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- the role of history in economic development:
 - developing countries differ significantly among themselves – the construct of one developing country that we will use is artificial because it does not reflect the diversity of developing countries, which may be important when looking at development
 - recently economists have tried to determine why some countries are rich and others are poor by analyzing both standard economic variables (tax levels, redistribution, etc.) and also political and social variables (social capital, quality of government, ethnic heterogeneity, etc.)
- the role of colonization:
 - one of the historical and social variables is whether a developing country was colonized and whether this led to the displacement of the indigenous population
 - the displacement of the indigenous populations had to do with their population density and the incidence of disease
 - for example, in Australia and North America the indigenous population density was low and the population density fell due to the incidence of disease (these regions became European offshoots as a result)
 - in contrast, India and Indonesia had dense indigenous populations resistant to European disease so there was no heavy settlement by Europeans and these populations still remain largely indigenous
 - some countries were under colonial rule, but others (like China) were not
 - Latin America gained independence in the 19th century from colonial rule roughly 100 years earlier than Asia and Africa which had effects on political institutions and economic policies
- the effect of cultural heterogeneity:
 - more dense populations tend to be more homogenous (such as China and Japan)
 - in contrast, sub-Saharan African nations tend to not have one dominant ethnic group
 - having many ethnic groups can lead to difficulties with national integration (such as by contributing to devastating civil wars)
- some of Professor Putterman's recent research has focused on the effect of pre-modern economic development and population growth on modern economic development and growth:
 - societies are classified by an anthropological system where they lie on a continuum from hunter-gatherer bands to large state organizations with intensive agriculture and high population densities
 - those societies that had "progressed" to high population densities and larger, centralized states have made the transition to industrialization more rapidly than countries without these characteristics
 - regression analysis using early development indicators such as population density, average farm size, degree of irrigation, and the antiquity of state (how early a central government was created, China has the earliest and Papua New Guinea the latest) are good predictors of economic growth rates during 1960-1995
 - the intuition behind this: a pre-existing state-level society and the use of intensive agriculture indicate an "economic culture" which is more similar to the modern industrial society and state than other societies, hence better able to carry out development policy
 - this indicates the importance of history when considering economic development – looking at recent economic policies, etc. might be insufficient when considering levels of development

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- this also emphasizes the capability of local populations and building “capacity”
- a link between formal education and development has been suggested
- economic history:
 - the textbook covers economic growth from about 1500 to the present
 - 1500 is commonly used as a starting point because colonization began around this time and modern capitalist institutions/market economies started to develop
 - economic historians estimate that at around 1500 the level of development was about the same throughout Eurasia, much of sub-Saharan Africa, and parts of North America
 - around 1500 income and technological development was slightly higher in China than in Europe
 - the following figures are from economic historian Angus Maddison
 - between 500-1500 the level of per capita income remained constant throughout the world (there was no growth) and this was probably true for the 1000 years before that also – thus, modern economic growth is a new phenomenon
 - according to Maddison, modern economic growth began around 1820 in Western Europe and its offshoots (Canada, the United States, Australia, and New Zealand)
 - page 36, figure 2-1 – shows that in the 19th Century, Western Europe grew faster than Eastern Europe, there was slow growth in Asia and the Pacific (Oceania), and there was very slow growth in Africa
- the debate over the Industrial Revolution:
 - in the late 19th century it was debated whether the Industrial Revolution had been beneficial because at the time it was not clear that the standard of living of the working population had improved
 - but by the early 20th century it was clear that the standard of living had improved
- economic growth through the 20th century:
 - note that the vertical axis of figure 2-1 uses a log scale
 - figure 2-1 shows that growth accelerated after 1940 and this continued in all groups of countries except Eastern Europe after 1989 (due to transitional changes) and sub-Saharan Africa after the mid-1970s
 - despite the downturn in the growth rates of Eastern Europe and sub-Saharan Africa after 1980, the per capita income of both regions increased between 1900 and the present
 - thus, income had stagnated for 1500 years or more – then growth began in Europe and its offshoots and then started in other countries
- GDP (gross domestic product) vs. GNP (gross national product) – common measures of the output of an economy:
 - GDP is defined as the sum of the value of finished goods and services produced by a society in a given year
 - note that the definition of GDP excludes intermediate goods (for example, the steel used to produce a car is not included by itself in GDP because it will be counted as part of the car)
 - GDP considers all output produced within the borders of a country, regardless of the citizenship of the producers
 - GNP is similar to GDP but counts only the income of the citizens of the country – thus, GNP does not include the income of foreign citizens within the country but does include the income of its citizens working outside the country
 - although GDP is more commonly used to refer to the value of what is produced while GNP is often refers to the value of what is earned, we will use them interchangeably

