A Short Course in Intermediate Microeconomics with Calculus

2nd edition

Solutions to Exercises – Short Answers[1]

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The purpose of this set of (mostly) short answers is to provide a way for students to check on their work. Our answers here leave out a lot of intermediate steps; we hope this will encourage students to work out the intermediate steps for themselves. We also have a set of longer and more detailed answers, which is available to instructors.

[1]We thank EeCheng Ong and Amy Serrano for their superb help in working out these solutions. We also thank Rajiv Vohra for contributing some nice improvements to our previous version.

Chapter 2 Solutions

1.(a) For this consumer, $6 \succ 0$. Show that $0 \sim 6$ if the transitivity assumption holds.

1.(b) Show that $x \succ y$, $y \succ z$, and $z \succ x$.

2.(a) The indifference curve corresponding to $u = 1$ passes through the points $(0.5, 2)$, $(1, 1)$, and $(2, 0.5)$. The indifference curve corresponding to $u = 2$ passes through the points $(0.5, 4)$, $(1, 2)$, $(2, 1)$, and $(4, 0.5)$.

2.(b) The MRS equals 1 along the ray from the origin $x_2 = x_1$, and it equals 2 along the ray from the origin $x_2 = 2x_1$.

3.(a) The indifference curves are downward-sloping parallel lines with a slope of -1 and the arrow pointing northeast.

3.(b) The indifference curves are upward-sloping with the arrow pointing northwest.

3.(c) The indifference curves are vertical with the arrow pointing to the right.

3.(d) The indifference curves are downward-sloping and convex with the arrow pointing northeast.

4.(a) The indifference curves are horizontal; the consumer is neutral about x_1 and likes x_2.

4.(b) The indifference curves are downward-sloping parallel lines with a slope of -1; the consumer considers x_1 and x_2 to be perfect substitutes.

4.(c) The indifference curves are L-shaped, with kinks along the ray from the origin $x_2 = \frac{1}{2}x_1$; the consumer considers x_1 and x_2 to be perfect complements.

4.(d) The indifference curves are upward-sloping and convex (shaped like the right side of a U); the consumer likes x_2, but dislikes x_1, i.e., good 1 is a bad for the consumer.

5.(a) $MU_1 = 6x_1x_2^4$.

5.(b) $MU_2 = 12x_1^2x_2^3$.

5.(c) $MRS = \frac{x_2}{2x_1}$.

5.(d) $MRS = 1$.

5.(e) $MRS = \frac{1}{8}$. The MRS has diminished because Donald has moved down his indifference curve. As he spends more time fishing and less time in his hammock, he is increasingly reluctant to give up hammock time for an extra hour of fishing.

5.(f) He is just as happy this week as he was last week.

6.(a) The MRS is the amount of money I am willing to give up in exchange for working an extra hour. My MRS is negative, meaning that someone would have to pay money to me in order to have me work more.

6.(b) Since work is a bad, the MRS should be negative. The MRS is negative because the indifference curves are upward-sloping, and the MRS is (-1) times the slope.

6.(c) The MRS is decreasing (increasing in absolute value) as the hours of work increase. The indifference curves are upward sloping and convex. As I work more and more hours, I would require ever higher rates of pay in order to be willing to work an additional hour.

7.(a) Negative.

7.(b) Its absolute value tells us her willingness to work (extra hours) as a function of increases in pay.

7.(c) The absolute value of the MRS is decreasing. The indifference curves are convex.

8. (a) They are L-shaped: the case of perfect complementts.

8. (b) They are monotonically decreasing, with a global satiation point at 0.

8. (c) The curves are horizontal.

8. (d) They are downward sloping and concave.

9. (a) The indifference curve through the point (1,1) consists exclusively of that point.

9. (b) These preferences are complete, transitive, and satisfy monotonicity.

10. (a) They are concentric circles around the point $(1, 2)$.

10. (b) These preferences are complete, transitive, and convex, but they violate monotonicity.

Chapter 3 Solutions

1.(a) The new budget line is $2p_1x_1 + \frac{1}{2}p_2x_2 = M$, and its slope is four times the slope of the original budget line.

1.(b) The new budget line is $2p_1x_1 + p_2x_2 = 3M$, and its slope is twice the slope of the original.

2.(a) $3x_1 + 2x_2 = 900$. Horizontal intercept at 300 and vertical intercept at 450.

2.(b) $(x_1^*, x_2^*) = (100, 300)$.

3.(a) $M = 60$ and $p_b = 1$.

3.(b) He will consume 0 apples and 60 bananas.

4.(a) The x_1 intercept is 27, the x_2 intercept is 12, and the kink is at $(20, 2)$.

4.(b) Peter's indifference curves are linear, with slopes of $-\frac{1}{3}$. His optimal consumption bundle is $(0, 12)$.

4.(c) The x_1 intercept is 11, the x_2 intercept is 4, and the kink is at $(4, 2)$.

4.(d) Paul's indifference curves are L-shaped, with kinks at $(2, 3)$, $(4, 6)$, etc. His optimal consumption bundle is $(2, 3)$.

5.(a) $c_1 + \left(\frac{1+\pi}{1+i}\right) c_2 = M$, or $c_1 + c_2 = 50$.

5.(b) $(c_1^*, c_2^*) = (25, 25)$.

5.(c) $(c_1^*, c_2^*) = (25, 22.73)$.

6.(a) The budget line is $c_1 + \left(\frac{1.05}{1.10}\right) c_2 = 190.91$. The c_1-intercept is 190.91, and the c_2-intercept is 200. The slope is $-\frac{1.1}{1.05} = -1.048$, reflecting the relative price of current consumption. The zero savings point is $(100, 95.24)$, the consumption plan he can afford if he spends exactly his income in each period.

6.(b) $(c_1^*, c_2^*) = (127.27, 66.67)$; Sylvester is a borrower. His optimal choice is a point of tangency between his indifference curve and the budget line, to the southeast of the zero-savings point.

6.(c) The budget line pivots counterclockwise through the zero savings point, and now has a slope of -1. The intercepts are 195.24 at both axes. The new consumption bundle is $(c_1^*, c_2^*) = (130.16, 65.08)$.

6.(d) Sylvester is better off than before.

7. (a) $x = 2,000,000$ and $y = 4,000,000$.

7. (b) $x = 1,000,000$, $y = 4,000,000$. His allowance should increase by 1.6 million, approximately.

7. (c) His initial optimal choice is $x = 4,000,000$, $y = 2,000,000$. After p_x rises, his new optimal choice is $x = y = 2,000,000$. His allowance must increase by 3.6 million approx.

7. (d) $\lambda^* = 16 \cdot 10^{12}$ in the initial situation, and exactly half of that in the final situation.

8. (a) $y = 6$ for $x \leq 1$, the line with equation $x + y = 7$ for $1 \leq x \leq 6$, and $x = 6$ for $0 \leq y \leq 1$.

8. (b) $x = 5$, $y = 2$.

8. (c) The budget line is now $y = 3.5$ for $0 \leq x \leq 1$, the line with equation $x + 2y = 8$ for $1 \leq x \leq 6$, and $x = 6$ for $0 \leq y \leq 1$. The optimal choice is the bundle $(6, 1)$.

9. (a) $u(x, y) = \min\{2x, y\}$. The indifference curves are L-shaped with vertices along the straight line $y = 2x$.

9. (b) $x = 10$, $y = 20$.

9. (c) $x = 100/11$, which is approx 9, $y = 200/11$, approx 18. Therefore, she would pay in taxes 3/11.

9. (d) Here, the solution is also $x = 100/11$, $y = 200/11$. That is, the same solution. This happens because the goods are perfect complements.

10. (a) $(10,000,0)$.

10. (b) $(1,0)$.

Chapter 4 Solutions

1.(a) Use the budget constraint and tangency condition to solve for $x_1(p_1, p_2, M)$.

1.(b) Good 1 is normal and ordinary. Goods 1 and 2 are neither substitutes nor complements.

2.(a) Show that the original consumption bundle is $(5, 5)$, and the new consumption bundle is $(2, 5)$.

