Signaling to a Network of Consumers (Job market paper)

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Abstract

This paper asks how localized communication affects quality signaling. I present a game in which a monopolist seeks to sell a product of hidden, exogenous quality to consumers who share information locally with their neighbors in their social network. Equilibria exist in which the monopolist forgoes price signaling in favor of restricting and locating early sales to allow communication to reveal quality. The most profitable equilibrium for a monopolist with a high quality product always belongs to this class and, for a natural refinement of the equilibrium notion, is the unique equilibrium of the game. More generally, in all equilibria satisfying the Intuitive Criterion, the number of sales in the first round is bounded above by the network graph's domination number, which is the smallest number that allows communication to reveal type to all other consumers.

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1 Introduction

A common way for people to find out about a product's quality before they try it is to gather information from others who have already tried, perhaps by talking to friends, family or colleagues or reading customer testimony online. In turn, this communication is relevant for a firm that is introducing a product whose quality is unknown to consumers.

Examples of firms' responses to the nature of communication can be seen in many industries. Pharmaceutical companies introducing a new drug typically concentrate their marketing effort, and especially their direct personal marketing, on physicians who they identify as 'opinion leaders'[1]. In 2005 a national portrait studio identified mothers who engage in civic activity and invited them to discussion panels, where they received a sample of the firm's product and were encouraged to spread the word about it[2]. The agency marketing a new video game in late 2008 mailed promotional items to bloggers and media members they considered 'influential'[3]. A young Philadelphia clothing line identified 'key influencers' among reviewers active on their website, selected their top reviewer and invited her to pioneer an online styling campaign for the brand[4]. LinkedIn, an online business networking site, has reported that advertisers ask for help in identifying such 'influentials' in their network who are most likely to engage in communication about a product with others[5]. Godes and Mayzlin (2009) documents several further examples of such strategies in industries including television, fashion and office supplies.

These examples raise the question of how firms strategically respond to the structure of communication among consumers in deciding how to signal the quality of new products. This paper takes a first step to address this question by proposing a stylized model in which a monopolist seeks to sell a product of exogenous quality to consumers with unit demand who cannot directly observe quality before buying. The innovative features of this model are the local communication structure among consumers and that firms can strategically target the location of sales. I model communication with a network graph whose links capture channels along which information flows, and in each of two sales rounds, the firm can choose the set of consumers to whom they will offer the product and the uniform (round-dependent) price at which the members of that set will be able to buy. A consumer who is offered the product decides to buy or not, given their belief on quality, which can come from either

[1] See Nair, Manchanda, and Bhatia (2006). Coleman, Katz, and Menzel (1966) is the seminal study in this area.

[2] "Ad agency looked for influential moms", by Mary Jo Feldstein, *St. Louis Post-Dispatch*, August 18 2005

[3] "Gears of War Stages Direct Mail Promo", by Brian Quinton, *Promo Magazine*, December 1 2008

[4] "Free People engages with their community and blogs their Top Reviewer", by Heather Brunner, *Bazaarblog*, February 9th 2009

[5] "Online Influencers: How The New Opinion Leaders Drive Buzz On The Web", by Alice LaPlante, *InformationWeek*, May 5 2007

Bayesian updating or direct observation of a neighbor's experience. There are two means by which signaling could occur in this game: first, the firm could undertake some costly action to demonstrate that it is of the high type, for example by offering a price discount to early buyers; second, communication can bear the signaling burden by revealing the firm's type directly to some consumers without the need for those consumers to make any indirect inference.

This framework reveals a class of equilibria, which I call *network signaling equilibria*, in which high type firms forgo any signaling through a pricing policy in favor of signaling that restricts early sales to generate word-of-mouth and exploit network communication. The structure of all equilibria in this game that survive the Intuitive Criterion is dependent on the structure of the network. In particular, as long as the firm is sufficiently patient, the number of equilibrium sales in the first period is bounded above, so that communication is always exploited. This upper bound is the *domination number* of the graph of the social network, which in turn is the cardinality of a *minimum dominating set* of the graph (Theorem 1 below)[6]. This set is such that if the consumers in that set learn the firm's true type today, all other consumers will have learned the firm's type through communication by tomorrow. The result therefore means that whenever consumers communicate and sales can be targeted and staggered, a monopolist selling a product whose quality is unknown to consumers will *always* stagger sales as much as communication allows.

The equilibrium prediction becomes sharper still under the *passive conjectures* refinement (Rubinstein (1985), Rasmusen (2001)). This refinement requires that when consumers see an out-of-equilibrium action by the firm, they retain the population distribution of types as their belief of the firm's type. In that case, the strategies in the set of equilibria of the game never include price signaling by the firm, so that price itself is uninformative. Instead, as Theorem 2 below shows, the firm restricts period 1 sales to precisely a minimum dominating set of the graph of consumers, at a price sufficiently low that those consumers are willing to buy despite inferring nothing about the firm's true type, and allows communication to reveal its type to all other consumers before the second round. For example, in a star network (as in Figure 1 in Appendix B) in which one consumer is connected to all other consumers, none of whom are connected to each other, the firm will target the central consumer today and offer him a price just low enough to induce him to buy. Then, if the product is of high quality, the firm will sell to all other consumers tomorrow after they learn from me the quality of the product. If the high type firm relies on communication among consumers, the firm will never choose a pricing policy that separates it before the first round of the game as long as

[6]See, for example, Haynes, Hedetniemi, and Slater (1998) for a discussion of the domination number and minimum dominating set.

consumers' beliefs permit; high and low types thus pool in the first stage, and word-of-mouth separates them between stages.

The aim of this model is not to map one-to-one to any single industry, but rather to isolate the effect of the communication structure on the decision of a firm. In particular, in real-world industries the manner in which sales are made will vary, so that targeting sales could take many forms, from a direct salesperson visit as in the pharmaceutical industry example, to opening a movie in select cities. Nevertheless, the key features of this model are shared by firms' efforts in settings like these, and those discussed in the examples given above.

2 Related literature

The model I present is related to three strands of literature: networks, quality signaling and repeat purchase.

2.1 Social structure and firm decisions

There are several existing papers in which an outsider makes strategic choices whose payoff is affected by the network structure of insiders. In Galeotti and Goyal (2007) a firm chooses an advertising strategy to inform a community about its product but does not know the precise structure of the network. Quality is observable by consumers, informing consumers is costly, and there are no prices or trade.

Campbell (2008) models a monopolist seeking to inform about and then sell a good of a single, fixed quality to heterogenous consumers who communicate across a social network. The firm may choose by choosing price to restrict sales and generate word-of-mouth. The firm is able to directly inform consumers about its product by engaging in costly advertising, but may not choose the number and location of sales. Instead, sales can be restricted by setting price appropriately: the consumers may have heterogeneous valuation of the good, so raising price makes some of them unwilling to buy.

Navarro (2006) features consumers whose network transmits quality information and a firm which can choose quality and price once-and-for-all. Consumers simultaneously decide whether or not to buy, where their willingness-to-pay is consistent in equilibrium with the decisions of all other consumers and with the social structure that will transmit quality information to them depending on those decisions.

To this literature I introduce the possibility of targeting by producers, allowing the firm to choose a set of consumers to whom to make offers and so directly generate word-of-mouth in locations of their choice. I am also interested explicitly in the signaling problem with fixed types, as opposed to either the problem of informing but not selling to the network, or of the firm choosing product quality.

2.2 Repeat purchase

The nature of the problem studied here has much in common with the repeat purchase literature: information about quality is asymmetric and firms with a high quality product receive a benefit tomorrow of sales today. For example, Wolinsky (1983) models the situation in which firms provide products to consumers with imperfect information about quality, although in that case quality is again itself a choice variable for the firm. A general difference between repeat purchase games and the model presented here is that in the repeat purchase literature high type firms generally receive the benefit of informing consumers today due to existing consumers learning quality and choosing to buy again. In this paper, each consumer seeks to buy only once, and the benefit to the high type firm accrues through informing neighbors to consumers who buy of the true quality of their product.

