-
proximately efficient outcome in such a market it would be sufficient to reduce search costs. As search costs approach zero, the patience of the buyers will force sellers to reveal their private information and sell at the “right price,” after a reasonably short period of negotiations.

It is, of course, pre-mature to draw any operational conclusions from the analysis. Many issues are still not sufficiently understood. The above reasoning is suggestive of the kinds of applications for which models with pairwise meetings can be of use.

References


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