2.(b) Show that the Hicks substitution effect bundle is $\left(\sqrt{10}, \frac{5\sqrt{10}}{2}\right)$.

3. With the Giffen good on the horizontal axis, the Hicks substitution effect bundle is to the southeast of the original bundle, and the final bundle is to the northwest of the original bundle. See Solutions-graphs file.

4.(a) $(x^*, y^*) = (8, 8)$.

4.(b) $(x^*, y^*) = \left(\frac{200}{33}, \frac{200}{33}\right)$. He will pay $\frac{16}{33}$ in taxes.

4.(c) The demand functions are $x = y = \frac{M}{p_x + p_y}$. The goods are normal, ordinary, and complements of one another.

5.(a) $(x^*, y^*) = (1, 1)$.

5.(b) $(x^*, y^*) = (0.5, 1)$.

5.(c) His parents would have to increase his allowance by $2\sqrt{2} - 2$, which is approximately \$0.83.

5.(d) All the answers are the same because v is an order-preserving transformation of u. That is, both consumers have identical preferences.

6.(a) The x-intercept is 8, and the y-intercept is 5. The budget line is horizontal between $(0,5)$ and $(3,5)$, and is downward-sloping with a slope of -1 beyond $(3,5)$.

6.(b) $(x^*, y^*) = (5.5, 2.5)$.

7. (a) The demand functions are $x = M/(p_x + 2p_y)$, $y = 2M/(p_x + 2p_y)$. The goods are normal, ordinary and of course complements (in fact, perfect complements).

7. (b) The substitution effect on either good is zero.

8. By monotonicity, the optimal choice must be on the budget line. See what would happen if we differentiate it with respect to income.

9. $D(p_y) : y^D = 650,000/p_y$.

10. (a) $-p_1/(p_1 + p_2)$. In absolute value, this is always less than 1, and hence this demand is price inelastic.

10. (b) $+1$. This is a normal good and always has unit income elasticity.

10. (c) $-p_2/(p_1 + p_2)$. The negative sign indicates that the goods are complements. In absolute value, this is always less than 1, and hence this demand is cross inelastic.

Chapter 5 Solutions

1. Use the budget constraint and tangency condition to solve for L^*.Note that this problem assumes that $T = 24$.

2. The budget line is downward-sloping between $\left(0, \frac{wT+M}{p}\right)$ and $\left(T, \frac{M}{p}\right)$ and vertical at T. The optimal bundle is $\left(T, \frac{M}{p}\right)$. See Solutions-Graphs file.

3.(a) The budget line has a kink at the zero-savings point. The slope is steeper to the right of the zero savings point, and flatter to its left.

3.(b) The budget line has a kink at the zero-savings point. This time the slope is flatter to the right of the zero-savings point, and steeper to its left. An indifference curve has two tangency points with the budget line, each one at either side of the zero-savings point.

4.(a) The budget line is $c_1 + c_2 = 195.24$. Both the intercepts are 195.24. The slope is -1. The zero-savings point is $(100, 95.24)$.

4.(b) Mr. A's optimal consumption bundle is $(65.08, 130.16)$; he is a lender. Mr. B's optimal consumption bundle is $(130.16, 65.08)$; he is a borrower.

4.(c) The savings supply curve places savings on the horizontal axis and the interest rate on the vertical axis. It is obtained from the savings supply function after fixing the other variables that determine the budget constraint.

Mr. A's savings supply curve is $s_A(i) = \frac{100}{3}\left(\frac{1+2i}{1+i}\right)$; $s = 33.33$ for $i = 0$ and $s = 50$ for $i = 1$. Mr. B's savings supply curve is $s_B(i) = \frac{100}{3}\left(\frac{-1+i}{1+i}\right)$; $s = -33.33$ for $i = 0$ and $s = 0$ for $i = 1$. The aggregate savings supply curve is $s_A(i) + s_B(i) = 100\left(\frac{i}{1+i}\right)$, an upward-sloping curve starting at the origin. See Solutions-Graphs file.

4.(d) Mr. A's optimal consumption bundle is $(63.64, 133.33)$; Mr. A is better off than before. Mr. B's optimal consumption bundle is $(127.27, 66.67)$; Mr. B is worse off than before.

5. One possible savings function in which the consumer switches from being a borrower to a saver at a given interest rate. See Solutions-Graphs file. Hint: Why must the savings supply curve be strictly increasing when the consumer is a borrower, but not necessarily when he is a saver? Why can't a saver ever become a borrower in response to a raise in the interest rate?

6. A decrease in π causes the budget line to rotate clockwise on the x-intercept while an increase in i causes the budget line to rotate clockwise on the zero savings point. Analyze the substitution effect and income effect on c_1 and c_2 in each case. In the first case, you can't predict the direction of change either for a borrower or a saver. In the latter case, it is ambiguous for a saver, but a borrower will definitely borrow less.

7. (a) The standard demand curve for current consumption is $c_1 = (2+i)/(2+2i)$.

7. (b) The compensated demand is $c_1 = \sqrt{1/(1+i)}$.

8. (a) Income and substitution effects go in the same direction.

8. (b) Yes, this is possible if the substitution effect is stronger than the income effect.

8. (c) Notice that the labor supply curve $l(w)$ is drawn for fixed values of the other relevant variables (two of which are non-labor income and unemployment benefits).

(I) If leisure is normal, the labor supply will shift to the left for each value of w. If leisure is inferior, $l(w)$ will shift to the right.

(II) If we denote by w^* that wage rate for which the tangency point is indifferent to $(T, U/p)$, we have that the labor supply is $l(w) = 0$ for $w \leq w^*$, $l(w) = w$ for $w \geq w^*$.

9. (a) $(L^*, c^*) = (1/10, 239)$.

9. (b) $l(w) = 24 - L(w) = 24 - \frac{1}{w}$. This is an upward sloping and convex curve, starting from the point $(0, 1/24)$ and asymptotic to the $l = 24$ vertical line as $w \to \infty$.

10. $l(w, M) = 12 - \frac{M}{2w}$.

Chapter 6 Solutions

1.(a) Her optimal consumption bundle is $(25, 50)$. Her utility is $1,250$.

1.(b) Her new consumption bundle is $(25, 40)$.

1.(c) The subsidy should be \$0.80 a pint or 20 percent.

2.(a) Her optimal consumption bundle is $(15, 10)$. Her utility is $1,600$.

2.(b) Her new consumption bundle is $(18, 9)$. Her new utility is $1,558 < 1,600$.

3. William is always made worse off by the tax, while Mary would be made worse off by the tax only if the original price of good x were less than the price of good y.

4.(a) His optimal consumption bundle is $(2, 16)$. His utility is $2,560$.

4.(b) His new consumption bundle is $(4, 16)$. His new utility is $40,960$.

4.(c) The income effect is 34.052.

5. The first program yields a utility of $5.324 \cdot 10^8$, and the second program yields a utility of $6.25 \cdot 10^8$; the couple prefers the second program. The first program costs \$3,000 and the second program costs \$5,000.

6. Pre-policy, $x_A^* = y_A^* = 25$, $x_B^* = y_B^* = 20$, and $x_C^* = y_C^* = 15$. Post-policy, $x_A^{**} = y_A^{**} = 24$, $x_B^{**} = y_B^{**} = 20$, and $x_C^{**} = y_C^{**} = 16$. The welfare of the median consumer (Group B) is unchanged. Lower-income consumers (Group C) are better off and higher-income consumers (Group A) are worse off.

7. (a) The typical indifference curve has a kink on the 45 degree line. For $u = 3$, the kink happens at the bundle $(1,1)$, and the extreme points of the curve are $(0,3)$ and $(3,0)$. For $u = 6$, the kink is at the bundle $(2,2)$, and the extreme points are $(0,6)$ and $(6,0)$. These are well-behaved preferences: downward sloping (more is preferred to less) and convex indifference curves (averages are always weakly preferred to extremes).

7. (b) The initial optimal choice is the bundle $(1,1)$ for a utility of 3. After the subsidy-induced price change, the optimal choice is $(6,0)$ with a utility of 6 The cost of the subsidy to the government is 4.