2.3 Quality signaling and word-of-mouth

The quality signaling literature has also frequently modeled word-of-mouth communication. Bagwell and Riordan (1991) presents a model in which firms with a high quality product exploit the production cost differential with low quality products to signal type, and the fraction of informed consumers in the population can change over time. Kennedy (1994) assumes overlapping generations of consumers and a firm whose only strategic variable is price and demonstrates that absent an appropriate cost differential, a low and rising price can be a signal of quality in settings where word-of-mouth communication can reveal type to future consumers. In this paper I also assume a similar cost structure but, by contrast, the firm faces a single, static population and is able to perfectly target specific consumers.

Experimental evidence on the topic of word-of-mouth is presented in Miller and Plott (1985) and more recently Godes and Mayzlin (2009). The latter reports results of both a field experiment and a laboratory experiment, and suggests that the most effective word-of-mouth to drive sales is that which comes from less loyal customers, but that *identifying* effective spreaders of information is more useful among more loyal customers. Similarly, Godes and Mayzlin (2004) is primarily concerned with the appropriate measurement of word-of-mouth communication, in order to draw inference on its effects on sales. The results imply that "more dispersed buzz may be better than concentrated buzz", which finds an analog here: while there is no intensity of communication in the model presented below, it is nevertheless true that the firm may find it profitable to initiate word-of-mouth at many locations throughout the network.

3 Describing the game

Consider a dynamic game with incomplete information. A firm will encounter a set of consumers arranged in a social network, and has two periods in which to make sales offers to consumers. Each time the firm can choose a price and a subset of consumers to whom the price will be offered, and the consumers who receive a price offer can choose to accept or reject it. Between the two rounds, communication will reveal details of round 1 transactions to neighbors of those consumers who transacted. I will formalize this game as follows:

3.0.1 Players

There is a set of $\mathcal{C} = \{1, ..., N\}$ risk-neutral consumers, arranged in some **social network** $\mathcal{S}$, defined as a connected graph with N vertices. Consumer j is considered a "neighbor" to consumer k if there is an edge between them. Denote some set of consumers Γ, where $\Gamma \subseteq \mathcal{C}$, and let $\gamma = |\Gamma|$, the cardinality of some such set.

The relevant parameters of the social network are as follows:

Definition 1. *Define:*

1. *The* ***network size****, denoted N, is the number of consumers in the social network.*

2. *Each consumer i is connected to a set Ω_i of* ***neighbors****, where $\omega_i = |\Omega_i|$. Denote the set of neighbors to any consumer in some Γ with $\Omega(\Gamma)$.*

3. *A dominating set of the network graph is some $\Gamma \subseteq \mathcal{C}$ such that $\Gamma \cup \Omega(\Gamma) = \mathcal{C}$. A* ***minimum dominating set****, denoted Γ^*, is a dominating set with the lowest possible cardinality.*

4. *The* ***domination number****, denoted $\gamma^* = |\Gamma^*|$, is the cardinality of a minimum dominating set; it is the smallest number of consumers such that marking γ^* would result in every single consumer either being marked or having a marked neighbor.*

For discussion of the concepts of dominating sets, minimum dominating sets and domination numbers, see, for example, Haynes, Hedetniemi, and Slater (1998).

While γ^* is unique for a given network, Γ^* is not. There will generally be multiple sets Γ^* that can satisfy γ^* in a network. Note that since $\mathcal{S}$ is connected, the domination number cannot exceed $\frac{1}{2}n$ (established in Ore (1962)).

Appendix B illustrates these concepts for the star and circular networks. For example, a star (or "hub and spoke") network would have $\gamma^* = 1$ since marking the hub consumer means that the spokes are then all connected to a marked consumer. A network in which every consumer is connected to every other consumer similarly has $\gamma^* = 1$. A circular network with, say, 6 members has $\gamma^* = 2$.

There is one firm which produces a good of quality $q \in \{q_L, q_H\}$, where q_H is realized with probability α. Call the expected quality $\bar{q} \equiv \alpha q_H + (1-\alpha) q_L$, where $q_L < 0 < \bar{q} < q_H$ (note that it would be possible to drop the assumption that consumers are risk-neutral provided we redefine this $\bar{q}$ as a willingness to pay for a product of unknown quality). Production costs are zero for both types of firm. This could be considered a normalization of production costs, or more literally that the goods to be sold have already been produced and the problem for the firm in this model is simply to sell them. Zero production costs does not imply a violation of the single-crossing property, since that requirement - that the high type has lower signaling costs than the low type - can here be satisfied by the existence of communication among consumers. The assumption of zero production costs guarantees that either type of firm is always willing to sell at any positive price if possible.

3.0.2 Information

Consumers are familiar with the distribution of quality but cannot directly observe the quality of good provided by the firm. It is not possible for the firm and consumer to write contingent contracts, such as warranties, legal recourse or money back guarantees. Each consumer follows a simple purchasing rule: they will always be willing and able to purchase the good when the price does not exceed expected quality.

The firm is fully informed of the whole game at all stages, including the quality of its own good and the structure of the social network.

3.0.3 Moves

The firm and consumers play a two-round market game. Let $t = 1, 2$ index the rounds. The structure of play is as follows:

0 Nature chooses $q \in \{q_L, q_H\}$.

1 a The firm chooses price p_1 and the set Γ_1 of consumers to whom the price will be offered.

b Sales round 1: consumers observe p_1. If consumer i was offered a price, she chooses to buy ($b = 1$) or not buy ($b = 0$).

c If consumer i accepts the sale, consumer i and each neighbor of consumer i observe the quality of the good.

2 a The firm chooses price p_2 and the set Γ_2 of consumers to whom the price will be offered.

b Sales round 2: consumers observe p_2. If consumer j was offered a price, she chooses to buy ($b = 1$) or not buy ($b = 0$).

The strategic variables for the firm are therefore the two price and visitation policies, making a strategy for the firm as follows: $\sigma_F = \{p_1, p_2, \Gamma_1, \Gamma_2\}$, where p_t specifies the price at time t and the set Γ_t of consumers to whom the price is offered (let the count of consumers in Γ_t be γ_t). The stage 2 strategy for the firm will depend on history (the firm's own stage 1 offer and the profile of consumer responses).

The firm can choose a policy that can vary price according to round and can specify the set of consumers to which each round's price will be offered. Stage 1c is the "communication" stage, in which any purchases made in stage 1 are observed by the neighbors of the consumer who made the purchase. To simplify the analysis we assume that a consumer may buy only once during the whole game. That is, if consumer i purchases during stage 1 she plays no part in stage 2.

A strategy for consumer i who is offered a price in stage 1 is $\sigma_{1,i} = \{b|p_1, \Gamma_1\}$. A strategy for consumer i who is offered a price in stage 2 also incorporates information set I gained in stage 1: $\sigma_{2,i} = \{b|p_1, \Gamma_1, p_2, \Gamma_2, I\}$. In round 2, let the set of informed consumers - those who have a neighbor who purchased in round 1 - be $\mathcal{I}$. Call the belief held by household i on the firm's quality at some time $\hat{q}$. In sales round 1 this will be $\hat{q}(p_1)$ and in sales round 2 $\hat{q}(p_2, I)$.

3.0.4 Payoffs

The firm discounts round 2 payoffs with the discount factor δ. The consumers may discount at any rate; this does not materially change the game, since the simplicity of the consumer side means that the consumers' discounting serves only to define the value of the consumers' maximal willingness to pay in round 2, which is throughout taken as given.

I will make the following assumption on the firm's discount factor:

Assumption 1. *Let $\delta > \frac{\bar{q}}{q_H}$, so that all else equal the firm prefers a sale at price q_H in round 2 to a sale at $\bar{q}$ in round 1.*

The ex ante payoff to a household i which buys at any point during the game at a price p is as follows:

$$E(U_i) = \hat{q} - p \tag{1}$$

$\hat{q}$ is the belief held by the consumer at the time of purchase, which may have been updated from their prior held at the start of the game. The payoff to the firm is as follows, where the γ_t is the number of buyers in each round:

$$\pi = \gamma_1 p_1 + \delta \gamma_2 p_2 \tag{2}$$

The game I describe differs slightly from a graphical game or a network game, as defined in Jackson and Yariv (2008); here, a consumer's location in the network determines what information she sees, but neighbors' actions do not *directly* affect the payoff earned by a consumer.

