Strength in Numbers:
Networks as a Solution to Occupational Traps

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1 Appendix

The main result of the model is that the new network strengthens more rapidly in communities with weak outside options (low $U_{NB}^j$). Recall that changes in network strength $W_j^t$ over time and across communities are driven entirely (up to a positive constant) by changes in $\Delta \omega_j^t$. It will thus be convenient to derive all the extensions to the model in terms of $\Delta \omega_j^t$.

1. $\lambda$ varies across communities: We might expect historically disadvantaged communities to have access to less favorable $NB$ occupations (low $U_{NB}^j$) as well as a smaller share of businessmen (lower $\lambda$). Adding a $j$ superscript to the $\lambda$ parameter, without loss of generality suppose that $U_{NB}^j$ and $\lambda^j$ are increasing in $j$. Let a constant measure $\Delta \mu_0 \equiv (1 - \lambda^j) \Delta \omega_0^j$ of individuals without a business background enter the network in period 0 across all communities. Moving forward in time, we arrive at an expression analogous to equation (1),

$$\Delta \omega_t^j = \frac{h(1 - \lambda^j) \Sigma_{\tau=0}^{t-1} \Delta \omega^j_{\tau}}{r_B - r_{NB}} - \left( \frac{U_{NB}^j}{r_B - r_{NB}} - 1 \right) \equiv \beta^j \Sigma_{\tau=0}^{t-1} \Delta \omega^j_{\tau} - \alpha^j,$$

which can be solved recursively to obtain an expression analogous to equation (2),

$$\Delta \omega_t^j = (\gamma \Delta \mu_0 - \alpha^j)(1 + \beta^j)^{t-1},$$

where $\gamma \equiv h/(r_B - r_{NB})$. Assuming $\Delta \mu_0 > \alpha^j/\gamma$ and noting that $d\alpha^j/dj > 0$, $d\beta^j/dj < 0$,

$$\frac{d\Delta \omega_t^j}{dt} = (\gamma \Delta \mu_0 - \alpha^j)(1 + \beta^j)^{t-1} \ln(1 + \beta^j) > 0$$

$$\frac{d^2\Delta \omega_t^j}{djdt} = -(1 + \beta^j)^{t-1} \ln(1 + \beta^j) \frac{d\alpha^j}{dj} + (\gamma \Delta \mu_0 - \alpha^j)(1 + \beta^j)^{t-2} \left[(t-1)\ln(1 + \beta^j) + 1\right] \frac{d\beta^j}{dj} < 0.$$

Notice that the second term on the right hand side of the preceding expression reinforces the divergence in network strength across communities derived in equation (4) (with constant $\lambda$ across communities).

2. The network technology is concave: One consequence of the linear network technology, together with the assumption that the length of the individual’s work life exceeds the number of cohorts, $M > N$, is that the strength of the network will be increasing at the margin over time. Suppose instead that the network technology is concave. The expression analogous to equation (1) is then

$$\Delta \omega_t^j = \phi(\Sigma_{\tau=0}^{t-1} \Delta \omega^j_{\tau}) - \alpha^j,$$

with $\phi'(\Sigma_{\tau=0}^{t-1} \Delta \omega^j_{\tau}) > 0$, $\phi''(\Sigma_{\tau=0}^{t-1} \Delta \omega^j_{\tau}) < 0$. While the network dynamics can no longer be derived analytically, it is straightforward to solve the model numerically, starting with $\Delta \omega_1^j$ and moving forward.
in time. Figure A1 describes the evolution of $\Delta \omega^j_t$ in two communities $H$ and $L$ with $\alpha^H > \alpha^L$, for the linear network technology $[H(L), L(L)]$ and a concave network technology $[H(C), L(C)]$.\footnote{The parameter values are $h = 0.1$, $\Delta \omega_0 = 0.2$, $r_B - r_{NB} = 0.6$, $\lambda = 0.07$, $u^L = 0.61$, $u^H = 0.615$. Ability is distributed uniformly on the unit interval.} The model is solved for 25 periods and the $\phi$ function is specified to be quadratic, with the coefficient on the linear term equal to the $\beta$ coefficient with the linear technology and the negative coefficient on the quadratic term set so that the network has zero effect in the final period.\footnote{This condition holds for the $L$ community, which has a larger network stock, so the network effect continues to be positive for the $H$ community in the final period.} Despite the fact that the $\phi$ function is assigned the maximum possible curvature, we see in Figure A1 that network strength continues to grow monotonically over time, more steeply in the community with worse outside options, matching the prediction from Proposition 1.

3. **Individuals benefit from additional cohorts:** The assumption that individuals benefit from the cohorts that precede them allows us to solve the dynamic model analytically. Given the linearity in the network technology, we can allow individuals to benefit from their own cohort as well without adding substantial complexity to the problem.\footnote{I am grateful to an anonymous referee for bringing this point to my attention.} The son of an individual in the $NB$ occupation who enters the workforce in period $t$ will choose the $D$ occupation in this case if

$$h(1 - \lambda) \left[ \sum_{\tau=0}^{t-1} \Delta \omega^j_\tau + (1 - \omega^j_t) \right] + r_B \omega^j_t \geq U^j_{NB} + r_{NB} \omega^j_t,$$

where $(1 - \lambda)(1 - \omega^j_t)$ is the measure of the individual’s own cohort that enters with him and subsequently supports him in the new occupation. There is now a strategic aspect to the individual’s entry decision since it depends on the decisions of other members of his cohort. The ability threshold $\omega^j_t$ thus solves a fixed-point, but one that is easy to solve by assuming that the preceding condition holds with equality and setting $\omega^j_t = \omega^j_t$, to obtain

$$\Delta \omega^j_t = \frac{\beta}{1 - \beta} \Sigma_{\tau=0}^{t-1} \Delta \omega^j_\tau - \frac{\alpha^j_t}{1 - \beta}.$$ 