7. (c) The total effect on the demanded amount of good 1, which was $6 - 1 = 5$ can be decomposed into substitution effect $3 - 1 = 2$, and income effect $6 - 3 = 3$. Using the compensating variation measure, this consumer has benefited by \$1.

8. (a) $L^* = 17$, $c^* = 70$, $\lambda^* = 17$. Thus, she chooses to work 7 hours.

8. (b) The overtime budget line will have a kink at the point $(L', c') = (16, 80)$. The equation of the overtime budget line for $L \leq 16$ is $c + w'L = 80 + 16w'$, whereas for $L \geq 16$, the equation is $c + 10L = 240$.

8. (c)

$$L(c + 100) = 2890;$$

$$c + w'L = 80 + 16w';$$

$$(c + 100)/L = w'.$$

This gives $c = 365/4 = 91.25$ in the good solution (there is a second one, but it is irrelevant). Then, $L = 136/9 = 15.11$, and then $w' = 405/32$. The cost of the subsidy per employee is $(405/32 - 10)(8/9) = 85/36$, which is approx 2.4.

9. By the compensating variation, her welfare has increased by $30,000$. By the equivalent variation, her welfare has increased by $40,000$.

10. This can be seen graphically, drawing the two budget lines as described.

Chapter 7 Solutions

1. The equation for the indifference curve where $u = 10$ is $x_2 = 10 - v(x_1)$, and the equation for the indifference curve where $u = 5$ is $x_2 = 5 - v(x_1)$. The vertical distance between the two curves equals the difference in the value of x_2, that is, the difference of the two equations, which is 5.

2. The utility function is quasilinear, so each unit of good 2 contributes exactly one unit of utility ($MU_2 = 1$). In addition, there is no income effect on the demand for good 1, so each additional unit of income will be spent on good 2. As a result, each additional unit of income increases utility exactly by one unit. It is as if utility were measured in dollars.

3. Decompose consumers' surplus in the graph at the far right into two triangles with areas $\frac{1}{2}ab$ and $\frac{1}{2}cd$.

4.(a) When $p = 0$, the net social benefit is \$1.5 million. When $p = \frac{5-\sqrt{5}}{2}$, the net social benefit is \$1.309 million.

4.(b) The price that maximizes revenue is $p = 2.5$, and the net social benefit is \$0.875 million.

4.(c) Net Social Benefit = Consumer Surplus+Government Revenue$-1,000,000 = 1,500,000-100,000p^2$. This function is maximized at $p = 0$.

5. Loss of consumer's surplus is 7.2984.

6.(a) His demand function for x is $x(p_x, p_y, M) = \sqrt{10 - \frac{p_x}{p_y}}$, and if $p_y = 1$, the demand curve is $x = \sqrt{10 - p_x}$. When $p_x = 1$, he consumes $x = 3$.

6.(b) His inverse demand function for x is $p_x = 10 - x^2$. His consumer's surplus from his consumption of x is 18.

6.(c) He now consumes $x = 2$. His consumer's surplus from his consumption of x is now $\frac{16}{3}$.

7. $x = 1/p_x$ whenever $M - 1 \geq 0$. Otherwise, that is, if $M < 1$, the optimal choice is $x = M/p_x$ and $y = 0$. Good x is normal for income levels $M < 1$, and is independent of income for higher levels of income. The Engel curve starts from the origin and is the 45-degree line until the point (1,1), and it switches to being a vertical line from then on.

8. (a) $x^* = 2$ and $y^* = 9$.

8. (b) Both are $x = 1/p_x$.

8. (c) With the new price of toothpaste, the optimal choice is $(3, 9)$. Therefore, the increase in utility is $\ln 3 - \ln 2$. The change in consumer's surplus is that same amount.

9. (a) $-1 + p_1 - \ln p_1$.

9. (b) $2\ln 2 - 2 - 2\ln p_1 + p_1$.

9. (c)

$$CS(p_1) = \int_{p_1}^{2} [\frac{2}{p_1} - 1] dp_1 \qquad \text{for} \qquad 1 \leq p_1 \leq 2$$

and

$$CS(p_1) = \int_{1}^{2} [\frac{2}{p_1} - 1] dp_1 + \int_{p_1}^{1} [\frac{3}{p_1} - 2] dp_1 \qquad \text{for} \qquad p_1 \leq 1.$$

10. This follows easily from the Slutsky equation, since $\partial x_1 / \partial M = 0$.

Chapter 8 Solutions

1.(a) Show that $\frac{d^2y}{dx^2} < 0$ for $x \neq 0$.

1.(b) $C(y) = y^2$; $AC(y) = y$; $MC(y) = 2y$.

1.(c) The supply curve is $y = \frac{1}{2}p$ for $p \geq 0$.

1.(d) $\pi = 25$.

2.(a) $C(y) = \left(\frac{1}{5}y + 6\right)^3$; $AC(y) = \frac{\left(\frac{1}{5}y+6\right)^3}{y}$; $MC(y) = \frac{3}{5}\left(\frac{1}{5}y + 6\right)^2$.

2.(b) The supply curve is $y = 0$ for $p < 48.6$, and $y = 5\sqrt{\frac{5p}{3}} - 30$ for $p \geq 48.6$.

3.(a) Show that $\frac{d^2x}{dy^2} > 0$.

3.(b) $MP(x) = \frac{1}{2\sqrt{x}}$; $AP(x) = \frac{1}{\sqrt{x}}$.

3.(c) $VMP(x) = \frac{5}{\sqrt{x}}$; $VAP(x) = \frac{10}{\sqrt{x}}$.

3.(d) Now we use the assumption $x \geq 1$. The input demand curve is $x = 0$ for $w > 10$, and $x = \frac{25}{w^2}$ for $w \leq 10$.

3.(e) $\pi = 25$.

4.(a) $MP(x) = \frac{1}{3}\left(\frac{2}{\sqrt[3]{x}} + 1\right)$; $AP(x) = \frac{1}{\sqrt[3]{x}} + \frac{1}{3}$.

4.(b) $VMP(x) = 2\left(\frac{2}{\sqrt[3]{x}} + 1\right)$; $VAP(x) = 2\left(\frac{3}{\sqrt[3]{x}} + 1\right)$.

4.(c) Now we use the assumption $x \geq 1$. The input demand curve is $x = 0$ for $w > 8$, and $x = \left(\frac{4}{w-2}\right)^3$ for $w \in (2, 8]$. If $w < 2$, the firm would like to hire an infinite amount of input.

5. In a (y_1, y_2)- quadrant, a typical isofactor curve is concave to the origin (using the same amount of input, the more additional units of output y_1 the firm wants to produce requires it to give up more units of output y_2). The isorevenue curves are downward-sloping straight lines of slope $-p_1/p_2$. The solution to the revenue maximization problem, conditional on a level of input x, is found at the tangency of the highest possible isorevenue line with the fixed isofactor curve. The solution to this revenue maximization problem yields the conditional output supply functions $y_1(p_1, p_2, x)$ and $y_2(p_1, p_2, x)$. Finally, the profit maximization problem is thus written:

$$\max_x \pi = p_1 \cdot y_1(p_1, p_2, x) + p_2 \cdot y_2(p_1, p_2, x) - wx.$$

Solving the maximization problem yields the input demand function, $x(p_1, p_2, w)$.

6.(a) $C(y_1, y_2) = y_1^2 + y_2^2 + y_1 y_2$; $MC_1(y_1) = 2y_1 + y_2$; $MC_2(y_2) = 2y_2 + y_1$.

6.(b) $y_1^*(p_1, p_2) = \frac{1}{3}(2p_1 - p_2)$; $y_2^*(p_1, p_2) = \frac{1}{3}(2p_2 - p_1)$.

6.(c) $y_1^*(1, 1) = \frac{1}{3}$; $y_2^*(1, 1) = \frac{1}{3}$; $\pi = \frac{1}{3}$.

6.(d) $y_1^*(1, 2) = 0$; $y_2^*(1, 2) = 1$; $\pi = 1$.

7. (a) $MP(x) = 2x$. $AP(x) = x$.