3.1 Consumer talk

3.1.1 Communication

Communication is modeled as a stage between the two rounds in which neighbors of consumers who accept an offer in sales round 1 observe truly and certainly the quality of the good sold by the firm who made the accepted offer. Recall that quality is not a choice variable for the firm; this reveals with certainty the quality of a good that the neighbor will receive from that firm if they should accept an offer in sales round 2. Communication is not a strategic choice by the consumers, and can travel no further than one degree, to direct neighbors.

Network links are very general here. A link from consumer i to another consumer could represent anything from a relationship with an acquaintance to readership of media. It may be reasonable to think of a very well connected "consumer" as a reviewer, critic or other journalist: their experience of the unknown product diffuses information to many more consumers than another individual simply talking to his friends.

The structure of communication is entirely exogenous and static. This rules out any strategic action by the firm that is designed to manipulate the information a consumer sees. In particular, real-world marketing choices that purport to recommend to a consumer items that other consumers like them have bought (especially popular among internet retailers like Amazon and Netflix) are ruled out here, although it would be possible to incorporate this into the model as the firm strategically choosing to create links between consumers in the network.

With communication potentially substituting for other means of signaling, the single-crossing property of different types having different signaling costs, familiar since Spence (1973), can be satisfied somewhat differently in the game with communication than it is in the game without. Communication itself implies satisfaction of the single-crossing property: when consumers talk, for firms with low quality products the revelation of quality by communication is a cost, but for firms with high quality goods it is a benefit.

3.1.2 Signaling

For the firm to 'signal' means to take some action that reveals its type to some or all consumers. In this game, firms choose price and visitation in each sales round, so signaling can take place using any of these four choices. In particular, it is possible for signaling to take place before or after the first round of sales. The communication among consumers that takes place between rounds facilitates signaling based on visitation, since it is possible that type is revealed to some consumers between sales rounds by this mechanism.

There is thus a distinction between 'active' signaling using price as the informative variable and 'passive' signaling using communication. In the latter case the set of consumers chosen by the firm is not necessarily informative in itself, but instead facilitates word-of-mouth that reveals the firm's type to other consumers.

4 Equilibria

Define a perfect Bayesian equilibrium in this game:

Definition 2. *A **perfect Bayesian equilibrium** in the quality signaling game is a strategy profile and belief structure such that:*

1. *All types of firm choose a strategy to maximize their payoff given the effect of their choice on the consumers' actions.*

2. *All consumers react optimally to the firm's action given their posterior beliefs about the firm's type.*

3. *Beliefs are obtained from equilibrium strategies and observed actions using Bayes' rule.*

The components of the equilibrium are strategies for each type of firm $\sigma_{F,j} = \{p_1, p_2, \Gamma_1, \Gamma_2\}$*,* $j = H, L$*, strategies for each consumer* $\sigma_{1,i} = \{b|p_1, \Gamma_1\}$*,* $\sigma_{2,i} = \{b|p_1, \Gamma_1, p_2, \Gamma_2, I\}$ *and supporting beliefs.*

Define also the Intuitive Criterion:

Definition 3. *By the **Intuitive Criterion** (Cho and Kreps (1987)), if there exists a type of firm who could not benefit from an off-equilibrium action no matter what beliefs are held by the consumers, then the consumers' beliefs must put zero probability on that type of firm being the one which takes the off-equilibrium action, when that action is observed.*

I will describe the equilibria in this game by class, and then apply the Intuitive Criterion. There are three broad classes:

Round 1	Round 2	Name
Separated	Separated	Price signaling
Pooled	Pooled	
Pooled	Separated	Network signaling

First, the case in which the two types are separated before the first sales round; second, the case in which the two types pool and both types make sales in both periods; third, the case in which the types pool in the first round and are separated by word-of-mouth between rounds. Of these, the third class is novel, being that class where the firm completely forgoes price signaling in favor of signaling using a visitation strategy that exploits network communication. By contrast, the other classes of equilibria are similar in spirit to pooling and separating equilibria in a traditional single-principal, single-agent signaling game.

Where an equilibrium can be supported by more than one strategy - for example if the low type firm is indifferent between making offers which will not be accepted in equilibrium and making no offers - I will restrict the description of the equilibrium to just one supporting strategy for convenience. Generally, the salient features of each equilibrium class can be supported by either pooling *or* separating strategies by the firms. Superscripts on strategic variables refer to the class of equilibria.

4.1 Equilibria surviving application of the Intuitive Criterion

In this section I will determine those equilibria which survive the application of the Intuitive Criterion. First I will illustrate those equilibria that correspond to signaling equilibria in typical one-stage signaling games, which in turn establishes a lower bound on the high type payoff. Next I show that there must be offers made in stage 2, and then establish an upper bound on the number of offers that are made in stage 1. Together these results define the set of equilibria that survive the application of the Intuitive Criterion.

4.1.1 Separating by up-front price signaling

There exist equilibria in this game analogous to separating equilibria in one-stage signaling games: the high type firm takes an action to distinguish itself from the low type firm. Here this involves setting round 1 price p_1 low enough so as to preclude profitable imitation by the low type. In these equilibria, the high type firm makes sales in both rounds and the low type makes no sales in either round.

Definition 4. *A* ***price signaling equilibrium*** *features price signaling by the high type before round 1, so that no sale takes place in either round to a consumer who does not know the true type of the firm.*

An example of a price signaling equilibrium in this game is:

$$\begin{aligned}
\sigma_{F,L} &: \{p_1^1, p_2^1, \emptyset, \emptyset\} \\
\sigma_{F,H} &: \{\hat{p}, q_H, \Gamma_1^1, N - \Gamma_1^1\} \\
\sigma_{1,i} &: \{(b = 1 | p_1(i) \leq \hat{p} \wedge \Gamma_1 = \Gamma_1^1), (b = 0 | p_1(i) > \hat{p} \vee \Gamma_1 \neq \Gamma^1)\} \\
\sigma_{2,i} &: \{(b = 1)\}
\end{aligned}$$

All consumers receive an expected payoff of zero. The payoff to each type of firm is as follows:

$$\pi_{L,S} = 0 \tag{3}$$

$$\pi_{H,S} = \gamma_1 \hat{p} + \delta(N - \gamma_1) q_H \tag{4}$$

With supporting beliefs:

$$(Pr(low) = 1 | p_1(i) \geq \hat{p} \vee \Gamma_1 \neq \Gamma_1^1) \tag{5}$$

$$(\hat{q}_2 = q | i \in \mathcal{I}), (\hat{q}_2 = q_H | i \notin \mathcal{I}) \tag{6}$$

In this equilibrium high types offer a discount to the households in Γ_1 sufficient to both preclude profitable imitation by low types in stage 1 itself and to inform all consumers of their true type before round 2, regardless of whether the consumer actually observed true quality during the communication stage.

Lemma 1. *At most one high type payoff value is realized in price signaling equilibria that survive the application of the Intuitive Criterion.*

The proof of this result, along with those of all others to follow, appears in the appendix. By separating types immediately, the strategy employed by the high type firm in the price separating equilibrium effectively collapses the two-stage market game to be equivalent to the traditional one-period signaling game. Lemma 1 is therefore analogous to the result in Cho and Kreps (1987) that in the two-type Spence signaling model, the only equilibrium surviving the Intuitive Criterion is the "Riley outcome", the separating equilibrium with the least amount of inefficient signaling. Here, in the class of equilibria in which separation occurs before the first sales round, only those with the least amount of inefficient signaling survive the Intuitive Criterion

What is the exact nature of the surviving price signaling equilibria?

