ECON 1810 Economics and Psychology
Assignment 2 — Answer keys

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Problem 1. (30 pt.) Let's check the assumptions in Rani's lecture notes.

(i) $u(0) = v(0) = 0$.

(ii) $u'(x) = \frac{1}{x+1} > 0$ and $v'(x) = \frac{9}{10(x+1)} > 0$.

(iii) $u''(x) = -\frac{1}{(x+1)^2} < 0$ and $v''(x) = -\frac{9}{10(x+1)^2} < 0$.

(iv) $\kappa = \frac{3}{4} < u'(0) = 1 < \infty$ and $\kappa < v'(0) = \frac{9}{10} < \infty$.

(v) $u'(1) = \frac{1}{2} < \kappa$ and $v'(1) = \frac{9}{20} < \kappa$.

(vi) $u'(x) - v'(x) = \frac{1}{10(x+1)} > 0$ for any $x \in [0, 1]$, which implies that this example is in Case (ii).

[Intuition] By (vi), the good is *less* attractive in Period 2 than Period 1. Since the consumer makes a contract in Period 1 but pays the price in Period 2, the firm might offer a *lower* price than the marginal cost κ. The difference between the sophisticated consumer and the naive consumer would appear in A. You could check this intuition in the following example and would use it in Reading assignment.

(a) First, Given a two-part tariff (A, p), we consider the consumer's optimal action $x^v \in \arg\max_x v(x) - t(x)$ in Period 2. Since $t(x) = A + px$, the first order condition (FOC) is[1]

$$v'(x^v) = t'(x^v) \;\Rightarrow\; \frac{9}{10(x^v + 1)} = p \;\Rightarrow\; x^v = \frac{9 - 10p}{10p}.$$

Second, since the consumer is fully sophisticated, he can predict this x^v in Period 1. He would accept the two-part tariff if and only if $u(x^v) - t(x^v) \geq 0$, i.e., $A \leq u(x^v) - px^v$.

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[1] We can know $p \neq 0$.

Finally, we optimize the firm's problem from the above facts. Given p, it is clear that A satisfies

$$A = u(x^v) - px^v.$$

The firm's profit is $\pi_s(p) = A + px^v - \kappa x^v = u(x^v) - px^v + px^v - \kappa x^v = u(x^v) - \kappa x^v$. Since we have already seen x^v is a function of p, we rewrite it as $\pi_s(p) = u(x^v(p)) - \kappa x^v(p)$. FOC is $\pi_s' = u' \frac{dx^v}{dp} - \kappa \frac{dx^v}{dp} = 0$ by the chain rule. Since $\frac{dx^v}{dp} \neq 0$,

$$u'(x^v(p)) = \kappa \;\Rightarrow\; \frac{1}{\frac{9-10p}{10p} + 1} = \frac{3}{4}.$$

Here, we can calculate (A, p) and x^v as real numbers.

$$p = \frac{27}{40} < \kappa$$
$$x^v(p) = \frac{9 - 10p}{10p} = \frac{1}{3} \in [0, 1]$$
$$A = u(x^v(p)) - px^v(p) = \ln(\frac{1}{3} + 1) - \frac{27}{40} \cdot \frac{1}{3} = \ln\frac{4}{3} - \frac{9}{40} > 0$$
$$t(x^v(p)) = A + px^v(p) = \ln\frac{4}{3}$$

The firm's profit is[2]

$$\pi_s = u(x^v(p)) - \kappa x^v(p) = \ln\frac{4}{3} - \frac{1}{4} = 0.03768... > 0.$$

Therefore, these can be the answers.[3]

[Optimal Contracts] As in Assignment 1, the optimal contract (x^*, T^*) satisfies

$$x^* \in \arg\max_x \; u(x) - \kappa x = \arg\max_x \; \ln(x + 1) - \frac{3}{4}x$$
$$T^* = u(x^*).$$

By FOC, it is easy to see $x^* = \frac{1}{3} = x^v(p)$ and $T^* = \ln\frac{4}{3} = t(x^v(p))$.[4] Moreover, the firm's profit from the optimal contract is

$$\pi_s^* = T^* - \kappa x^* = \ln\frac{4}{3} - \frac{3}{4} \cdot \frac{1}{3} = \ln\frac{4}{3} - \frac{1}{4} = \pi_s.$$

Thus, there is no difference between the two cases.

(b) Although the consumer is fully naive, we employ the similar arguments to (a).

First, given a two-part tariff (A, p), we consider the consumer's optimal action $x^v \in \arg\max_x \; v(x) - t(x)$ in Period 2, which is the firm's prediction. By the same arguments as in (a), FOC is

$$v'(x^v) = t'(x^v) \;\Rightarrow\; x^v = \frac{9 - 10p}{10p}.$$

[2] You would need a calculator here.