7. (b) $C(y) = \sqrt{y}$. $MC(y) = (1/2)y^{-1/2}$. $AC(y) = y^{-1/2}$.

7. (c) There is no supply, as this firm would not be able to maximize profit.

8. (a) $MP(x) = AP(x) = 5$.

8. (b) $C(y) = y/5$.

$MC(y) = AC(y) = 1/5$.

8. (c) For $p < 1/5$, the profit-maximizing level of output is $y(p) = 0$. If $p = 1/5$, the supply is infinitely elastic. If $p > 1/5$, there is no supply.

9. (a) $MP(x) = 2x$ for $0 \leq x \leq 1$, and $(1/2)x^{-1/2}$ for $x \geq 1$. $AP(x) = x$ for $0 \leq x \leq 1$ and $x^{-1/2}$ for $x \geq 1$.

9. (b) $C(y) = \sqrt{y}$ for $0 \leq y \leq 1$, and y^2 for $y \geq 1$.

   $MC(y) = (1/2)y^{-1/2}$ for $0 \leq y \leq 1$, and $2y$ for $y \geq 1$.

   $AC(y) = y^{-1/2}$ for $0 \leq y \leq 1$, and y for $y \geq 1$.

9. (c) For $p < 1$, the profit-maximizing level of output is $y(p) = 0$. If $p \in [1, 2]$, $y(p) = 1$, and for $p \geq 2$, $y(p) = p/2$.

10. This follows from

$$\frac{dAP}{dx} = \frac{f'(x)x - f(x)}{x^2} = \frac{1}{x}[MP(x) - AP(x)].$$

Furthermore, if there is a unique gloabal maximum of AP, to its left $MP(x) > AP(x)$. Similarly, to the right of the maximum, $MP(x) < AP(x)$.

Chapter 9 Solutions

1. Returns to scale are related to the way in which isoquants are labeled as we increase the scale of production moving out along a ray from the origin. Returns to scale is a meaningful concept because the labels on isoquants represent a firm's level of output, and output is a cardinal measure. In contrast, indifference curves represent a consumer's utility level and utility is an ordinal measure. The rate at which the utility number rises along a ray is therefore not particularly meaningful.

2. A price-taking firm will produce where price equals marginal cost. Furthermore, at profit-maximizing points the marginal cost curve cannot be downward sloping. Therefore, the firm's output rises.

3.(a) Use the production function and tangency condition to solve for the long-run conditional factor demands.

3.(b) $C(y) = w_1 x_1 + w_2 x_2$.

3.(c) The supply curve is the $MC(y)$ curve, if $p \geq \min AC(y)$, and $MC(y)$ is rising.

4.(a) This technology shows increasing returns to scale. The isoquants are symmetric hyperbolas; the isoquants get closer and closer to each other away from the origin.

4.(b) $L^*(y) = 10\sqrt{y}$; $K^*(y) = \sqrt{y}$.

4.(c) $C(y) = 200\sqrt{y}$.

4.(d) If $y = 1$, then $L^*(1) = 10$, $K^*(1) = 1$, and $C(1) = 200$. If $y = 2$, then $L^*(2) = 10\sqrt{2}$, $K^*(2) = \sqrt{2}$, and $C(2) = 200\sqrt{2}$.

4.(e) $AC(y) = \frac{200}{\sqrt{y}}$; $MC(y) = \frac{100}{\sqrt{y}}$. Both $AC(y)$ and $MC(y)$ are decreasing hyperbolas, and $MC(y) < AC(y)$. There is no long-run supply curve.

5.(a) $x_1^*(y) = 0$; $x_2^*(y) = y$; $C(y) = y$.

5.(b) The long-run supply curve is $y = 0$ for $p < 1$, and $y \in [0, \infty)$ for $p = 1$.

5.(c) $C(y) = 2y$; $AC(y) = 2$; $MC(y) = 2$. The long-run supply curve is $y = 0$ for $p < 2$, and $y \in [0, \infty)$ for $p = 2$.

6.(a) $x_1^*(y) = x_2^*(y) = x_3^*(y) = y^{5/3}$.

6.(b) $C(y) = 3y^{5/3}$.

6.(c) The long-run supply curve is $y = (\frac{p}{5})^{3/2}$ for $p \geq 0$.

7. Consider any ray vector from the origin. Take a point on that ray vector, and multiply by the same constant $\lambda > 0$ both inputs. After such an operation, one remains on the same ray vector. The TRS at the new point is $\alpha(\lambda x_2)/\beta(\lambda x_1)$, which equals the TRS at the original point. The implication for the conditional input demands is that, for constant relative input prices, the firm will always minimize cost on the same ray, for any level of output.

8. (a) This is a quarter ellipse, a downward-sloping concave curve from the point $(0, 3)$ to the point $(2, 0)$.

8. (b)

$$TRS = \frac{9x_1}{4x_2}.$$

What is unusual about this TRS is that it is increasing in absolute value as one moves down and to the right along an isoquant.

8. (c) The point in question is $(4/\sqrt{13}, 9/\sqrt{13})$. The total cost at that point is $\sqrt{13}$. At that point, however, the firm is not minimizing cost. The cost-minimizing combination is the extreme point $(2, 0)$, with a total cost of 2.

9. (a) $y = \min\{x_1, 2x_2\}$. The isoquant of level 1 is an L-shaped curve with vertix at $(1, 1/2)$.

9. (b) $x_1(w_1, w_2, y) = y$ and $x_2(w_1, w_2, y) = y/2$. What is unusual about them is that, due to the fixed proportions technology, they are completely insensitive to the relative price of inputs.

10. For $p < 2.75$, $y(p) = 0$; and for $p \geq 2.75$,

$$y(p) = 1 + \frac{\sqrt{3p - 6}}{3}.$$

Chapter 10 Solutions

1.(a) $AP(x_1|1) = \frac{\left[243+\frac{1}{3}(x_1-9)^3\right]}{x_1}$; $MP(x_1|1) = (x_1-9)^2$.

1.(b) $AP(x_2|x_1^0) = 243 + \frac{1}{3}(x_1^0-9)^3$; $MP(x_2|x_1^0) = 243 + \frac{1}{3}(x_1^0-9)^3$.

1.(c) In (a), $AP(x_1|1)$ and $MP(x_1|1)$ vary with x_1. In (b), $AP(x_2|x_1^0) = MP(x_2|x_1^0)$, and both average product and marginal product curves are constant for a given level of input 1.

2.(a) If w_1 rises, the firm produces less y, x_1^* falls, and π falls.

2.(b) If w_2 falls, this is a fall in the fixed cost, x_1^* is unchanged, and π rises.

2.(c) If p rises, the firm produces more output, x_1^* rises, and π rises.

3.(a) $C^S(y) = y^4 + 2$.

3.(b) The supply curve is the $MC^S(y)$ curve, if $p \geq \min AVC(y)$, and $MC^S(y)$ is rising.

4.(a) $C^S(y) = 100y + 100$.

4.(b) $ATC(y) = 100 + \frac{100}{y}$; $AVC(y) = 100$; $MC^S(y) = 100$. The short-run supply curve is infinitely elastic at $p = 100$. That is, the firm's short-run supply is flat at that price.

5.(a) $ATC(y) = \frac{100}{y} + 10 - 2y + y^2$; $AVC(y) = 10 - 2y + y^2$; $MC^S(y) = 10 - 4y + 3y^2$. $ATC(y)$ is a U-shaped parabola starting at $(0, \infty)$ with a minimum around $y = 4$. $AVC(y)$ is a U-shaped parabola starting at $(0, 10)$ with a minimum at $(1, 9)$. $MC^S(y)$ is a U-shaped parabola starting at $(0, 10)$ with a minimum at $y = \frac{2}{3}$.

5.(b) The short-run supply curve is $y = 0$ for $p < 9$, and $y = \frac{2}{3} + \frac{\sqrt{3p-26}}{3}$ for $p \geq 9$.

6. If the output price is below min $ATC(y)$ but above min $AVC(y)$, the firm recoups some of the fixed cost if it produces output. Therefore, in the short run, the firm produces output as long as the output price is above min $AVC(y)$.