Lemma 2. *All price signaling equilibria surviving the Intuitive Criterion feature the same high type strategy and payoff as the following:*

$$\begin{aligned}
\sigma_{F,L} &: \{p_1^1, p_2^1, \emptyset, \emptyset\} \\
\sigma_{F,H} &: \{0, q_H, \Gamma^*, N - \Gamma^*\} \\
\sigma_{1,i} &: \{(b = 1|p_1(i) \leq 0), (b = 0|p_1(i) > 0))\} \\
\sigma_{2,i} &: \{(b = 1|p_2(i) \leq q_H), (b = 0|p_2(i) > q_H))\}
\end{aligned}$$

All consumers receive an expected payoff of zero. The payoff to each type of firm is as follows:

$$\pi_{L,S} = 0 \tag{7}$$

$$\pi_{H,S} = \delta(N - \gamma^*)q_H \tag{8}$$

With appropriate supporting beliefs.

Again, the surviving price signaling equilibria take on exactly the same form as the familiar Intuitive Criterion-compatible separating equilibria in standard signaling games: the high type firm undertakes the least-cost action to distinguish itself from the low type. The least-cost action is to offer a price of 0 to consumers in Γ^* in round 1 and to exploit the revelation of type by offering a price of q_H to all other consumers. Note also that the strategy employed by the high type firm to realize price signaling equilibria is unique in the case of networks with a unique minimum dominating set.

As a corollary, Lemma 2 guarantees a lower bound on the payoff of the high type under the Intuitive Criterion, which in turn guarantees that no equilibrium in which no sales are made survives the Intuitive Criterion.

Lemma 3. *No equilibrium in which the high type receives a payoff lower than $\pi_H = \delta(N - \gamma^*)q_H$ survives the application of the Intuitive Criterion.*

The price signaling payoff serves as a 'fallback' to which the high type firm can deviate.

4.1.2 Pooling

Next, consider the cases in which both types of firm make sales in both rounds. This can only be equilibrium play either if a 'pooling' price - one which does not exceed average quality - is set by both types in both rounds, or if there are no sales in the second round. In either case this means that in these equilibria the high type does not attempt to exploit its distinction from a low type, even though neighbors of consumers who buy from the high type in round 1 learn of its type before round 2.

Definition 5. *A **pooling** equilibrium has* $p_1^2 \leq \bar{q}, p_2^2 \leq \bar{q}$.

An example of an equilibrium of this type is:

$$\begin{aligned}
\sigma_{F,L} &: \{p_1^2 \leq \bar{q}, p_2^2 \leq \bar{q}, \Gamma_1^2, \Gamma_2^2\} \\
\sigma_{F,H} &: \{p_1^2 \leq \bar{q}, p_2^2 \leq \bar{q}, \Gamma_1^2, \Gamma_2^2\} \\
\sigma_{1,i} &: \{(b = 1|p_1(i) = p_1^2 \wedge \Gamma_1 = \Gamma_1^2), (b = 0|p_1(i) \neq p_1^2 \vee \Gamma_1 \neq \Gamma_1^2)\} \\
\sigma_{2,i\in\Omega_{\Gamma_1}} &: \{(b = 1|q = q_H), (b = 0|q = q_L)\} \\
\sigma_{2,i\notin\Omega_{\Gamma_1}} &: \{(b = 1|p_2(i) \leq \hat{q}), (b = 0|p_2(i) > \hat{q})\}
\end{aligned}$$

All consumers receive an expected payoff of zero. The payoff to each type of firm is as follows:

$$\pi_{L,P2} = \gamma_1^2 p_1^2 + \delta(\omega_{\gamma_1} - \gamma_2^2)p_2^2 \tag{9}$$

$$\pi_{H,P2} = \gamma_1^2 p_1^2 + \delta\gamma_2^2 p_2^2 \tag{10}$$

With supporting beliefs:

$$(Pr(low) = 1|p_1(i) \neq p_1^2 \vee \Gamma_1 \neq \Gamma_1^2) \tag{11}$$

$$(\hat{q}_2 = q|i \in \mathcal{I}), (\hat{q}_2 = \bar{q}|i \notin \mathcal{I}) \tag{12}$$

We can quickly rule out cases in which no offers are made in the second round. Pooling equilibria in which $\Gamma_2^2 = \emptyset$ do not survive the application of the Intuitive Criterion:

Lemma 4. *No equilibrium in which no offers are made in the second sales round survives the application of the Intuitive Criterion.*

In the surviving pooling equilibria, both types of firm offer some price to a given group in round 2, and some price to a given nonempty group in round 2. The high type firm makes no attempt to exploit the revelation of information about its type.

Equilibria of this type may well survive the Intuitive Criterion, provided that the high type firm prefers making sales at p_2 to Γ_2 rather than making sales at q_H to informed neighbors of Γ_1. Conversely, however, if the high type firm prefers to sell at q_H to neighbors of Γ_1 than to sell at some p_2 to Γ_2, that equilibrium in this class does not survive application of the Intuitive Criterion.

4.1.3 Separating by between-round word-of-mouth

This class of equilibria exclusively uses communication rather than price signaling. In this class, firms with a high quality product make no attempt to distinguish themselves via price from low type firms before sales round 1, but instead choose to restrict round 1 sales so that their type is revealed by word-of-mouth during the communication stage. They can then exploit this in their round 2 strategy. High type firms sell in both rounds, and low type firms sell only in round 1.[7]

Definition 6. *A* ***network signaling equilibrium*** *has a common, pooled price p in round 1 for both types of firm, and a price q_H in round 2 for at least the high type firm.*

An example of a network signaling equilibrium is:

$$\begin{aligned}
\sigma_{F,L} &: \{p, q_H, \Gamma_1^3, N - \Gamma_1^3\} \\
\sigma_{F,H} &: \{p, q_H, \Gamma_1^3, N - \Gamma_1^3\} \\
\sigma_{1,i} &: \{(b = 1 | p_1(i) = p \wedge \Gamma_1 = \Gamma_1^3), (b = 0 | p_1(i) \neq p \vee \Gamma_1 \neq \Gamma_1^3)\} \\
\sigma_{2,i} &: \{(b = 1 | p_2(i) \leq \hat{q}), (b = 0 | p_2(i) > \hat{q})\}
\end{aligned}$$

All consumers receive an expected payoff of zero. Let $\omega(\gamma_1^3)$ be the count of all consumers who are neighbors of consumers in Γ_1^3. The payoff to each type of firm is as follows:

$$\pi_{L,P2} = \gamma_1^3 p \tag{13}$$

$$\pi_{H,P2} = \gamma_1^3 p + \delta\omega(\gamma_1^3) q_H \tag{14}$$

With supporting beliefs:

$$(Pr(low) = 1 | p_1(i) \neq p, \Gamma_1 \neq \Gamma_1^3) \tag{15}$$

$$(\hat{q}_2 = q | i \in \mathcal{I}), (\hat{q}_2 = \bar{q} | i \notin \mathcal{I}) \tag{16}$$

Both types of firm set a price of p in sales round 1 and make this offer to those consumers who belong to the set Γ_1^3, and in sales round 2 the high type firm offers a price of q_H to neighbors of Γ_1^3, that is, those consumers who learn the firm's type during the communication stage. This strategy is also that to which a high type firm could deviate to kill the no-signaling equilibria, should the firm wish to do so.

[7] Note that although the firms are 'separated' by word-of-mouth and experience different round 2 outcomes, in equilibria of this class the two types of firm may choose identical strategies in round 2, since the low type firm will be indifferent between mimicking the high type strategy (thus making no sales) and setting some different strategy.

This word-of-mouth driven class of equilibria helps to dictate an upper bound on the number of sales in round 1 in any equilibrium that survives the application of the Intuitive Criterion:

Lemma 5. *No equilibrium in which $\gamma_1 > \gamma^*$, $\gamma_2 < N - \gamma^*$ survives the application of the Intuitive Criterion.*

This result completes the characterization of surviving equilibria.