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- problems of measurement:
 - income distribution: not only are averages such as GDP per capita important, but so is the distribution of income
 - nonmarketed (nontraded) goods and services: they do not go through a market and so are not counted as part of GDP/GNP:
 - however, nonmarketed goods can be a significant part of what is produced and consumed by the producer of the goods
 - for example, if subsistence agriculture is widespread then much of what is produced will never be counted as part of GDP/GNP if we followed the usual rule
 - because we want to use GDP/GNP as a measure of the standard of living to partially correct for the exclusion of nontraded goods in output, economists estimate the amount of nontraded agricultural products produced and add their value based on market prices to GDP and GNP
 - however the choice of agricultural goods is arbitrary because other goods and services could also be included, such as domestic labor services (thus the classic paradox: if a man marries his housekeeper, then GDP decreases because she is no longer paid a wage although she does the same amount of work as before)
 - there is no general solution to correct for all such goods that are not traded
 - the use of exchange rates (we have already addressed this with purchasing power parity)
 - index number problems:
 - in order to determine the value of GDP/GNP the value of different kinds of things are added together (machines, food, clothes, etc.)
 - this requires using money prices which may change over time
 - for example, between the beginning and end of a decade the price of computers may drop relative to the price of food – in this case, which price of computers should be used for GDP/GNP (the one at the beginning or the end of the period)? If computer output rises rapidly and the price of computers falls, the measured rate of growth will be larger if we use the initial prices than if we use the end-of-period prices.
 - economists have used the value at the beginning of the period, end of the period, or the weighted average
 - Adam Smith tried to use labor as a common value (an unchanging yardstick) which contributed to the rise of “labor theories of value” in Ricardo and Marx, important in the history of economic thought but an “aside” for our purposes
- aggregate production function:
 - the output considered is a composite of all of the different goods produced
 - output (Y) is a function of the amounts of capital (K) and labor (L) used – if either the amount of capital or labor is increased then Y will increase:

$$Y = F(K, L)$$

Y = output

K = capital

L = labor

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- we will modify the aggregate production function in the future (for example, we will modify it because labor is not homogenous - it depends on how hard laborers work, how much skill they possess, how educated they are, etc.)
- the Harrod-Domar model and equation - this model will use a number of equations to derive an equation for the growth rate of output:
 - we are looking for the annual growth rate of output, defined as:

$$\text{growthrate} = \frac{\Delta Y}{Y}$$

ΔY = change in income between year t and year t+1

Y = income of year t

for example, the growth rate between year t and year t+1 is defined as:

$$\frac{Y_{t+1} - Y_t}{Y_t}$$

- an equation for total national savings:

$$S = sY$$

S (capital S) = total national savings

s (lower case S) = saving rate, $0 < s < 1$

Y = output

Y is the total national income (national output) and s is a constant expressing what percent of national output is saved - for example if $s = 0.1$ then 10% of annual national output is saved

- the relationship between savings and investment:

$$S = I$$

S = total national savings

I = investment

we assume that all savings are channeled into investment (which is done through some intermediary or directly by entity who saves)

- capital, investment, and depreciation:

$$\Delta K = I - dK$$

ΔK = the annual change in the capital stock

I = total investment (the amount by which K increases)

d = the constant rate of depreciation

K = the capital stock

now, we substitute in $I = sY$ because $I = S = sY$, so:

$$\Delta K = sY - dK$$

if ΔK is the amount by which the capital stock increases, then:

$$K_{t+1} = K_t + sY - dK$$

K_t = the capital stock at time = t

K_{t+1} = the capital stock at time = t+1 (t plus one year)