7. The short run supply curve is $y^S(p) = 0$ for $p \leq 1/3$, $= 3$ for $p \in [1/3, 2/3]$, and $= 9p/2$ for $p \geq 2/3$.

8. The short run supply curve is infinitely elastic at $p = 5$.

9. (a) In the long run, this technology exhibits increasing returns to scale. In this first short run case, $AVC(y) = w_l/\sqrt{y}$, which is always decreasing.

9. (b) In this second short run case, the short run supply curve is infinitley elastic at the level $p = w_k$.

10. Suppose $x_1 = x_1^0$. Then, the output can never be greater than x_1^0/a_1, even if one increases the use of all other inputs.

Chapter 11 Solutions

1.(a) The equilibrium price and quantity both increase.

1.(b) The equilibrium price increases and the quantity decreases.

1.(c) The equilibrium price decreases and the quantity increases.

2.(a) Use the production function and tangency condition to solve for the conditional factor demands, and find the cost curves. You should get that $C(y) = 4y$.

2.(b) $y = 996,000$.

3.(a) $K^*(h) = \frac{1}{64}h$; $L^*(h) = 8h$; $C(h) = 12h$. The long-run individual and market supply are infinitely elastic at $p = 12$.

3.(b) $p = 12$; $h^* = 3,000$. Each producer earns zero profit. The amount produced by each firm is indeterminate; all we know is that the total amount supplied is 3,000.

3.(c) $p = 12$; $h^{**} = 2,000$. As before, each producer earns zero profit and the amount produced by each firm is indeterminate. It is possible that the number of firms has changed, but there is not enough information to give an exact answer.

4.(a) The representative firm's supply curve is $y = 0$ for $p < 24$, and $y = \frac{1}{6}\left(1 + \sqrt{1 + 12p}\right)$ for $p \geq 24$.

4.(b) In the long run, the number of firms adjusts to drive the market to the zero-profit equilibrium. The long-run market supply curve is horizontal at $p = \min AC(y) = 24$. Each firm produces $y_i = 3$. At equilibrium, market supply equals market demand: $y = 900$. Therefore, there are 300 firms in the market.

5.(a) $\pi(y) = 100y - \frac{1}{2}y^2 - 2,450$.

5.(b) $y^* = 100$.

5.(c) $\pi^* = 2,550$.

5.(d) The number of firms in the rocking horse industry will rise since profits are positive.

6.(a) $y^{**} = 70$.

6.(b) $p^{**} = 110$.

6.(c) Calculate profits when $y^{**} = 70$.

6.(d) The firm's profits are unaffected by the tax, but Dakota may drop out of the market. In either case, its profit will be zero.

7. (a) $D(p) : y^D = 650,000/p$.

7. (b) Output exchanged is $21,667$ approximately. Each firm is supplying $y(30) = 10/3$. There are 6,500 active firms.

Short run individual profits are $-4000/27$, which is negative, but greater than negative fixed cost, and hence, it is optimal to open the firm and produce.

8. In the long run equilibrium, $p = 65$. The total output exchanged will be 10,000, and since each firm produces 5 cakes, there will be 2,000 active firms in the long run. Of course, long run equilibrium profits are zero.

9. $p^* = 80$.

10. In the long run equilibrium $p^* = 40$, and only type-3 firms will remain active. There will be 216 active firms.

Chapter 12 Solutions

1. The markup (and thus price) will be higher for the group with the lower elasticity of demand, which is Group B.

2. $h = 950$; $p = 31$; $\pi = 18,050$.

3. The monopolist will produce where $MR_1 = MC_1$ and $MR_2 = MC_2$.

4.(a) $x_B = 49.50$; $x_F = 7.25$; $p_B = 50.50$; $p_F = 15.50$; $\pi = 2,555.37$.

4.(b) Consumers' surplus = 1,277.69; Producer's surplus = 2,555.37.

4.(c) $x = 49.50$; $p = 50.50$; $\pi = 2,450.25$. Note that the monopolist serves only one group of consumers.

4.(d) Consumers' surplus $= 1,225.12$; Producer's surplus $= 2,450.25$. Society is worse off after this change is introduced.

5.(a) $y^* = 30$.

5.(b) $p^* = 85$.

5.(c) $\pi^* = 1,350$.

5.(d) Consumers' surplus = 225.

6.(a) $y^{**} = 36 \ > \ 30 = y^*$.

6.(b) $p^{**} = 82 \ < \ 85 = p^*$.

6.(c) $\pi^{**} = 1,296 \ < \ 1,350 = \pi^*$.

6.(d) Consumers' surplus $= \frac{1}{2}(36)(100-82) = 324 \ > \ 225$.

6.(e) Total welfare $= 1,620 \ > 1,575$.

7. (a) $h^M = 625$, $p^M = 37.5$, $\pi = 7812.50$.

7. (b) In the graph, we find the amount at the intersection of the marginal revenue curve $(50 - h/25)$ and the MC (25). The price is obtained taking at that output level the vertical to the demand. Profits are $ph - ACh$. Policies suggested: fight to try to reduce the demand and to increase the monopolist's costs. The former might be based on education campaigns, while the latter may be accomplished through tighter enforcement of laws by police (compare for example the solution here to that for Exercise 2).

8. (a) $p = 10^{-2}$, $y = 99$, profits are zero, and the equilibrium number of firms is not determined.

8. (b) $y = 9$, $p = 0.1$, $\pi = 0.81$.

8. (c) Under perfect competition, producers' surplus is zero, and consumers' surplus is $2\ln 10 - 0.99$. This is then total social surplus.

Under monopoly, we have $CS + PS = \ln 10 - 0.09$, clearly lower than the social surplus under competition.

The deadweight loss caused by the presence of the monopolist is precisely $\ln 10 - 0.9 > 0$.

8. (d) No, the name is fine.

9. (a) $y_i = 50$, $p = 42$.

9. (b) $y_i = 40$, $p_i = 142$, $n = 50$.

10. (a)

$$(1+i)c_1 + (1+\pi)c_2 = (2+i+\pi)M_1.$$

The absolute value of the slope is $(1+i)/(1+\pi)$. The horizontal intercept is $M_1(2+i+\pi)/(1+i)$. The vertical intercept is $M_1(2+i+\pi)/(1+\pi)$. The zero-savings point is $(M_1, M_2/(1+\pi)) = (M_1, M_1)$.

10. (b) $s_i = M_1 - c_1 = 500(i-0.2)/(1+i)$. This is always increasing, concave [The inverse function is convex, which is what you see visually if i is on the vertical axis.] At $i = 0$, $s_i = -100$, it crosses the vertical axis at $i = \pi = 0.2$, and as $i \to \infty$, $s_i \to 500$.

10. (c) $i = \sqrt{1.2} - 1$, approx 0.1, with $S = -50000/11$. Profit is approx $K + 5000/11$.

Chapter 13 Solutions

1.(a) $h_1 = 500 - \frac{1}{2}h_2$; $h_2 = 400 - \frac{1}{2}h_1$.

1.(b) $h_1^* = 400$; $h_2^* = 200$; $p^* = 30$; $\pi_1^* = 8,000$; $\pi_2^* = 2,000$.

2.(a) $h_1^* = 500$; $h_2^* = 0$; $p^* = 35$; $\pi^* = 12,500$; $\pi_1^* = 7,500$; $\pi_2^* = 5,000$.

2.(b) In principle, both firms have an incentive to cheat since the cartel agreement is not on either reaction function. In this case, clearly Corleone has an incentive to break the agreement by producing the monopoly output but not sending the check to the other family. Chung would be happy to receive the check from Corleone, but his best response to Corleone's cartel output is not zero, and hence he would also like to deviate. The answer may be more complex, as a function of the assumptions on observability of deviations and the kind of game that is actually being played after such deviations take place.

3.(a) $y_1^* = 3.75$; $y_2^* = 249,998.12$; $p^* = 7.50$; $\pi_1^* = 14.06$; $\pi_2^* = 624,995.31$.