4.2 Surviving equilibria

Theorem 1. *The set of surviving equilibria consists of:*

1. *the price signaling equilibria described in Lemma 2, in which the high type plays $p_1 = 0$, $\Gamma_1 = \Gamma^*$, $p_2 = q_H$, $\Gamma_2 = N - \Gamma^*$.*

2. *the set of network signaling equilibria that satisfy Lemma 3 and Lemma 5, in which the high type play $p_1 \leq \bar{q}$, $\gamma_1 \leq \gamma^*$, $p_2 = q_H$, $\gamma_2 = N - \gamma_1$ and $\pi_H > \delta(N - \gamma^*)q_H$.*

3. *the set of pooling equilibria that satisfy Lemma 3 and Lemma 5, in which the high type play $p_1 \leq \bar{q}$, $\gamma_1 \leq \gamma^*$, $p_2 \leq \bar{q}$, Γ_2 and $\pi_H > \delta(N - \gamma^*)q_H$.*

This follows directly from the Lemmas. It is apparent that applying the Intuitive Criterion does not select a unique equilibrium in this game; nevertheless, we can draw some concrete conclusions from Theorem 1. No equilibrium features more than γ^* offers in sales round 1, so that the monopolist never visits more than a minimum dominating set of consumers in any equilibrium that survives the application of the Intuitive Criterion. The lower the domination number γ^*, the more work the social network structure does to spread news of the firm's type, and so the lower the number of consumers who are targeted in round 1 as part of a signaling strategy to reveal type before round 2.

Comparing the surviving equilibria, we can see that the equilibrium with up-front price signaling is less profitable to the high type firm than at least some of the network signaling equilibria in which the high type pools on some group with the low type in round 1 and allows communication to signal their type to other consumers.

If the payoff to the high type firm in the surviving price signaling equilibria *was* higher than in any network signaling equilibrium, the price signaling equilibrium will be the unique perfect Bayesian equilibrium in this game that survives the Intuitive Criterion. However, it is emphatically *not* the case that the price signaling equilibrium is more profitable than any network signaling equilibrium.

To see this, consider the most profitable network signaling equilibrium, in which the two types of firm 'pool' on Γ^* at the maximal pooled price, $\bar{q}$, in round 1, and the high type sells at q_H in round 2 to $N - \Gamma^*$; this can be supported, for example, by the following strategies and suitable beliefs:

$$\sigma_{F,L} : \{\bar{q}, q_H, \Gamma^*, N - \Gamma^*\}$$
$$\sigma_{F,H} : \{\bar{q}, q_H, \Gamma^*, N - \Gamma^*\}$$
$$\sigma_{1,i} : \{(b = 1 | p_1(i) = \bar{q}), (b = 0 | p_1(i) \neq \bar{q})\}$$
$$\sigma_{2,i} : \{(b = 1 | p_2(i) \leq \hat{q}), (b = 0 | p_2(i) > \hat{q})\}$$

All consumers receive an expected payoff of zero. The payoff to each type of firm is as follows:

$$\pi_L = \gamma^* \bar{q} \tag{17}$$
$$\pi_H = \gamma^* \bar{q} + \delta(N - \gamma^*) q_H \tag{18}$$

This is clearly more profitable for the high type firm than the price signaling equilibrium. The fundamental difference between the two classes of equilibria is, again, that in the network signaling case the types are not separated in round 1, while in the price signaling case the high type firm prices at a discount sufficient to separate before round 1. This means that the analog of the Riley outcome in this game is *not* the efficient price signaling equilibrium, but rather the efficient network signaling equilibrium. Minimizing inefficient signaling here means exploiting word-of-mouth rather than simply engaging in the least amount of inefficient price signaling.

On the consumer side, all surviving equilibria yield zero payoff to consumers in equilibrium, since the bargaining power in the pricing game was given to the firm.

The Intuitive Criterion clearly admits a large class of equilibria. In particular, it cannot put a *lower* bound on Γ_1, the set of consumers to whom offers are made in round 1. In the extreme, if consumers believe that an out-of-equilibrium $\gamma_1 > 0$ must come from a low type, $\Gamma_1 = 0$ is admitted as a network signaling or no-signaling equilibrium, despite the firm's clear incentive to generate word-of-mouth, if the round 2 component of the equilibrium makes it more profitable than the fallback price signaling equilibrium. This extreme case could be 'blamed' on the extreme pessimism of those supporting beliefs. In the following section I will explore an equilibrium refinement that will restrict further consumers' beliefs and select a unique equilibrium in this game.

5 Equilibria surviving passive conjectures

The divinity criterion of Banks and Sobel (1987) offers no further help in refining the set of equilibria in this game. In particular, it is not possible to use this further refinement to select out the "inefficient" network signaling equilibria, despite the intuitive appeal of the most profitable of that class. If consumer beliefs support $p_1 < \bar{q}$ or $\gamma_1 < \gamma^*$, any deviation to a higher round 1 price or a larger round 1 clientele will always be equally beneficial to both the high and low types, and thus the divinity refinement (or, indeed, the related D1 criterion) cannot restrict consumer beliefs in the face of such a deviation in a way that would rule out the original strategies as equilibrium play.

This is certainly related to the abandonment of static single-crossing in the setup of the game. The class of network signaling equilibria feature pooling in round 1 and separating in round 2, and while the nature of single-crossing in this game has ruled out those network signaling equilibria that do not conform to Lemma 5, it cannot be of any further help now.

Consider again the "efficient" network signaling equilibrium:

$$\begin{aligned}
\sigma_{F,L} &: \{\bar{q}, q_H, \Gamma^*, N - \Gamma^*\} \\
\sigma_{F,H} &: \{\bar{q}, q_H, \Gamma^*, N - \Gamma^*\} \\
\sigma_{1,i} &: \{(b = 1 | p_1(i) = \bar{q}), (b = 0 | p_1(i) \neq \bar{q})\} \\
\sigma_{2,i} &: \{(b = 1 | p_2(i) \leq q_H), (b = 0 | p_2(i) > q_H)\}
\end{aligned}$$

All consumers receive an expected payoff of zero. The payoff to each type of firm is as follows:

$$\pi_L = \gamma^* \bar{q} \tag{19}$$

$$\pi_H = \gamma^* \bar{q} + \delta(N - \gamma^*) q_H \tag{20}$$

It is unambiguously true that this is preferred by the high type firm to the price signaling equilibrium, since the round 1 pooling is certainly preferred to incurring the cost of making the price discount to separate in round 1. Again, however, this is not evidence that the price signaling case does not survive refinement: since any deviation from the price signaling case to this network signaling case is beneficial for either type, neither the Intuitive Criterion nor divinity can restrict consumer beliefs in the face of such a deviation.

What restriction *would* be sufficient to guarantee that this equilibrium was unique in the game? One possibility would be to impose a **passive conjectures** property, as discussed in, for example, Rubinstein (1985) and Rasmusen (2001). It is defined here as follows:

Definition 7. ***Passive conjectures***: *When consumers see an out of equilibrium action* *that would be profitable for either type of firm if it was accepted, they maintain their prior beliefs, which place probability α on the deviation coming from a high type and probability $1-\alpha$ on the deviation coming from a low type, the true exogenous propensities of each type of firm.*

Passive conjectures can be motivated as the consumers perceiving out-of-equilibrium play as a mistake that is equally likely to be made by either type of firm, or as uninformative about type for some other reason. The key feature of passive conjectures is that when a consumer is offered the good under a price and visitation menu they did not anticipate, they are still willing to purchase the good if the unexpected price does not exceed mean product quality in the population of firms.