- an equation for the increase in labor:

$$\Delta L = nL$$

ΔL = the annual change in the labor supply

L = the labor supply

n = the net population growth rate

we assume the labor force grows at the same rate as the population - this is a good assumption if the age structure is level; so:

$$L_{t+1} = L_t(1 + n)$$

L_t = the labor supply at time = t

L_{t+1} = the labor supply at time = t+1 (t plus one year)

- putting these equations together:

$$\frac{\Delta Y}{Y} = \frac{Y_{t+1} - Y_t}{Y_t} = \frac{F(K_t + sY_t - dK_t, L_t(1 + n)) - F(K_t, L_t)}{F(K_t, L_t)}$$

note that the critical variables in determining growth are s , d , and n

- an equation expressing the ratio of capital and output:

$$v = \frac{K}{Y}$$

v = capital-output ratio (also known as the incremental capital to output ratio, or ICOR for short)

we derive the following equations from this one:

$$Y = \frac{K}{v} \quad \text{and} \quad \Delta Y = \frac{\Delta K}{v}$$

- the Harrod-Domar equation:

$$g = \frac{\Delta Y}{Y} = \frac{\Delta K / v}{Y} = \frac{\Delta K}{Yv} = \frac{sY - dK}{Yv}$$

g = the growth rate, defined as $\Delta Y/Y$

this equation is simplified by the assumption that depreciation is near zero, that is $d \approx 0$:

$$g = \frac{sY}{Yv} = \frac{s}{v}$$

the following equation is the Harrod-Domar equation:

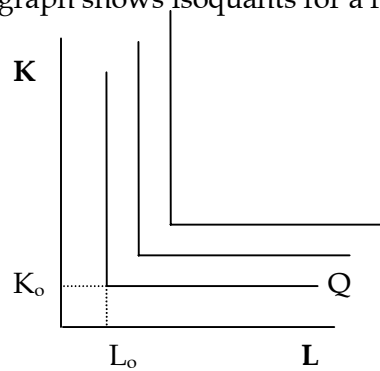
$$g = \frac{s}{v}$$

again, g is the growth rate, s is the saving rate, and v is the capital-output ratio

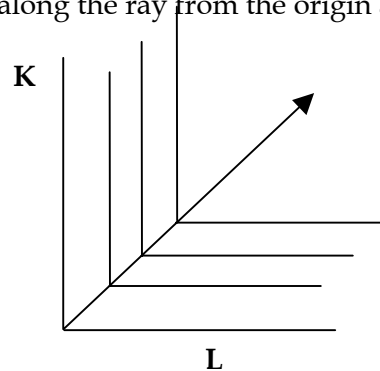
- the intuition behind the Harrod-Domar equation:
 - a larger saving rate (a higher s) will lead to a larger growth rate (g) because a higher saving rate leads to greater investment (thus, greater capital formation) and output (Y) grows faster – for example, the rate of growth will be twice as much if 20% of output is saved instead of 10% (assuming the ICOR is the same)
 - the ICOR (v) is a measure of the efficiency of the use of capital – a smaller capital-output ratio means less capital is needed to create a certain amount of output (or that a given amount of capital will create more output)
 - as a matter of policy, to increase the growth rate either the saving rate or ICOR could be targeted
- the ICOR:
 - the ICOR is the incremental capital-output ratio – although often the average capital-output ratio is discussed, it is often looked at on the margin (how much will an additional amount of capital increase output); the ICOR measures how much extra capital is needed to produce an additional unit of output
 - a higher ICOR (v) leads to a lower growth rate (g)
 - a determinants of the ICOR is the technology used, where technology is defined as a combination of factors of production (for example, a certain ratio of capital to labor)
 - countries that use technology efficiently and use capital-saving technology have lower ICOR's than countries that invest in capital-intensive industries
 - page 49, table 2-5 lists the ICOR for several economies – the ICOR ranges from 1.8 - 9.8
 - China as a case study of the ICOR:
 - in the 1960s and 1970s, China had high saving rates of about 30% (up to 35%), so $s = 0.30$ or so; but the growth rate was only about 4.5% per year because the capital-output ratio was 6.7 (which is relatively high)
 - if China's ICOR had been 2.8 (like South Korea in the 1980s) then g (the growth rate) would have been 10.7% per year
 - when China undertook market reforms the rate of growth did roughly double while the saving rate did not increase – this is attributed to a greater efficiency in the use of capital