3.(b) $y_1^C = 3.75$; $y_2^C = 249,998.12$. Note that the two answers are basically the same. The explanation is that MBI suffers from such a large cost disadvantage compared to Pear that MBI benefits very little from its first-mover advantage.

4.(a) $p_1^* = 128$; $p_2^* = 192$; $y_1^* = 48$; $y_2^* = 32$; $\pi_1^* = 2,304$; $\pi_2^* = 1,024$.

4.(b) The joint maximization makes no sense, as both firms could agree to price one good infinitely high to sell zero of it, but sell infinite amounts of the other good.

5.(a) $p_1^* = 36$; $p_2^* = 39$; $y_1^* = 11$; $y_2^* = 19$; $\pi_1^* = 132$; $\pi_2^* = 361$.

5.(b) $p_1^* = 35.62$; $p_2^* = 39.73$; $y_1^* = 11.62$; $y_2^* = 18.08$; $\pi_1^* = 135.02$; $\pi_2^* = 356.72$.

6. For homogeneous goods markets, we rank the models in order of increasing output and decreasing price: collusion (if successful), the Cournot and Stackelberg models, and the Bertrand model. For differentiated goods, collusion, price leadership, and the Bertrand model.

7. (a) $C(x) = 20x$.

7. (b) $x_1^* = x_2^* = 800,000/3$; $p = 100 - (2/3)80 = 53.33$; $\pi_1 = \pi_2 = (53.33 - 20)800,000/3 = 80,000,000/9$; $CS = (1/2)(2/3)800,000 \cdot (100 - 53.33)$, which equals $46.66 \cdot 800,000/3$.

7. (c) By symmetry, this gives $x_i^* = 800,000/(n+1)$; $p = 100 - [n/(n+1)]80$; $\pi_i = 80/(n+1)800,000/(n+1) = 64,000,000/(n+1)^2$; $CS = (1/2)[n/(n+1)]800,000 \cdot 80n/(n+1)$.

As $n \to \infty$, this gives $x_i^* \to 0$, with total output nx_i^* going to 800,000. The market price converges to $p = 20$, which would be the competitive price. Profit for each firm would go to zero. Consumers' surplus goes to $(1/2)800,000 \cdot 80$. These are the values in the competitive equilibrium. That is, we would get the competitive limit as $n \to \infty$.

8. (a) $y_1 = y_2 = \sqrt{2}$; $p = 16 - (2\sqrt{2})^2 = 8$; $\pi_1 = \pi_2 = 8\sqrt{2}$.

8. (b) In equilibrium, $p_1 = p_2 = 0$, and each firm sells to half of the market, i.e., 2 units. Profits are zero.

9. (a) $y = y_1 + y_2 = 4/\sqrt{3}$; $p = 16 - 16/3 = 32/3 = 10.67$; $\pi = 128/3\sqrt{3}$. Of course, there are infinite solutions to the cartel problem: any split of that total output between the two firms. We are not told either how the cartel profits are to be divided, so all we can give is the total profit for the cartel.

9. (b) $y = y_1 + y_2 = 4$; Of course, there are now also infinitely many prices, unit by unit, and cartel profits now are the value of that integral:

$$\int_0^4 (16 - y^2)dy = (16y - y^3/3)_0^4 = 64 - 64/3 = 128/3.$$

10. (a) First argue that there is no equilibrium where the firms post different prices. Treat all cases under this assumption. Finally, argue that there is no equilibrium where both firms post the same price. Again, argue all cases.

10. (b) You can check that $(p_1^*, p_2^*) = (0.50, 0.49)$ and $(p_1^{**}, p_2^{**}) = (0.27, 0.26)$ are both equilibria in the Bertrand game. There are many others, though.

Chapter 14 Solutions

1.(a)

		Player 2	
		Macho	Chicken
Player 1	Macho	0, 0	7, 2
	Chicken	2, 7	6, 6

1.(b) Neither player has a dominant strategy.

1.(c) The Nash equilibria are (Macho, Chicken) and (Chicken, Macho).

2.(a)

		Jill	
		Build	Don't Build
Jack	Build	1, 1	-1, 2
	Don't Build	2, -1	0, 0

2.(b) Both of them have a dominant strategy, which is "Not Build." The Nash equilibrium is (Not Build, Not Build).

2.(c) This game resembles the Prisoner's Dilemma. The Nash equilibrium (Not Build, Not Build) is not socially optimal. Both of them would be better off at (Build, Build).

3.(a) This is a sequential game where Sam is the first mover.

3.(b) $X < 0$.

4.(a) This is a sequential game with payoffs occurring only when the game is terminated at $t = 1, ..., 99$, and $t = 100$.

4.(b) Use backward induction to show that player 1 chooses "Terminate" at $t = 1$.

5.(a) Use backward induction to show that the winning strategy for player X is to take the total to $100 - 11a$ at $t = n - 2a$.

5.(b) There is a first-mover advantage in this game.

6. Show that the unique Nash equilibrium is $\left(p_1 = \frac{1}{2}, p_2 = \frac{1}{2}\right)$ and $y_1 = y_2 = \frac{1}{2}y$.

7. (a) In this game, there are no dominant strategies: there does not exist any strategy that is always a best response.

7. (b) There is a unique Nash equilibrium in pure strategies, and it is (Q, Q), with zero payoffs.

8. The backward induction solution has the two prisoners choosing D in every period, regardless of what has happened previously in the game.

9. (a) By symmetry, there will be exactly 3 units on each of the ABD and ACD routes. Total travel time will be 83 minutes.

9. (b) Let y be the number of cars travelling on the segment AB. Let z be the number of cars travelling on the segment BC. Then the number of cars travelling on the segment AC is $6 - y$ and the number on CD is $6 - y + z$. In equilibrium, the time on all three possible routes must be the same and thus we get $y = 4$ and $z = 2$. The total travel time is now 92 minutes. What is the explanation? Perhaps it can be related to the lesson learned in the expanded battle of the sexes.

10. First, suppose we have only the initial configuration. The efficient solution is $y = 3$, which is the same as the equilibrium solution.

With the new road built, the optimum is achieved when $z = 0$, $y = 3$, as if the new highway would not have been built.

Chapter 15 Solutions

1.(a) The Edgeworth box is a square, and each side is 6 units long, with Michael's origin on the lower left hand corner. The endowment point would be $(x_M, y_M) = (5, 1)$; $(x_A, y_A) = (1, 5)$

1.(b) Their indifference curves are L-shaped, with kinks where $x_i = y_i$.

1.(c) Any allocation in the area bounded by the indifference curves through the initial endowment is a Pareto improvement. There are many such allocations.

2.(a) Use the tangency condition and the feasibility condition to show this.

2.(b) Show that $x_g = \alpha$, $x_f = \alpha p_y/p_x$. Use the market-clearing condition to show this.

3.(a) Duncan's suggested allocation has the right totals: $x'_r + x'_t = x^0_r + x^0_t = 5$; $y'_r + y'_t = y^0_r + y^0_t = 5$.

3.(b) Rin's utility is unchanged at 4, and Tin's utility falls from 27 to 16. Duncan's suggested equilibrium allocation is not a Pareto improvement over the endowment.

3.(c) Rin's budget constraint is $x_r + y_r = 4$. His optimal consumption bundle is $(x''_r, y''_r) = (2, 2)$.

3.(d) Tin's budget constraint is $x_t + y_t = 6$. His optimal consumption bundle is $(x''_t, y''_t) = (2, 4)$.

3.(e) Duncan's suggested allocation and prices is not a competitive equilibrium. The totals add up, but consumers are not maximizing their utilities.

4.(a) Rin's budget constraint is $x_r + p_y y_r = 2 + 2p_y$. His optimal consumption bundle is $(x^*_r, y^*_r) = \left(1 + p_y, \frac{1+p_y}{p_y}\right)$.

4.(b) Tin's budget constraint is $x_t + p_y y_t = 3 + 3p_y$. His optimal consumption bundle is $(x^*_t, y^*_t) = \left(1 + p_y, \frac{2(1+p_y)}{p_y}\right)$.

4.(c) $x_r^* + x_t^* = x_r^0 + x_t^0 \quad \Leftrightarrow \quad p_y = \frac{3}{2}$.