Comparing this equation with equation (1), the only difference is the $\frac{1}{1 - \beta}$ multiplier. Thus, the entry game has a unique solution and the dynamic process is well defined as long as $\beta < 1$. If that condition is satisfied as with the dynamic paths reported in Figure A2, $[H(t), L(t)]$, it is straightforward to verify that Proposition 1 continues to hold. Note that Figure A2 uses the same parameter values as Figure A1. The divergence across communities over time actually gets larger relative to the baseline model, $[H(t-1), L(t-1)]$, once the additional cohort is added to the network.
Now suppose that individuals receive support from the cohort that follows them as well. The equation corresponding to equation (1) takes the form

$$\Delta \omega_j^t = \beta \left[ \sum_{\tau=0}^{T-1} \Delta \omega_j^\tau + \Delta \omega_j^{\tau+1} \right] - \alpha_j^t / (1 - \beta).$$

Given the linearity in the network technology, it is straightforward to verify that there is a unique equilibrium dynamic path even when individuals are forward looking. To derive this path I solved for a fixed-point, starting with an arbitrary sequence \(\Delta \omega_1, ..., \Delta \omega_T\) and using that sequence to compute \(\Delta \omega_1, ..., \Delta \omega_{T-1}\), based on the preceding equation, in the first iteration. This sequence was used to compute \(\Delta \omega_1, ..., \Delta \omega_{T-2}\) in the second iteration, and this procedure continued until the sequence converged from one iteration to the next. Figure A2 plots the dynamic path for \(H\) and \(L\) communities, \([H(t+1), L(t+1)]\), using the procedure described above. It is evident from the Figure that Proposition 1 continues to hold as more cohorts are added to the network.

4. Delayed entry: The model assumes that individuals without a business background make an irreversible choice between the \(D\) and the \(NB\) occupation when they enter the workforce. Without entry costs, there is no reason to delay entry into the \(D\) occupation strategically. However, an individual belonging to community \(j\) who enters the workforce in period \(t\) and has ability below \(\omega_j^t\) could still choose to enter later once the network strengthens and the threshold has dropped sufficiently.

To allow for this possibility, I solved the model numerically with individuals below the threshold when their cohort entered the workforce entering occupation \(D\) with a lag if their ability exceeded the threshold in the subsequent period. We see that Proposition 1 continues to hold in Figure A3, which uses the same parameter values as Figure A1, although the network strengthens more rapidly over time in both types of communities when individuals can delay entry and establish their firms at different ages \([L(+1), H(+1)]\) relative to the baseline model \([L, H]\).

4Because \(\Delta \omega_j^t\) is increasing monotonically over time, there exists a period \(T\) at which it reaches one, remaining at one thereafter. \(\Delta \omega_j^t\) for all periods from one to \(T - 1\) must then satisfy the preceding equation. This provides us with a nonhomogeneous system of \(T - 1\) linear equations in \(T - 1\) unknowns, \(\Delta \omega_1, ..., \Delta \omega_{T-1}\). It is easy to verify that the equations are independent, which implies that the system has a unique solution for a given \(T\). It is also easy to verify from the structure of the preceding equation that a unique solution would continue to be obtained if individuals were looking a finite number of periods forward. To pin down \(T\), we take advantage of the additional condition that the entire cohort must choose to participate in the network in period \(T\), which implies from the preceding equation that \(\Sigma_{\tau=0}^{T-1} \Delta \omega_j^\tau = (1 + \alpha - 2\beta) / \beta\).

5The fixed-point procedure always converged to the same sequence, regardless of \(T\) or the starting sequence, indicating that the equilibrium dynamic path is indeed unique. Notice that one period is lost with each iteration. Let the system converge in \(N\) periods and let \(\Delta \omega_j^\tau\) reach one in \(T\) periods. \(T\) must be set so that \(T - N > T\). \(T = 75\) in the numerical solution reported in Figure A2, where we see that \(T < 20\). The number of iterations required for convergence, regardless of the starting sequence, never exceeded 35 so the required condition was easily satisfied.
5. Ability is normally distributed: Figure A4 plots changes in network strength across communities with both a uniform and a normal ability distribution. With normality, the measure of entrants in period $t$ is $(1 - \lambda)[1 - F(\omega_j^t)]$, where $F$ describes the distribution of ability among the entrants. The ability threshold then satisfies the condition

$$h(1 - \lambda) \sum_{\tau=0}^{t-1} \left[1 - F(\omega_{\tau}^j)\right] + r_B \omega_j^t = U_{NB}^j + r_{NB} \omega_j^t.$$

We can no longer derive the dynamic path analytically, but it is straightforward to solve the model numerically, computing the ability threshold and the measure of entrants in each period and moving forward in time. I use the same parameter values as in Figure A1 and the normal distribution has the same mean (0.5) and the same standard deviation (0.29) as the uniform distribution. It is apparent from Figure A4 that network strength diverges for the $H$ and $L$ communities with the normal distribution $[H(N), L(N)]$ as well, matching Proposition 1, although not as dramatically as with the uniform distribution $[H(U), L(U)]$.6

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6 When ability is distributed uniformly on the unit interval, changes in the ability threshold from one period to the next map one-for-one into the measure of entrants, for both communities and at each point in time. In contrast, the density of the distribution, which is less than one, changes differentially over time and across communities with normality.
Figure A1: Concave Network Technology
Figure A3: Delayed Entry

Network strength vs Period

L(+1)
L
H(+1)
H