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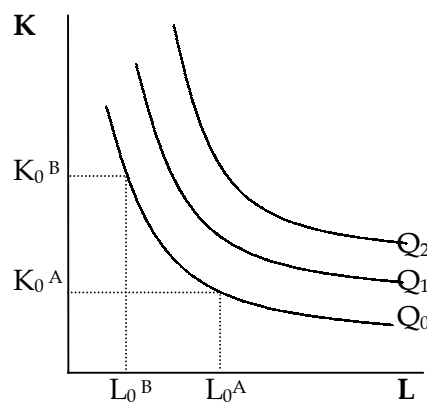
- isoquants – fixed proportions technology:
 - this graph shows isoquants for a fixed proportions technology:



- if a producer is using (K_0, L_0) to produce amount Q , then adding only capital or labor will not increase output – capital and labor must be added in the same proportion to increase output
- if using fixed proportions technology then to produce more output, the producer must move along the ray from the origin as shown:



- the Harrod-Domar model assumes fixed-proportions technology and also assumes that the additional required labor is added with an increase in capital
- isoquants – variable proportions technology:
 - with variable proportions technology the ratio of capital to labor is not predetermined
 - for example, to produce amount Q_0 , either technology (L_0^A, K_0^A) or (L_0^B, K_0^B) could be used:



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- the most efficient combination of capital and labor used to produce a given amount depends on the relative scarcity of capital and labor
- where capital is relatively abundant and labor relatively scarce – technology B should be used; where capital is relatively scarce and labor relatively abundant – technology A should be used
- for example, technology B should be used in richer countries where wages tend to be high and technology A should be used in poor countries where wages tend to be low
- isocost lines:
 - a firm will seek to maximize profit (minimize cost) and will choose a technique based on the relative prices of labor and capital
 - an isocost line is the set of all combinations of capital and labor that add to the same total cost
 - the isocost line is given by the equation:
$$TC = p_K K + p_L L$$
$$TC = \text{total cost}$$
$$p_K = \text{the price of capital (the rental rate)}$$
$$K = \text{the amount of capital used}$$
$$p_L = \text{the price of labor (the wage rate)}$$
$$L = \text{the amount of labor used}$$

the total cost is some constant which is the sum of the cost of the factors used

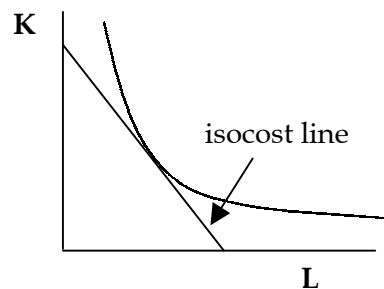
- if L is on the horizontal axis and K on the vertical axis, the slope of the total cost line (which can be derived from the above equation) is given by the expression:

$$-\frac{p_L}{p_K} = -\frac{w}{r}$$

$r = \text{the rental rate} = p_K$

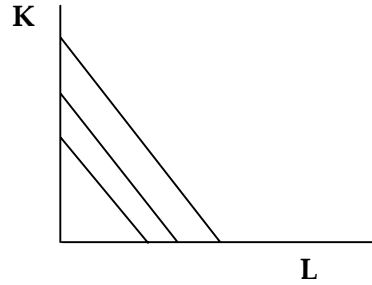
$w = \text{the wage rate} = p_L$

- the relative scarcity of capital and labor will determine the prices of labor and capital
- the isocost line graphed:

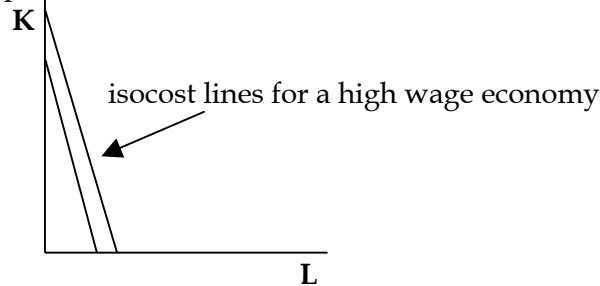


- the further an isocost line is from the origin, the more expensive it is; also all isocost lines will be parallel because they have the same slope ($-w/r$) which is determined by the wage rate and rental rate:

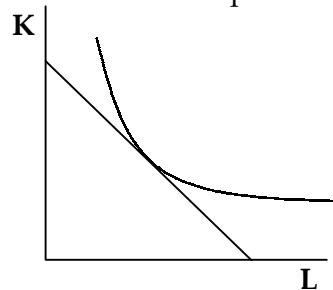
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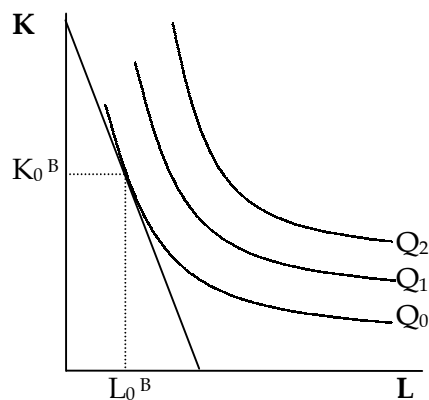
- so if the price of labor is high relative to the price of capital then the isocost lines will be steeper:



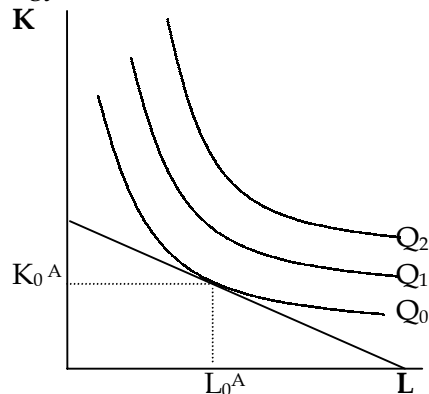
- cost-minimization (profit maximization): the cheapest isocost line for an isoquant is the isocost line tangent to the isoquant because it is the isocost line closest to the origin that still touches the isoquant:



- if the value of w/r is high then a more capital-intensive (labor-saving) technology will be used:



- however, if the value of w/r is low then a more labor-intensive (capital-saving) technology will be used:



- technology and the ICOR:
 - when labor is abundant and capital scarce then less capital-intensive (but more labor-intensive) technology will be used and the ICOR will be lower (because less capital is required for more output)
 - economies where capital is scarce and labor abundant should have a lower ICOR but distortions might prevent prices from properly reflecting the relative scarcities of labor and capital
 - for example, a poor country (where labor is abundant and cheap) that uses capital-intensive technology (because it borrows the technology from a country where capital is abundant) will require a lot of capital for more output and will not fully employ labor – inappropriately capital-intensive technology can be a reason for a high ICOR
 - for instance, China in the 1950s through 1970s used capital-intensive technology, originally borrowed from the Soviet Union, and consequently had a high ICOR
 - if an economy has a policy that prevents enterprise decision-makers from choosing technologies based on prices that reflect scarcity, then the consequence might be a high ICOR
 - wages should be based on scarcity; having high wages in an enclave can benefit a few workers but will hurt the economy, notwithstanding any “trickle-down” from the workers helped – instead it is better to have broader job creation even if it is at a lower wage
- the Harrod-Domar model and technology:
 - the Harrod-Domar model is based on fixed proportions
 - however, because the capital-labor ratio is variable the ICOR is also variable
 - the Harrod-Domar model has two policy implications. To increase growth: either (a) raise the savings rate, or (b) use capital more efficiently (by using capital-saving, labor-intensive technology) (or do both (a) and (b)).
- the Solow growth model:
 - the Solow growth model has become popular over the past ten years although it was developed in the 1950s
 - in 1989 the Solow growth model became the basis for a branch of macroeconomics that studies growth
 - first, the aggregate production function (which aggregates all factors of production and output):

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$$Y = F(K, L)$$

Y = total output (total income)

K = capital stock

L = labor supply

- the production function is put in per worker terms (which is similar to income per capita):

$$\frac{Y}{L} = F\left(\frac{K}{L}, \frac{L}{L}\right) = F\left(\frac{K}{L}, 1\right)$$