[3] Please check the consistency with the results in the lecture notes.

[4] FOC is $\frac{1}{x+1} - \frac{3}{4} = 0$.

Second, since the consumer is fully naive, in Period 1, he predicts his utility function in Period 2 is still u. He would accept the two-part tariff if and only if $u(x^u)-t(x^u) \geq 0$, i.e., $A \leq u(x^u)-px^u$, where $x^u \in \arg\max_x u(x)-t(x)$. Since $t(x)=A+px$, FOC is

$$u'(x^u)=t'(x^u) \;\Rightarrow\; x^u=\frac{1-p}{p}.$$

Finally, we optimize the firm's problem from the above facts. Given p, it is clear that A satisfies

$$A=u(x^u)-px^u.$$

The firm's profit is $\pi_n(p)=A+px^v-\kappa x^v=u(x^u)-px^u+px^v-\kappa x^v$. Since we have already seen x^v and x^u are functions of p, we can rewrite the profit as $\pi_n(p)=u(x^u(p))-px^u(p)+px^v(p)-\kappa x^v(p)$. FOC is

$$\begin{aligned}\pi_n'(p)&=u'(x^u(p))\frac{dx^u(p)}{dp}+(x^v(p)-x^u(p))+p(\frac{dx^v(p)}{dp}-\frac{dx^u(p)}{dp})-\kappa\frac{dx^v(p)}{dp}\\&=x^v(p)-x^u(p)+\frac{dx^v(p)}{dp}\cdot(p-\kappa)+\frac{dx^u(p)}{dp}\cdot[u'(x^u(p))-p]\\&=0.\end{aligned}$$

Here, we can calculate (A,p), x^u, and x^v as real numbers. Note that $\frac{dx^u(p)}{dp}=-\frac{1}{p^2}$, $\frac{dx^v(p)}{dp}=-\frac{9}{10p^2}$, and $u'(x^u(p))-p=0$.

$$\begin{aligned}p&=\frac{27}{40}<\kappa\\x^u(p)&=\frac{13}{27}\in[0,1]\\x^v(p)&=\frac{1}{3}\in[0,1]\\A=u(x^u(p))-px^u(p)&=\ln\frac{40}{27}-\frac{13}{40}>0\\t(x^u(p))=A+px^u(p)&=\ln\frac{40}{27}\\t(x^v(p))=A+px^v(p)&=\ln\frac{40}{27}-\frac{1}{10}\end{aligned}$$

The firm's profit is

$$\pi_n=A+px^v-\kappa x^v=\ln\frac{40}{27}-\frac{7}{20}=0.043...>\pi_s.$$

Therefore, these can be the answers.[5]

[Optimal Contracts] As in Assignment 1, the optimal contract (x^u,T^u,x^v,T^v) satisfies

$$\begin{aligned}\max_{x^u,x^v}\;&[v(x^v)-\kappa x^v]+[u(x^u)-v(x^u)]\\&u(x^u)-v(x^u)\geq u(x^v)-v(x^v),\qquad\text{(IC}_2\text{U)}\end{aligned}$$

[5] p is the same as in the previous case.

that is,

$$\begin{aligned}
x^v &\in \arg\max_x v(x) - \kappa x = \arg\max_x \frac{9}{10}\ln(x+1) - \frac{3}{4}x \\
x^u &\in \arg\max_x u(x) - v(x) = \arg\max_x \frac{1}{10}\ln(x+1) \\
T^v &= v(x^v) + u(x^u) - v(x^u). \\
T^u &= u(x^u)
\end{aligned}$$

By FOC for x^v, it is easy to see $x^v = \frac{1}{5} < x^v(p)$.[6] For x^u, the ordinary FOC is not appropriate since $u'(x) - v'(x) = \frac{1}{10(x+1)} > 0$ for any $x \in [0,1]$ by assumption (vi). However, in this case, we can easily find $x^u = 1 > x^u(p)$ since $u - v$ is strictly increasing on $[0,1]$. Thus, $u(x^u) - T^u = \ln 2 - \ln 2 = 0 = u(x^u(p)) - t(x^u(p))$. Moreover,

$$\begin{aligned}
T^v = v(x^v) + u(x^u) - v(x^u) &= \frac{9}{10}\ln(\frac{1}{5}+1) + \ln(1+1) - \frac{9}{10}\ln(1+1) \\
&= \frac{1}{10}\left(9\ln\frac{6}{5} + \ln 2\right) \\
u(x^v) - T^v &= \frac{1}{10}\left(\ln\frac{6}{5} - \ln 2\right) \\
&= -0.051... < u(x^v(p)) - t(x^v(p)) = \ln\frac{4}{3} - \ln\frac{40}{27} + \frac{1}{10} = -0.005... \\
\pi_n^* = T^v - \kappa x^v &= \frac{1}{10}\left(9\ln\frac{6}{5} + \ln 2\right) - \frac{3}{4}\cdot\frac{1}{5} \\
&= \frac{1}{10}\left(9\ln\frac{6}{5} + \ln 2\right) - \frac{3}{20} = 0.0834... > \pi_n.
\end{aligned}$$

Problem 2. (20 pt.)