4.(d) $(x_r^*, y_r^*) = \left(\frac{5}{2}, \frac{5}{3}\right)$; $(x_t^*, y_t^*) = \left(\frac{5}{2}, \frac{10}{3}\right)$; $p_y = \frac{3}{2}$.

5.(a) The original endowment is Pareto optimal. Shepard cannot be made better off without making Milne worse off.

5.(b) Milne's new budget constraint is $x_m + p_y y_m = 4p_y$. His optimal consumption bundle is $(x_m^*, y_m^*) = (p_y, 3)$.

5.(c) Shepard's new budget constraint is $x_s + p_y y_s = 4$. His optimal consumption bundle is $(x_s^*, y_s^*) = \left(2, \frac{2}{p_y}\right)$.

5.(d) $(x_m^*, y_m^*) = (2, 3)$; $(x_s^*, y_s^*) = (2, 1)$; $p_y = 2$

5.(e) Since $MRS^m = MRS^s = \frac{1}{2}$ and all goods are being consumed, the new equilibrium allocation is Pareto optimal.

6. Consider the case of an individual starting out with nothing. To induce him to consume positive amounts with per unit subsidies, you would have to sell him one or both goods at negative net prices.

7. (a) The Edgeworth box is a square of sides 1. The initial endowments is the bottom right corner. The indifference curves are hyperbolas for both consumers.

7. (b) The Pareto efficient allocations are found in the diagonal of the box that connects the two origins. The initial endowments are not P-e.

7. (c) $x_1 = 1/2$; $y_1 = 1/2$; $x_2 = 1/2$; $y_2 = 1/2$; and $p = 1$. (Notice that the competitive equilibrium is one of the P-e allocations).

8. (a) The Edgeworth box is a square of sides 1. The endowments are located at the center. Both indifference maps are hyperbolas, the one for 1 has steeper curves than the symmetric map of agent 2.

8. (b) $y_1 = x_1/(2 - x_1)$. This is a convex hyperbola that connects both origins. The initial endowment is not P-e: at it, $MRS_1 = 2$ while $MRS_2 = 1$.

8. (c) $p = 7/5$; $x_1 = 4/7$; $x_2 = 3/7$; $y_1 = 2/5$; $y_2 = 3/5$.

9. (a) The Edgeworth box is a square of sides 2. The endowment is at the center point. The indifference maps consist of linear curves, with slope -1 for 1 and -1/2 for 2.

9. (b) The P-e allocations are all the points at the bottom and right wall of the box.

9. (c) $p = 1$, $x_1 = 2$, $x_2 = 0$, $y_1 = 0$, $y_2 = 2$.

10. $p = 1/2$; $(x_1, y_1) = (2, 0)$ and $(x_2, y_2) = (0, 1)$. Donald ends up consuming the same bundle as before, while Vladimir is worse off.

Chapter 16 Solutions

1.(a) $x = 1$; $l = 1$.

1.(b) $x = 1$; $l = 1$; $w = \frac{1}{2}$; $\pi = \frac{1}{2}$.

2.(a) $x = \frac{16}{9}$; $l = \frac{64}{27}$.

2.(b) $x = \frac{16}{9}$; $l = \frac{64}{27}$; $w = \frac{1}{2}$; $\pi = \frac{16}{27}$.

3.(a) The Pareto efficient allocation is the tangency of the indifference curve and the production function.

3.(b) $x = \frac{3}{2}$; $l = \frac{1}{4}$; $w = 1$; $\pi = \frac{5}{4}$.

4.(a) $l_x = l_y = 2$.

4.(b) She studies $\sqrt{2}$ chapters of economics and $\sqrt{2}$ chapters of mathematics.

4.(c) $u = 0$.

4.(d) $u = 1.17$.

5.(a) $x^S = \frac{3}{5}p_y - \frac{2}{5}p_x$; $y^S = \frac{3}{5}p_x - \frac{2}{5}p_y$.

5.(b) $x^D = 1$; $y^D = 1$; $l^S = 5$.

5.(c) $p_x = p_y = 5$.

6. Robinson solves the following profit maximization problem:

$$\max_{x,y,z,l} \pi = p_x x + p_y y + p_z z - wl \quad \text{subject to} \quad l = l(x, y, z).$$

He solves the following utility maximization problem:

$$\max_{x,y,z,l} u = u(x, y, z, l) \quad \text{subject to} \quad p_x x + p_y y + p_z z \leq wl + \pi.$$

7. (a) $x^* = y^* = z^* = \sqrt{24/5} = \sqrt{4.8}$. From the inverse production function, we get that $l^* = 3 \cdot 4.8 = 14.4$.

7. (b) $x = y = z = \sqrt{24/5} = \sqrt{4.8}$; $l = 14.4$; $p_x = p_y = p_z = 2\sqrt{4.8}$.

8. (a) The production function is a straight line with slope 2 that comes out of the origin. The indifference curves are asymmetric hyperbolas whose preference direction goes in the northwest.

8. (b) $(l^*, f^*) = (8, 16)$.

8. (c) $w = 2$, $(l^S, f^D) = (l^D, f^S) = (8, 16)$. National income is equal to just labor income because profits are 0. Labor income is 16 (which of course also equals the value of the national product - output. As one can see, the equilibrium allocation is efficient (first welfare theorem). There is only one efficient allocation, so the second welfare theorem is less relevant here.

9. (a) The technologically feasible combinations are the 45-degree line of the quadrant and all points below it.

9. (b) The unique Pareto efficient allocation is the origin $(0, 0)$.

10. (a) There are infinitely-many Pareto efficient allocations: all combinations (l, x) where $x = l$ for $0 \leq l \leq 24$. Each of these is also a competitive equilibrium allocation, along with a relative price equal to 1, which yields zero profit for the firm.

10. (b) Now there is only one Pareto efficient allocation: $(l^*, x^*) = (24, 24\alpha)$. This allocation, along with a relative price p_l/p_x equal to α, is the only competitive equilibrium.

Chapter 17 Solutions

1.(a) $u_m(2) = 2$; $u_l(2) = 4$; $u_c(2) = -5$.

1.(b) $u_m(3) = 5$; $u_l(3) = 9$; $u_c(3) = -10$.

1. (c) Neither situation is Pareto superior to the other.

2.(a) $p^M = 18$; $h = 9$.

2.(b) $u_S = 324$; $u_G = 135$.

2.(c) The efficient number of plants is $p^* = 26$. Also, for example, a transfer of $65 from Gam to Sam would make both better off than at the equilibrium allocation.

3.(a) Each person blasts music 2 hours per day. Music is blasting 22 hours per day.

3.(b) $u_i = 2$.

3.(c) $m^* = \frac{5}{4}$. Note that everyone's utility is 3.125.

4.(a) $f^M = 8$; $b^M = 9$; $\pi_f^M = 275$; $\pi_b^M = 650$.

4.(b) $f^* = 17.2$; $b^* = 15.6$; $\pi_f^* = 140$; $\pi_b^* = 1,070$.

4.(c) Beatrice transfers between $135 and $420 to Flo.

5.(a) $c^M = 60$; $b^M = 90$.

5.(b) $c^* = 50$; $b^* = 100$.

5.(c) Since $\pi^* > \pi^M$, market profits are lower than the joint profit maximization outcome from (b), which gives the highest possible joint profit. The equilibrium is not Pareto optimal.

6.(a) Clyde pays the tax, which should be set at the marginal external cost at the efficient outcome: $t = b^* = 100$. The negative externality imposed on Bonnie by Clyde is now internalized by Clyde.

6.(b) Bonnie pays Clyde an amount between 500 and 950 to cut his output back to 50 units, from c^M to c^*.

7. $x_1 = 10$; $x_2 = 20$.

8. (a) $b^M = 1/2$, $a^M = 1/2$; profits of each firm are 1/4 for the bee keeper and 3/4 for the apple orchard.

8. (b) $a^* = 1/2$, $b^* = 1$. Profits of the joint ownership firm are 5/4 (greater than the earlier sum of profits).