Theorem 2. *When consumers' beliefs satisfy passive conjectures, all equilibria in any network are efficient network signaling equilibria.*

Proof. With passive conjectures, a unilateral deviation by the high type firm to the strategy specified in an efficient network signaling equilibrium will result in all consumers who are offered a price in round 1 under that strategy accepting the offer: the round 1 price of $\bar{q}$ is acceptable under the assumption that consumers place probability α on the unknown firm being of high type. Since efficient network signaling equilibria are the most profitable for the high type firm, this unilateral deviation is always attractive to the high type firm. □

In the case in which the network has a unique minimum dominating set, this equilibrium is unique, and in the case in which there are multiple minimum dominating sets the equilibria are qualitatively distinct in that they all feature the firm visiting some minimum dominating set in round 1. In order to select these "efficient" network signaling cases as the unique set of equilibria in the game, it must be the case that the high type firm can deviate from a strategy with $p_1 < \bar{q}$ or $\gamma_1 < \gamma^*$ - a lower round 1 price or fewer round 1 customers - to $p_1 = \bar{q}$, $\Gamma_1 = \Gamma^*$ and have this menu be accepted by round 1 consumers. Under passive conjectures, consumers will indeed be willing to accept such a deviation, since it would be equally profitable to either type of firm and remains weakly profitable for the consumers who accept. I emphasize that such a restriction is, of course, very strong, and not a particularly satisfying way to restrict consumer beliefs, but I feel it is nevertheless instructive to consider the conditions under which a unique equilibrium would be selected in this game.

This suggests that the high type firm would, in the ideal, prefer to take a signaling action that lets the communication mechanism reveal its type rather than to undertake costly price signaling. The communication among network members acts as a strong substitute for price discounting in this signaling game. The parameters Γ^* and γ^* are critical throughout.

Representing, respectively, the set and number of consumers who must be sold to in round 1 in order to inform all other consumers in the network of product quality before sales round 2, they represent the most profitable signaling strategy using visitation. They are therefore the analog in this two-period model of the kind of opinion leaders and key influencers that a practical 'network analysis' strategy of the type discussed in the opening sections seeks to identify in order to most successfully generate word-of-mouth.

6 Comparative statics

Under passive conjectures, which selects a qualitatively unique equilibrium in this game, how does the equilibrium depend on the network parameters?

The selection of an efficient network signaling equilibrium is entirely independent of the structure of the social network, despite the fact that the material outcome of the equilibrium varies with the network structure. The firm's discount factor and the consumers' willingness to pay for a product of unknown quality are important, however, since through the analysis above I assumed that the firm is sufficiently patient that a sale tomorrow at the high quality price is preferred to a sale today at the unknown quality price. If there is little difference between a high and low quality product or if the firm is sufficiently impatient, this may not be the case. With that caveat, provided we are not dealing with a singleton network, the quantitative result changes but qualitatively the solution to the problem is the same for all networks. Any equilibrium satisfying the passive conjectures is always an efficient network signaling equilibrium, for any network.

This quantitative change, however, depends entirely on γ^*. The relationship between the average number of neighbors in a given network and the domination number γ^* is not straightforward. For example, in the examples of the star network and circular network seen in Appendix B, the circle network has a higher average number of neighbors per consumer yet has a larger domination number. Networks with a smaller domination number will have less round 1 sales in equilibrium; that is, more offers will be postponed to round 2 since fewer consumers must be 'convinced' today in order to reveal type to all tomorrow, which naturally enhances the signaling value of restricting round 1 activity.

The parameter γ^* was also crucial in defining those equilibria which survive the Intuitive Criterion, and, again, in determining the qualitative nature of those equilibria is not affected by the network structure. The set of equilibria satisfying the Intuitive Criterion is identically characterized for any network, even though the number of consumers visited in each round depends on the precise structure of the network.

It is perhaps surprising that the strength of the network does not influence the selection between up-front signaling strategies and during-play signaling; this indicates the extreme value of the communication signal over price signaling. Even the weakest version of an N-person network will still support the during-play network signaling equilibrium as the unique outcome, and it is unambiguously more costly to signal by price discounting than by exploiting consumers' talk.

The volume of high quality trade is higher here than in the benchmark case in Akerlof (1970) in which high quality products are driven out of the market by low quality products. This model belongs to the class which demonstrate that quality signals (including, for example, guarantees) can mitigate the adverse selection problem, although with the caveat that here low quality goods are traded and low types make positive profit. Here, communication helps to ensure not only that trade exists, but that high quality products are more traded than low quality in equilibrium. Information revelation is also high in this model, although some consumers buy before learning the true type of the firm; in equilibrium, though the high type firm will not take an action to unambiguously distinguish itself from the low type before any trade takes place, all consumers in the network nevertheless learn the true type of the firm by the end of the communication stage.

6.1 The value of communication

Although the equilibrium strategy for a profit-maximizing firm is qualitatively unique under the assumption of passive conjectures, the level of profit for the firm depends on the structure of the network of consumers. Two networks of the same size N will generally yield different profit in equilibrium. It is therefore generally true that the firm would find value in altering the structure of communication, if such a thing were possible.

The equilibrium profit for the high-type firm in a general network was as follows:

$$\pi_H = \gamma^* \bar{q} + \delta(N - \gamma^*) q_H \tag{21}$$

Consider adding one more node to the graph of the network, and let that node be connected to each of the N existing nodes. By Theorem 2 the firm's equilibrium strategy remains $\sigma_{F,H} : \{\bar{q}, q_H, \Gamma^*, N - \Gamma^*\}$, but now γ^* is no greater than 1, since the new node is connected to all existing nodes and therefore forms a minimum dominating set of the new graph. Equilibrium profit in this new network is therefore given by:

$$\pi_H = \bar{q} + \delta N q_H \tag{22}$$

Since Assumption 1 guarantees that $\delta > \frac{\bar{q}}{q_H}$, this is certainly greater than before. The firm would therefore be willing to up to this difference in order to alter the structure of the network, albeit with the caveat that such an action would undoubtedly change the game since it is a strategic choice and therefore potentially informative to the consumers, should they observe it. The same logic would apply to less extreme expansions of the network, and not only in the case of adding a node; adding a link would operate similarly. In both the case of adding a node and the case of adding a link, provided that the count of a minimum dominating set of the new network graph is lower than that of the old, the high-type firm will be more profitable in the new network.

This is clearly not the case for the low-type firm. In the equilibria surviving passive conjectures, the low-type firm earns profit of:

$$\pi_H = \gamma^* \bar{q} \tag{23}$$

Networks with a lower value of γ^* will therefore yield lower profit for the low-type firm. This makes the net welfare effect of an exogenous change in the network structure non-obvious, a problem compounded by the question of whether we should care about the ex ante or ex post utility for consumers. Ex ante utility is zero for all consumers in equilibrium, but ex post utility may be positive or negative depending on the quality of the product the consumer has purchased.

If we are willing to be flexible in defining the real-world analog of the nodes and links in the network graph in this model, one interpretation of this simple result is as a justification of firms' willingness to pay to support the media. Provided that product quality is exogenous and objective, as it is in the model here, any disseminator of information can be treated as a network node. If, for example, a universally read newspaper exists in the network, it can be modeled as a node connected to every other consumer. Then 'selling' the product to that node will form at least part of the round 1 strategy for a high-type firm, and information about the product will spread to every other node in the network by tomorrow.

There then grows the issue of dividing the surplus from this service. Real-world media clearly do not typically operate as non-profit organizations, and they derive at least part of their monetary value from their role as disseminators of information (as well as, for example, providers of entertainment). Although the case of objectively valued products being reported or reviewed is an imperfect approximation of subjectively valued products being advertised, it is nevertheless possible to see in this model the spirit of the mutually beneficial relationship between the media and other firms.

In a similar vein, if some exogenous shift in the structure of social interaction or of the media was to occur, it would operate similarly. Consider, for instance, the hypothesis that the advent of the internet has brought an increase in the fragmentation and specialization of media sources (for example, less time spent consuming general-audience newspapers and more time spent consuming special-interest or single-issue websites or blogs) and an increase in the level of available information on specialist topics. This might be interpreted as an increase in the density of the network graph, especially around those nodes whose interests are similar; the value of additional nodes and links may thus fall to the point where non-media firms are not willing to support the existence of traditional wide-media to the same extent as in a more disparate network.