Observation 1. For any $z \in Z$ and any nonempty sets $A, B \subseteq Z$, if $z \in A \subseteq B$, then $U(A,z) \geq U(B,z)$, where $U(X,z) = [u^*(z) + v^*(z)] - \max_{y \in X} v^*(y)$ for any $X \subseteq Z$ with $z \in X$.

It is easy to show Observation 1 since $\max_{y \in B} v^*(y) \geq \max_{y \in A} v^*(y)$ if $A \subseteq B$.

Suppose that $\succ^*$ can be induced by U and we will see some contradiction. In this answer, we use the observation. Note that i is the most tempting good in Period 2.

First, we consider $\{b, y\} \succ^* \{y\}$. Since $U(\{y\}, y) \geq U(\{b,y\}, y)$ by Observation 1, $U(\{b,y\}, b) > U(\{y\}, y)$, which implies that $u^*(b) + v^*(b) > u^*(y) + v^*(y)$.

Second, similarly to the previous case, $\{i, y\} \succ \{i\}$ implies $u^*(y) + v^*(y) > u^*(i) + v^*(i)$.

[6] FOC is $\frac{9}{10(x+1)} - \frac{3}{4} = 0$.

Third, we consider $\{b, i, y\} \succ^* \{b, i\}$. Note that $U(\{b, i\}, b) \geq U(\{b, i, y\}, b)$ and $U(\{b, i\}, i) \geq U(\{b, i, y\}, i)$ by Obseervation 1. If $U(\{b, i, y\}, b) > U(\{b, i\}, i)$, we can also see $U(\{b, i\}, b) > U(\{b, i, y\}, i)$. Then, we can say $\{b, i\} \succ^* \{b, i, y\}$, which is a contradiction. If $U(\{b, i, y\}, y) > U(\{b, i\}, b)$, this contradicts $u^*(b) + v^*(b) > u^*(y) + v^*(y)$. If $U(\{b, i, y\}, i) > U(\{b, i\}, b)$, this contradicts that $u^*(b) + v^*(b) > u^*(y) + v^*(y)$ and $u^*(y) + v^*(y) > u^*(i) + v^*(i)$. From the above facts, we can derive $U(\{b, i, y\}, y) \leq U(\{b, i\}, b) \leq U(\{b, i, y\}, i) \leq U(\{b, i\}, i)$, which implies that $U(\{b, i, y\}, y) > U(\{b, i\}, i)$ is impossible. Therefore, $\succ^*$ cannot be captured by the model of costly self-control.

Problem 3. (20 pt.)

This can be regarded as a variant model of self-control. Given $x \in \{b, c\}$ and a nonempty set $M \subseteq \{b, c\}$, let

$$U(M, x) = \frac{1}{2}[u_1(x) - R(x, M, u_1)] + \frac{1}{2}[u_2(x) - R(x, M, u_2)].$$

We can easily calculate $U(\{b, c\}, b) = 2$, $U(\{b, c\}, c) = 1$, $U(\{b\}, b) = 3$, and $U(\{c\}, c) = 2.5$. Therefore, he will go to Restaurant 2.

[Intuition] Imagine a common situation in a restaurant which serves a beef steak and a chicken teriyaki. After you order beef and your friend orders chicken, i.e. you think the state is u_1, you might find the taste of your dish is worse than your friend's, i.e., you find the true state is u_2. If you can predict this "regret" like this model, you might try to go to Restaurant 2 to avoid the regret.

Reading assignment. (30 pt.)

Case (i): $v' > u'$

[Intuition] This is the opposite case to Problem 1.

[Example] *"Leisure Goods"*

[The Author's Explanation] page. 379. "This high interest rate (by credit card companies) could reflect above-marginal-cost pricing or high default costs.[7]" ... "Ausubel (the name of the researcher) suggests that overconfidence about future borrowing may explain the high rates of interest. Our model embeds this prediction in a general theory of leisure good pricing for individuals with time-consistent preferences and naiveté."

Case (ii): $u' > v'$

[7] Remember that in case (i), p is higher than κ.

[Intuition] This is the similar case to Problem 1.

[Example] *"Investment Goods"*

[The Author's Explanation] page. 377. "A contract with low price per week of holiday and high upfront fees appeals to time-inconsistent consumers—either because they demand a commitment device (sophisticated consumers) or because they overestimate the actual number of holidays they will book (naive consumers)."