Homework assignment on *Modeling Self-Control*

Problems

1. Consider a person on a diet who faces the following food items: broccoli (b), high-fat chocolate ice-cream (i) and low-fat chocolate frozen yogurt (y). This person prefers the menu $\{b, y\}$ to $\{y\}$ and $\{b, i, y\}$ to $\{b, i\}$. In other words, he prefers a chance of sticking to his diet to committing himself to violating it (hence, $\{b, y\} \succ^* \{y\}$). In addition, if this person cannot avoid having ice-cream available, it's better to also have the low-fat frozen yogurt around. If so, then when temptation strikes, the person may be able to resolve his hunger for chocolate in a less fattening way.

Show that these preferences cannot be captured by the model of costly self-control studied in class.

2. Consider a student on a diet who has to choose what to do *and* what to eat. There are only two available activities: homework (h) or movie (m). And there are only two available snacks: baby carrots (c) or ice-cream (i). Formally, the student chooses a consumption bundle (x, y), where $x \in \{m, h\}$ and $y \in \{c, i\}$. In this set-up the student makes choices from *pairs* of menus: a subset of $\{m, h\}$ and a subset of $\{c, i\}$.

We therefore need to amend the definition of *Betweeness* to this set-up. We say that a preference relation $\succ^*$ over pairs of menus satisfies "pair-betweeness" if the following holds. Let (A, B) and (C, D) be two pairs of menus such that $A, C \subseteq \{m, h\}$ and $B, D \subseteq \{c, i\}$. If $(A, B) \succ^* (C, D)$ then $(A, B) \succsim^* (A \cup C, B \cup D) \succsim^* (C, D)$.

Assume that the student most prefers to commit to do homework and to eat carrots (i.e., his most preferred *pair* of menus is $(\{h\}, \{c\})$), and least prefers to commit to watch a movie and eat ice-cream (i.e., his least preferred *pair* of menus is $(\{m\}, \{i\})$).

(a). Assume next that the student can exercise self-control along one dimension at most: either resisting the temptation to eat ice-cream, *or* resisting the temptation to watch a movie, *but not both.* Assume also that when the student is faced with two *pairs* of menus from which he makes the same choice, he strictly prefers the pair in which he does not need to exercise self-control in order to make his choice. What is the student's ranking of all possible pairs of menus? Does this ranking satisfy pair-betweeness?

(b). Assume now that when the student faces temptation both in his choice of activity and in his choice of snack, he succumbs to the temptation along *both* dimensions (i.e., he watches a movie *and* eats ice-cream). However, when he does this, he reasons as follows: well, I've already blown my diet, so I might as well watch a movie and not feel bad about myself. Propose a ranking over menu pairs that captures this reasoning. Does this ranking satisfy pair-betweeness?

3. Consider a student who needs a computer (c) for his studies but is tempted to buy an i-phone (i). Assume the student has enough money to buy only one of these items. Let $X = \{c, i\}$. Assume the student's utility from a menu $A \subseteq X$ and a price p is given by $W(A) - p$, where W is given by

$$W(A) = \max_{x \in A}[u(x) + v(x)] - \max_{y \in A} v(y)$$

with

$$\begin{aligned} u(c) &= 800 \\ u(i) &= 600 \\ v(c) &= 0 \\ v(i) &= 100 \end{aligned}$$

(a) Suppose this student decides to purchase one of the two items online. One website offers only a computer, and another website offers only the i-phone. Both websites sell their items using a sealed-bid second-price auction. In such an auction the highest bidder wins the item and pays the second highest bid. Assume our student's willingness-to-pay is given by the utility representation of the self-control model studied in class, and is independent of the willingness-to-pay of other bidders. Under this assumption, if our student participates in one of the two auctions, it is optimal for him to bid his utility from that item, regardless of what other bidders do. In light of this, in which of the auctions would our student participate and how much would he bid?

(b) Assume next that there is only one website that offers a computer and an i-phone. This website sells these items through a single-round, second-price "right-to-choose" auction. In this type of auction each buyer submits a single bid, the highest bidder wins the right to choose a *single* item from $\{c, i\}$, and he pays the second-highest bid. The item not chosen by the winner is kept by the seller. Since this auction is also held using the second-price rule, it is optimal for a buyer to bid his true willingness-to-pay regardless of the actions of the other bidders. However, here, in contrast to an auction for a single item, the willingness-to-pay is equal to the buyer's utility from the menu of items.

4. Consider a *monopolistic* market in which there is a *single* store that can provide both broccoli (b) and ice-cream (i) at no cost. Consumers' preferences over menus $A \subseteq \{b, i\}$ and prices $p \in \Re$ is represented by the utility function $W(A) - p$, where

$$W(A) = \max_{x \in A}[u(x) + v(x)] - \max_{y \in A} v(y)$$

The values of u and v depend on the type of consumer. There are two types, b and i, such that

$$\begin{aligned} u_b(b) &= 2 \\ u_b(i) &= 0 \\ v_b(i) - v_b(b) &= \delta < 1 \end{aligned}$$

and

$$\begin{aligned} u_i(i) &= 1 \\ u_i(b) &= 0 \\ v_i(i) - v_i(b) &= 0 \end{aligned}$$

(a) Assume that the monopolist can observe which consumer type he faces. Furthermore, assume the monopolist can set up two separate shops, one which caters only to consumers of type i and another, which caters only for consumers of type b. In other words, the shop to which only b types may enter offers a menu A_b and a price $p_b(a)$ for each element $a \in A$. The shop to which only i types may enter may offer a different menu A_i and a different price per item $p_i(a)$. Assuming the monopolist is interested in maximizing its profits, what menus and prices would it choose for each shop?

(b) Assume next that the monopolist cannot distinguish between consumer types, and hence, cannot prevent consumers of both types from entering any of his shops. Assume the monopolist believes that the population of consumers is evenly divided between types b and types i (in other words, there is a $50 - 50$ chance that a given consumer is of type i or of type b). Assume also that the monopolist can still set up separate shops that cater to all consumers, but given these shops, consumers decide optimally where to shop. How many shops would the monopolist set up and what would each shop offer (menu and prices)? Explain why your answer maximizes the monopolist's profits, taking into account that consumers are free to choose where to shop.

(c) How would your answer to (b) change if $v_b(i) - v_b(b) = 0$ (i.e., if consumers had *no* self-control problems)?