The spirit of the word-of-mouth strategy is also mirrored in experience goods with subjective values, such as movies, for which criticism and reviews play a key role: the marketer of an experience good typically provides the good at no cost to critics, who then go on to spread information about the product and its quality to their audience. In that interpretation, the critic's 'audience' is the set of consumers to whom she is linked in the network graph. The criticism industry is in some sense a formalization and institutionalization of the use of this targeting strategy. There are also parallels with the type of persuasion attempted by producers who give away their product to celebrities or tastemakers in the hope of generating buzz among a wider market.

7 Extensions

The preceding section speculated on the incorporation of the media as nodes and links in the network graph in this model. There exist other areas of the model which could be magnified to further illuminate more issues.

The game presented has two rounds of sales. This could naturally be extended, and a visitation strategy equivalent in spirit to the Γ^* strategy in the network signaling equilibrium above would exist for every finite game of this type (although it would not necessarily be an equilibrium strategy). For example, if the firm could sell over three periods rather than two, it would be relevant to identify that set of consumers such that no other consumer is more than two degrees away from a member of the set.

The price signaling equilibrium was unappealing to the high type in this game precisely because it was less profitable than the network signaling equilibrium in which the high type was temporarily pooled with the low. This is strongly dependent on the nature of competition in this game, where there was only one firm; the high and low types do not coexist. If, in fact, the game was played with high and low types competing directly and making price offers to consumers simultaneously, the network signaling class would be less appealing to the high type firm, since temporarily pooling would then not just reduce the maximum price consumers are willing to accept from an unknown firm but would also invoke a cost of sharing the market with the low type. This is the most direct argument for the strength of the social network influencing equilibrium selection: the cost of temporary pooling will depend on the strength of the network, so in the competitive version of this game equilibrium selection as well as characteristics may well depend on network parameters. A second extension of this framework would thus be to ask how this type of direct competition between types would influence the result.

The communication stage was here very simple: communication was not strategic, and information traveled only one degree before dying. Strategic, probabilistic or costly communication may operate differently than the simple model of communication I have adopted. Magnifying the mechanics of the communication stage could also be fruitful. In particular, what if the communication was in fact transmitted by another player in the game? In markets with information transmission, critics and the media play key roles, and it would be possible to model their role in the framework presented here without necessarily treating them identically to consumer nodes in the network graph.

8 Concluding comments

This paper has studied a quality signaling game in which consumers can communicate locally across links in a social network and the firm can perfectly target sales to particular consumers. When consumers talk, the hidden type problem is indeed altered. In the terminology of the quality signaling game, communication among consumers can be exploited by a high type firm which chooses a particular visitation strategy that reveals its type during play in the most efficient possible way. The model presented above isolates the effect of network parameters on the quality signaling game as much as possible, at the expense of any discussion of production costs, quality as a choice variable, richer signaling mechanisms (for example, advertising or licensing) or heterogeneous consumers. The model allows a monopolist to make sales offers in two stages to consumers who cannot directly observe the quality of the firm's product but can communicate such information between stages. The set of consumers to whom the firm offers prices in each stage is thus a strategic variable.

When the communication structure is parameterized by modeling talk as taking place across exogenous links in a social network, we can formally link the degree and location of communication to the outcome of the quality signaling game. First we can show that if the firm is patient enough, a passive signaling strategy featuring strategically chosen visitation order is preferred by the high type firm to an equilibrium in which no separation takes place.

There is also an upper bound to the number of households that will be visited in the first round of the two-round game I have described above. It is the domination number of the network's graph, that particular number of households such that visiting a particular set with that cardinality will leave all remaining households at most one degree removed from those visited. That is, the number and set such that tomorrow the communication across links in the social network will have informed all remaining consumers of the quality of a firm's product. For the purveyor of a high quality product, visiting more than this number of households today is wasteful, assuming that the patience condition holds: restricting sales today allows such a firm to make more valuable sales tomorrow once its type has been revealed.

In the two-round game, one equilibrium featuring separation before round 1 survives the application of the Intuitive Criterion: it features a visitation strategy of precisely the set of consumers just described and a round 1 price low enough to preclude profitable imitation by low type firms. A set of equilibria featuring signaling *during* round 1 and *between* rounds survive the application of the Intuitive Criterion; all of them feature a round 1 visitation strategy that visits weakly less than that special set and a round 1 price weakly less than the willingness to pay by a consumer for a product of unknown quality.

In the particular case in which consumers' beliefs on the distribution of firm types in the absence of a separating signal is identical to the true, exogenous distribution of types (passive conjectures), the equilibria of this quality signaling game are efficient network signaling equilibria. These are the equilibria in which the two types pool on a minimum dominating set of consumers at a common price in round 1, and then the high type firm makes sales at the high quality price to all other consumers in round 2. In the case in which the network has a unique minimum dominating set, this yields a unique equilibrium. This result is analogous to the uniqueness of the Riley outcome in the Spence signaling game. Here the result shows the superiority of using communication among consumers to do the legwork of signaling rather than enduring the cost of sending a price signal.

In other words, high type firms seeking to signal their type will always find it cheaper to allow their type to be revealed through communication than to undertake a directly costly price signaling strategy, provided that it is not outright loss-making to temporarily indistinguishable from a low type firm. No non-atomistic network will support an equilibrium with costly price signaling by a monopolistic firm: in the case when a firm can identify those consumers most valuable in generating word-of-mouth and can perfectly target them, price signaling is dominated by a strategy that exploits consumers' local communication.

References

Akerlof, G. A. (1970): "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, 84(3), 488–500.

Bagwell, K., and M. H. Riordan (1991): "High and Declining Prices Signal Product Quality," *The American Economic Review*, 81(1), 224–239.

Banks, J. S., and J. Sobel (1987): "Equilibrium Selection in Signaling Games," *Econometrica*, 55(3), 647–661.

Campbell, A. (2008): "Tell Your Friends! Word of Mouth and Percolation in Social Networks," Job Market Paper.

Cho, I.-K., and D. M. Kreps (1987): "Signaling Games and Stable Equilibria," *Quarterly Journal of Economics*, 102(2), 179–221.

Coleman, J. S., E. Katz, and H. Menzel (1966): *Medical Innovation*. Bobbs-Merrill.

Galeotti, A., and S. Goyal (2007): "Network multipliers and the optimality of indirect communication," Working paper.

Godes, D., and D. Mayzlin (2004): "Using Online Conversations to Study Word-of-Mouth Communication," *Marketing Science*, 23(4), 545–560.

——— (2009): "Firm-created Word-of-mouth Communication: Evidence from a Field Test," *Marketing Science*, 28(4), 721–739.

Haynes, T. W., S. T. Hedetniemi, and P. J. Slater (1998): *Fundamentals of Domination in Graphs*. New York: Marcel Dekker, Inc.

Jackson, M. O., and L. Yariv (2008): "Diffusion, Strategic Interaction, and Social Structure," in *Handbook of Social Economics*.

Kennedy, P. W. (1994): "Word-of-Mouth Communication and Price as a Signal of Quality," *The Economic Record*, 70(211), 373–380.

Miller, R. M., and C. R. Plott (1985): "Product Quality Signaling in Experimental Markets," *Econometrica*, 53(4), 837–872.

Nair, H., P. Manchanda, and T. Bhatia (2006): "Asymmetric Peer Effects in Physician Prescription Behavior: The Role of Opinion Leaders," Stanford Graduate School of Business Research Paper No. 1970.

Navarro, N. (2006): "Asymmetric information, word-of-mouth and social networks: from the market for lemons to efficiency," CORE Discussion Paper 2006/02.

Ore, O. (1962): *Theory of Graphs*. Providence, American Mathematical Society, Colloquium publications v.38.

Rasmusen, E. (2001): *Games and Information*. Blackwell, 3rd edn.

Rubinstein, A. (1985): "Choice of conjectures in a bargaining game with incomplete information," in *Game-theoretic models of bargaining*.

Spence, M. (1973): "Job Market Signaling," *The Quarterly Journal of Economics*, 87(3), 355–374.

