Homework assignment on "*Dynamically Inconsistent Preferences*"

Problems

1. Consider a parent who brings his child to a street fair to buy him a toy. It is available in two versions - G and B. The parent is unable to assess the quality of the toy and therefore relies on his child's judgement. there are T stands in a row and the parent plans to walk from one edge of the row to the other. each stand sells of the two versions and the parent believes there's a 50-50 chance that any stand sells the G version.

Both the parent and the child prefer the G version to the B version. However, the child has a present bias and will settle for the B version even if he expects to find the G version with very high probability at another stand. Without the parent's intervention, the trip will end at the first stand and the child will buy whatever version the first stand is selling. The parent is not happy about this and seeks a way to "trick" the child into making the right choice.

Suppose the parent has \$1 in his pocket. The child prefers $G + \$1$ in the future over B today. On the other hand, he prefers G right away to $G + \$1$ in the future. A strategy determines, for every stand t, whether or not to pay the child a dollar if he buys the toy at the t-th stand. The parent's aim is to maximize the probability that the child buys G.

(a) Assume the child is fully naive. Explain why is T is large enough, then an optimal strategy would be to promise to pay the dollar if the child buys the toy in one of the last K stands, where K is sufficiently small.

(b) Assume mext thart the child is sophisticated. Explain why the best the parent can do is guarantee that the child buys G with probability $\frac{3}{4}$ by promising to give him the dollar if he buys the toy at the second stand.

2. Consider a liquidity-constrained consumer, whose only means of making purchases at period 2 is a credit card. Let $a \in [0, 1]$ denote the amount of period 2 consumption. Let u and v represent the consumer's *net* value of period 2 consumption, from his period 1 and period 2 perspectives. Let $u(a) = a$ for $a \leq \frac{1}{2}$ and $u(a) = \frac{1}{4} + \frac{1}{2}a$ for $a > \frac{1}{2}$. Let $v(a) = a$ for every $a \in [0, 1]$. That is, the consumer's two selves agree on the net value of consumption up to $a = \frac{1}{2}$, but they diverge for $a > \frac{1}{2}$: the second period self is tempted to increase the consumption level.

A credit card company is able to extend credit to the consumer. The credit is directly tied to consumption. An interest rate schedule is thus a function t that assigns a payment to each choice of $a \in [0,1]$ such that $t(a)$ is the amount paid to the credit card company for a consumption level of a. The company's cost of providing credit is $c(a) = c \cdot a$, where $\frac{1}{2} < c < 1$.

(a) Suppose the company knew it was facing a fully *naive* consumer. What is the optimal interest rate schedule to offer this consumer?

(b) Suppose the company knew it was facing a fully *sophisticated* consumer. What is the optimal interest rate schedule to offer this consumer?

(c) Suppose the company did not know whether the consumer it faces is fully naive or fully sophisticated. Imagine the company offered a menu of the two interest rate schedules you found in (a) and (b) above. Would the fully naive consumer choose the schedule you found in (a)? Would the fully sophisticated consumer choose the schedule you found in (b)?

3. Consider a situation in which a consumer exhibits a "reference-point effect": his evaluation of available actions depends on whether he is *choosing* a contract or *cancelling* (or modifying) an existing contract. When choosing a contract, the consumer cannot find a good enough reason for taking any action $a > 0$, hence $u(a) = 0$ for all a. After signing a contract, the consumer's reference point changes, and he strictly prefers 'higher' actions, such that $v(a) = -(1-a)^2$. Assume zero costs.

The following scenario fits this specification. The consumer considers insuring himself against a set of possible damages. The range of possible actions represents the amount of coverage offered by some insurance policy. In the absence of insurance, the consumer believes that the contingencies specified in the policy are so unlikely, that thinking about them is not worth his while. However, once the consumer obtains some insurance plan, he starts viewing the contingencies as realistic. Consequently, he prefers greater coverage.

Consider a monopolistic company that offers the consumer a contract t that specifies an amount to be paid for every action a the consumer chooses.

(a) Suppose the company knew it was facing a fully *naive* consumer. What is the optimal contract to offer this consumer?

(b) Suppose the company knew it was facing a fully *sophisticated* consumer. What is the optimal contract to offer this consumer?

(c) Suppose the company did not know whether the consumer it faces is fully naive or fully sophisticated. Imagine the company offered a menu of the contracts you found in (a) and (b) above. Would the fully naive consumer choose the contract you found in (a)? Would the fully sophisticated consumer choose the contract you found in (b)?

4. In this question we examine the role of differences in beliefs in our analysis. We describe a situation with dynamic *consistency* but where the consumer and the monopolist disagree on the likelihood of future events.

A hungry customer enters a restaurant, unsure of how much he will have to eat in order to curb his hunger. In one state (the "hungry" state), he will have to eat both a main course and a dessert in order to feel satiated. In the other state (the "satiated" state), a main course will suffice.[1] The restaurant manager believes that the states are equally likely. While the cost of preparing each course is \$5, the customer is willing to pay up to \$20 for a meal that curbs his hunger, and only \$10 for a course that leaves him hungry. Stated differently, the customer's willingness to pay for a main course is \$10 in the hungry state and \$20 in the satiated state.

(a) Show that if the customer was known to share the manager's prior, the latter could offer him an "all you can eat" buffet for a fixed price of \$20.

(b) Suppose next that the manager knows that the customer *underestimates* his hunger - specifically, that he assigns probability $\frac{3}{4}$ to the satiated state. Show first that the manager could still offer the above "all you can eat" buffet, which would be accepted by the customer and generate the same expected profit. Show next that the manager can do strictly better by offering the customer an "a la carte" deal, in which the prices of a main course and a dessert are \$17.50 and \$10. In showing this, explain why this contract would induce the consumer to eat a dessert in the hungry state.

(c) Now suppose that the manager knows that the customer *overestimates* his hunger - specifically, that he assigns probability $\frac{3}{4}$ to the hungry state. Show that in contrast to the previous case, an "a la carte" menu cannot generate a higher expected profit for the restaurant manager than the above "all you can eat" buffet.

(d) Show that if the manager does *not* know which of the three customer types he is facing, he could simply ask the customer to choose between the "all you can eat" and "a la carte" deals: the customer who underestimates his hunger would weakly prefer the speculative "a la carte deal", while the other two types would strictly prefer the non-speculative, "all-you-can-eat" buffet.

Readings

[1] Assume that the consumer can have dessert only after eating a main course ("you can't have your pudding if you don't eat your meat...").