8. (c) After the subsidy, $b = 1$ and the allocation is efficient. (You can check that the apple orchard's problem is unaffected).

Other solutions to the market failure problem: one is joint ownership of the two firms (part)b)). A third one would be to create a market for the externality.

9. (a)

$$p_x = p_y = 1; \qquad x_A^M = y_A^M = x_S^M = y_S^M = 5.$$

9. (b)

$$y_A = 1 + 0.9x_A.$$

9. (c) The Walrasian equilibrium allocation is not efficient. Indeed, at the equilibrium allocation, $MRS_A < MRS_S$, indicating that Alterix would like to transfer more good x to Selfy, because of altruism.

10. (a)

$$p_x = p_y = 1; \qquad x_H^M = y_H^M = x_S^M = y_S^M = 5.$$

10. (b)

$$y_H = -1 + 1.1x_H.$$

10. (c) The Walrasian equilibrium allocation is not efficient. Indeed, at the equilibrium allocation, $MRS_H > MRS_S$, indicating that Haterix would like to have Selfy consume less of good x, because of spite.

Chapter 18 Solutions

1.(a) The Nash equilibrium is (Don't Pay, Don't Pay).

		Paolo	
		Pay	Don't Pay
Fabio	Pay	50, 50	-100, 200
	Don't Pay	200, -100	0, 0

1.(b) There are now two Nash equilibria: (Pay, Don't Pay) and (Don't Pay, Pay).

		Paolo	
		Pay	Don't Pay
Fabio	Pay	250, 250	100, 400
	Don't Pay	400, 100	0, 0

2.(a) $(x_f, y_f) = (0.25, 9.75)$; $(x_p, y_p) = (0.25, 19.75)$; $u_f = 10.25$; $u_p = 20.25$.

2.(b) $u_f = 10.46 > 10.25$; $u_p = 20.46 > 20.25$.

2.(c) Use the Samuelson optimality condition to show that the allocation from part (a) is not Pareto optimal. If each pays half of the public good, $u_f = 10.5$ and $u_p = 20.5$.

3.(a) The maximum amount each pig is willing to contribute is as follows: $h_1 = \frac{1}{4}M_1$; $h_2 = \frac{1}{3}M_2$; $h_3 = \frac{1}{2}M_3$. To do this, one should find the point at which each pig is indifferent between getting the house making the payment, and having no house.

3.(b) What we can say is that, if each pig pays the maximum amount he is willing to pay, each would be indifferent between buying and not buying the house. There are no incentives to free-ride.

4.(a) $MRS_{x,y_i} = \frac{1}{10\sqrt{x}}$.

4.(b) $x^* = 10,000$.

5. If agent 2 were to contribute 0, agent 1 would like to contribute 1/4. But if agent 1 contributes 1/4, agent 2 would not like to contribute 0, so a zero contribution from agent 2 could never happen in equilibrium.

6. (a) It is efficient to build the park.

6.(b) $T_1 = 50$; $T_2 = 150$; $T_3 = 250$; $T_4 = 350$.

6.(c) The park gets built. $T_4 = 350$. Family 4's net benefit is zero, just as in part (b).

6.(d) The park does not get built. $T_4 = 0$. Family 4's net benefit is zero, whereas it was positive in part (b).

6.(e) $K = 37.50$.

7. A Nash equilibrium of the voluntary contributions game without the free-riding feature would be for each pig to contribute \$300 (pig 1), \$200 (pig 2), and \$50 (pig 3), but there are many others.

There are also many equilibria with the free-riding feature; for example, let pig 1 contribute 550 and the other two contribute 0.

8. (a) There are multiple market equilibria. Each of them is as follows: villager i contribute 0.01, i.e., one cent, while all the others contribute zero. The equilibrium expenditure 0.01 is obviously dismal, compared to the efficient expenditure, which was $10,000$.

8. (b) $x^E = 10,000$, which is the efficient production of festivals. Each villager would contribute 1/1000 of the cost, that is, 10 euros.

9.(a) The efficient decision is to purchase the medium quality unit.

9.(b) In this mechanism applied to this example, our prediction would be that free-riding would occur, although it would not interfere with efficiency. That is, the efficient purchase would still take place, but it would not be funded by all individuals.

10. (a) $x^* = 7$.

10.(b) Choose an arbitrary dwarf i. Then, $x_i = 7^{1/3}$ and $x_j = 0$ for all $j \neq i$.

Chapter 19 Solutions

1. Show that if consumer i pays consumer k \$13 to bear the risk, both of them would be better off.

2.(a) $E(L) = 185$

2.(b) $u_g(L) = 42,087.5$.

2.(c) $P \approx 290.13$.

2.(d) George is risk loving.

3.(a) $E(L) = 2.5$.

3.(b) $u_a(L) = 11$; $u_i(L) = 5$.

3.(c) Jack prefers to hike up the hill, and Jill prefers to draw water at the foot of the hill.

3.(d) Jack is risk loving; $P_a > E(L)$. Jill is risk neutral; $P_i = E(L)$.

4.(a) $E(L) = 4$.

4.(b) $u_a(L) = 1$; $u_m(L) = 4$; $u_s(L) = 256$.

4.(c) Adam and Michael will buy protection. Stella will not buy protection.

4.(d) $P \leq 4.95$.

5. He does not accept the lottery.

6. The net expected value of Will's lottery ticket is $-\frac{1}{3}$.

7. (a) He is risk-loving.

7. (b) $w \leq 82$.

8. The mistake Henry seems to be making is that the probabilities are not independent of the number of men that he will have.

9. Infinite.

10. (a) Because $11,000 < 990,000$.

10. (b) It is dominant to take both boxes, for any belief of this kind.

Chapter 20 Solutions

1.(a) Harry is willing to pay \$2,000, and he ends up buying type B and type C cars.

1.(b) Harry is willing to pay \$1,500, and he ends up buying type C cars only.

2.(a) The market price for used cars would be \$1,500.

2.(b) The market price for uninspected used cars would be $\$X/2$.

2.(c) $X = 600$.

2.(d) Any car worth less than \$600 will not get inspected, and there are 200 such cars. Each uninspected car will be sold for \$300.

3.(a) The expected loss for Placido is 600,000; for Jose is 200,000; and for Luciano is 100,000.

3.(b) Placido and Jose are risk loving; Luciano is risk averse.

3.(c) The expected payout per person is 300,000.

3.(d) Only Placido will buy insurance. The insurance company will make a loss because $E(Payout) = 600,000 \; > \; 300,000 =$ Price.

4.(a) Kevin will lock his door 100 percent of the time.

4.(b) \$100.

4.(c) \$300.

5.(a) In equilibrium, insurance policies cost \$300. Kevin locks his door 80 percent of the time, and his net money-like benefit is $H - 200$.

5.(b) If homeowners lock their doors 100 percent of the time, policies cost \$100, and each homeowner's net money-like benefit is $H - 100 \ > H - 200$. The benefit from having to lock the door only 80% of the time is only \$100, and yet it raises the cost of insurance by \$200. Even so, once insured, everyone will lock the door only 80% of the time.

6.(a) $c \approx 0.1844$; $E(\pi, e = 1) \approx 4.49$.

6.(b) $c \approx 0.4245$; $E(\pi, e = 1) \approx 5.47$.

6.(c) $c \geq 182.04\%$. The scheme is not workable, as the agent would have to be paid more than 100% of the output.

7.(a) Wage $= 1,600$, the expected productivity.

7.(b) $X < 600$ and $Y < 600$.

7.(c) $X > 400$ and $Y < 400$.

8. For any $\alpha \leq 1/3$, the equilibrium consists of all cars being sold at a price somwhere between 2,000 and $2,400 - 1,200\alpha$. However, when $\alpha > 1/3$, the equilibrium has only lemons trading, at a price somewhere between 1,000 and 1,200.

9. $c \geq 1400$.

10. The equilibrium in this example is one where nobody buys insurance, and nobody uses their cell phones while driving. Getting insurance would be too expensive, given the increase in premiums associated with the differential in the accident probabilities. In this sense, the market outcome does not display moral hazard (which would mean that agents are taking riskier actions because they have insurance).