Wolinsky, A. (1983): "Prices as Signals of Product Quality," *Review of Economic Studies*, 50(4), 647–658.

A Proofs

A.1 Lemma 1

Proof. We require $\hat{p}$ such that a low-type firm imitating the high-type strategy completely would receive zero profit in expectation. The potential credible imitator would make γ_1 sales at $\hat{p}$ in round 1 and $(N-\gamma_1-\Sigma_1)$ sales at q_H in round 2, where Σ_1 is the number of neighbors to consumers in Γ_1 (since these neighbors observe the true type, they will not buy from the imitator in round 2). $\hat{p}$ must thus satisfy:

$$\begin{aligned} \pi_{L,imitate} &= \gamma_1\hat{p} + \delta(N-\gamma_1-\Sigma_1)q_H && (24) \\ &\leq 0 && (25) \\ \Rightarrow \hat{p} &\leq -\frac{\delta(N-\gamma_1-\Sigma_1)q_H}{\gamma_1} && (26) \end{aligned}$$

The profit to the high type is thus:

$$\pi_H = \gamma_1\hat{p} + \delta(N-\gamma_1)q_H \quad (27)$$

All price separating equilibria that do not maximize this profit with respect to visitation γ_1 and price $\hat{p}$ fail to survive the application of the Intuitive Criterion. By the Intuitive Criterion consumers will place zero probability on a deviation from any such equilibrium to the profit-maximizing price separating equilibrium coming from the low type firm; the deviation is profitable for the high type and will be made. □

A.2 Lemma 2

Proof. Consider the price signaling equilibrium in which $\gamma_1 = \gamma^*$. In that case, Σ_1, the number of neighbors to consumers in Γ_1, is equal to $N-\gamma^*$, since by the definition of γ^* all remaining consumers are neighbors of those in Γ^*. The maximal price in that case is:

$$\begin{aligned} \hat{p} &= -\frac{\delta(N-\gamma^*-\Sigma_1)q_H}{\gamma^*} && (28) \\ \hat{p} &= -\frac{\delta(N-\gamma^*-(N-\gamma^*))q_H}{\gamma^*} && (29) \\ \hat{p} &= 0 && (30) \end{aligned}$$

The profit to the high type is thus:

$$\pi_{H,1} = \delta(N-\gamma^*)q_H \quad (31)$$

Now consider any other price signaling equilibrium, with generic γ_1. This has a maximal price:

$$\hat{p} = -\frac{\delta(N - \gamma_1 - \Sigma_{1,\gamma_1})q_H}{\gamma_1} \tag{32}$$

Which yields profit to the high type:

$$\pi_{H,2} = -\delta(N - \gamma_1 - \Sigma_{1,\gamma_1})q_H + \delta(N - \gamma_1)q_H \tag{33}$$

Now comparing the two:

$$\pi_{H,2} > \pi_{H,1} \tag{34}$$

$$\text{if } \Sigma_{\gamma_1} > N - \gamma^* \tag{35}$$

But $\Sigma_{1,\gamma_1} = N - \gamma^*$ if $\gamma_1 \geq \gamma^*$ and $\Sigma_{1,\gamma_1} < N - \gamma^*$ if $\gamma_1 < \gamma^*$, so it is never true that $\pi_{H,2} > \pi_{H,1}$. The most profitable price separating equilibrium thus features $\gamma_1 = \gamma^*$ and has profit given by:

$$\pi_{H,1} = \delta(N - \gamma^*)q_H \tag{36}$$

□

A.3 Lemma 3

Proof. The price signaling strategy $\sigma_{F,H} : \{0, q_H, \Gamma^*, N - \Gamma^*\}$ is profitable for the high type but not profitable for the low type firm, so when consumers see this strategy they must place zero probability on it coming from the low type (by the Intuitive Criterion). The high type earns payoff $\delta(N - \gamma^*)q_H$ by playing this strategy, so will prefer to deviate to it from any other strategy that yields a lower equilibrium payoff. □

A.4 Lemma 4

Proof. Assume not, so that there exists an equilibrium with some Γ_1 and $\Gamma_2 = \emptyset$. Consider three cases:

1. Take $\Gamma_1 = N$. Consider a deviation to $\Gamma_1 = N - i$, $\Gamma_2 = i$ by the high type firm. Consumer i will buy since she will learn that quality is q_H; this is profitable for the high type firm under Assumption 1, but not profitable for the low type firm, which would lose consumer i.

2. Take $\Gamma_1 \subset N$, $\Gamma_1 \neq \emptyset$. Consumers in Ω_{Γ_1} learn that quality is q_H. A deviation to $\Gamma_2 = \Omega_{\Gamma_1}$, $p_2 = q_H$ is profitable for the high type firm, but (weakly) not profitable for the low type firm since those consumers learn true quality before stage 2 and so will not buy from the low type.

3. Take $\Gamma_1 = \emptyset$. Consider a deviation to $p_1 = 0$, $\Gamma_1 \neq \emptyset$, $\Gamma_2 = \Omega_{\Gamma_1}$, $p_2 = q_H$. This is profitable for the high type firm, but (weakly) not profitable for the low type firm, which makes no profit in stage 1 and no sales in stage 2.

In each case there is a deviation to $\Gamma_2 \neq \emptyset$ that is profitable for the high type but not the low type. By the Intuitive Criterion, consumers must place zero probability on that deviation coming from the low type and so will accept the deviation's offers; this is profitable for the high type. $\Gamma_2 = \emptyset$ could not have been part of an equilibrium. □

A.5 Lemma 5

Proof. Assume not, so that $\gamma_1 > \gamma^*$ in an equilibrium that survives the Intuitive Criterion. Consider two cases:

1. If consumers in Γ_1 choose not to buy in round 1 or if $p_1 < 0$, the high type firm cannot realize a higher payoff than in the equilibria of Lemma 2. By that result, the high type can profitably deviate to a price signaling equilibrium, and since the low type cannot profit from such a deviation consumers place zero probability on the deviant being of low type.

2. If consumers in Γ_1 choose to buy in round 1, the high type firm can deviate to $\Gamma_1 = \Gamma^*$, $\Gamma_2 = N - \Gamma^*$, $p_2 = q_H$ and p_1 unchanged. This deviation is profitable for the high type firm under Assumption 1, but is not profitable for the low type since it reduces sales in round 1 and does not increase sales in round 2, since word-of-mouth reveals type to all in $N - \Gamma^*$.

$\gamma_1 > \gamma^*$ could not have been part of an equilibrium surviving the Intuitive Criterion. □

B Examples of networks

To illustrate the γ^* and Γ^* concepts, let us consider some examples of networks. In the following figures, nodes in the network that are square belong to a set Γ^* whose count satisfies γ^*, the minimum number of consumers such that marking those γ^* would result in every single consumer either being marked or having a marked neighbor. Nodes represented

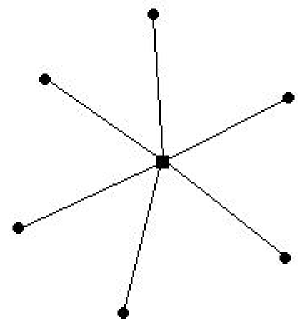

Figure 1: Star network

by circles are not in Γ^*. The hub-and-spoke, or star, network has $\gamma^* = 1$: since all consumers are at most one degree removed from the central consumer, marking the central consumer means that every consumer is either marked or has a marked neighbor. In this case, Γ^* is unique: there is only one set of one consumer that satisfies the requirement of γ^*.

The circular network shown has 6 consumers: In this case, $\gamma^* = 2$, as shown: marking

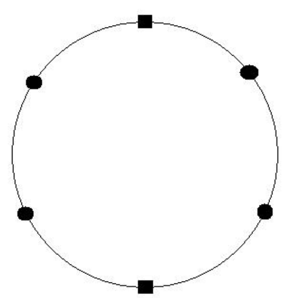

Figure 2: Circular network

these consumers leaves all consumers either marked or neighbor to a marked consumer. There are three sets Γ^* that satisfy the γ^* requirement in this case.