

Please answer all of the following questions.

1. Suppose there are only four alternatives, w, x, y, z . The following table shows in what choice problem each alternative is chosen:

w	x	y	z
$\{w, x\}$	$\{x, y\}$	$\{w, y\}$	$\{w, z\}$
	$\{x, z\}$	$\{y, z\}$	$\{w, x, z\}$
	$\{x, y, z\}$	$\{w, y, z\}$	
	$\{w, x, y\}$		
	$\{w, x, y, z\}$		

Recall the following properties that we studied in class:

Expansion: *An alternative chosen from each of two sets is also chosen from their union.*

Weak WARP: *If an alternative a is chosen when only b is also available and when b and a set S of other alternatives are also available, then b is not chosen when x and a subset of S are available.*

Show that this choice behavior satisfies *Expansion* but not *Weak WARP*.

2.¹ Consider a consumer who can watch a total of three movies over the next four weeks. This consumer faces the problem of deciding which movie to skip. In the week in which he skips a movie he gets a payoff of zero (measured in utility units called *utils*). His payoff (in utils) for seeing the movies are as follows:

- Week 1 (mediocre movie): 3
- Week 2 (good movie): 5
- Week 3 (great movie): 8
- Week 4 (Angelina Jolie movie!): 13

The consumer has present-biased preferences and is dynamically inconsistent. Specifically, from the point of view of the current week, his payoff in any period beyond the

¹Copied from Vincent Crawford, who gives credit to Matthew Rabin.

current week are valued half as much the current week's payoff. E.g., from the point of view of week 1, the following stream of utils,

Week 1	Week 2	Week 3	Week 4
4	2	3	5

has a total value of $4 + (\frac{1}{2})(2 + 3 + 5) = 9$. But from the point of view of week 2, this stream of payoffs from week 2 on has a total value of $2 + (\frac{1}{2})(3 + 5) = 6$.

(a) From the point of view of week 1, what is the consumer's ideal plan, assuming he could follow? Explain your answer.

(b) Suppose the consumer is naïve, in the sense that he ignores the present bias of his future selves. Thus, when deciding what to do each week, he incorrectly believes that in future weeks his future selves will adhere to his plan. what will such a consumer do?

(c) Suppose next that the consumer is fully sophisticated, in the sense that he can correctly predict his own future choices, taking into account the present bias of his future selves. What will this consumer type do?

3. Consider the following example, which is a variation on the costly self-control model we studied in class. Imagine a decision-maker who needs to decide between three restaurants:

<i>Restaurant 1</i>	<i>Restaurant 2</i>	<i>Restaurant 3</i>
beef	beef only	chicken only
chicken		

At the moment, the decision-maker is unsure of whether he likes beef (*b*) to chicken (*c*) or vice versa, and so he is unsure of how much he would be willing to pay for either option. However, he believes that there are two equally likely states of the world, where in one his utility from each option is given by the function u_1 and in the second state his utility is given by the function u_2 . The following table displays the values these functions assign to *b* and *c*:

	u_1	u_2
<i>b</i>	4	2
<i>c</i>	1	4

Assume that when the decision-maker chooses x from a menu M , he experiences a *cost*

of regret equal to

$$R(x, M, u_1) = \max_{y \in M} u_i(y) - u_i(x)$$

where u_i is the utility function that was realized at the time he made his decision. Hence, the value he assigns to eating in a restaurant that offers menu M is given by

$$\max_{x \in M} \left\{ \frac{1}{2} [u_1(x) - R(x, M, u_1)] + \frac{1}{2} [u_2(x) - R(x, M, u_2)] \right\}$$

What restaurant will the decision-maker go to?

4. Consider two sellers, 1 and 2, who compete in quantities. That is, each chooses a quantity from the interval $[0, 1]$, and the payoff to seller i when he chooses q_i and his rival chooses q_j is given by

$$u_i(q_i, q_j) = q_i(1 - q_i - q_j)$$

(assume there are no costs in production).

(a) What is the Nash equilibrium in this game between the two sellers?

(b) Assume that seller of level-0 simply chooses a quantity at random from the interval $[0, 1]$. What quantities will be chosen by sellers of level-1? level-2? level-3?