- this gives equations for output per worker and capital per worker:

$$y = \frac{Y}{L}$$

y = output per worker

$$k = \frac{K}{L}$$

k = capital per worker

thus, output per worker (y) is a function of capital per worker (k):

$$F\left(\frac{K}{L}, 1\right) = y = f(k)$$

- the change in the capital stock is given by the equation:

$$\Delta K = sY - dK$$

ΔK = the change in the capital stock

s = saving rate

Y = total income (total output)

d = depreciation rate

K = capital stock

the equation for the change in the capital stock per worker:

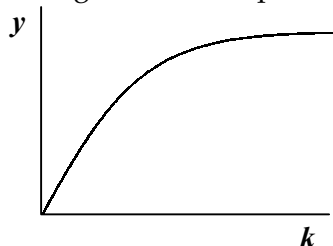
$$\Delta k = sy - (n + d)k$$

Δk = change in the capital stock per worker

n = the labor supply growth rate (or the population growth rate)

the capital stock per worker increases due to savings and decreases due to depreciation and an increase in the labor supply

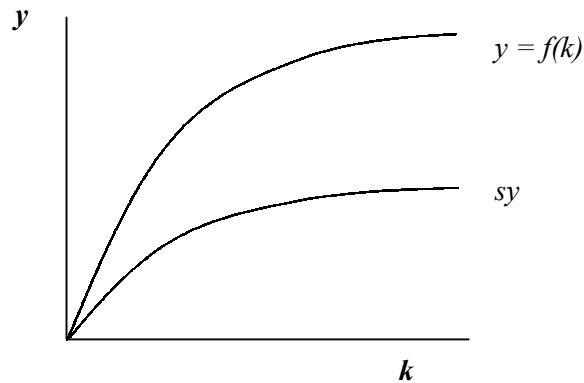
- page 54, figure 2-4 - the production function in the Solow model:



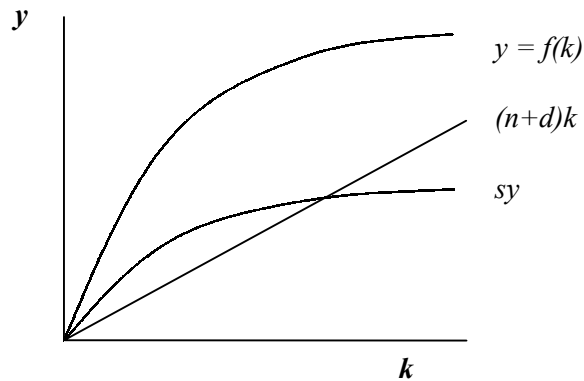
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output (income) per worker is an increasing function of capital per worker, but output per worker increases at a decreasing rate (due to diminishing returns to a variable factor, which is capital here)

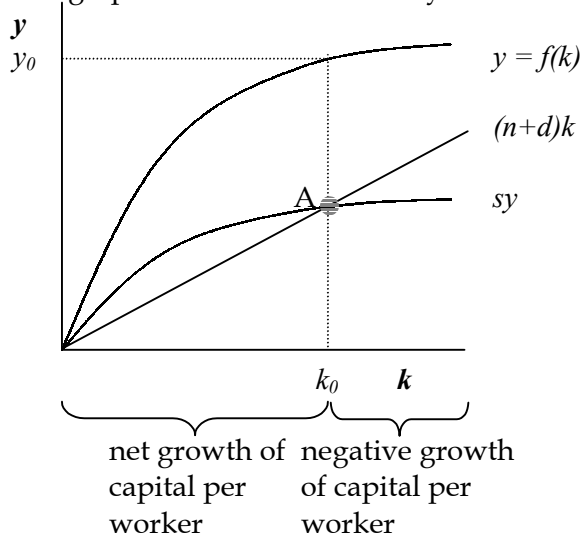
- this leads to the Solow growth diagram:
the savings curve (sy in the diagram) is the same shape as the production function but is scaled downward (because savings is equal to output multiplied by the savings rate, a constant between 0 and 1):



a ray from the origin is drawn in for the factors of production retarding the growth of capital per worker (the term $(n+d)k$):



this graph demonstrates a steady-state for the economy:



to the left of k_0 : if the capital stock is less than k_0 then the sy curve lies above the $(n+d)k$ curve so the amount of savings per worker more than offsets depreciation and population growth – thus, there is an increase in the amount of capital per worker because savings more than make up for the loss of capital due to depreciation and population growth

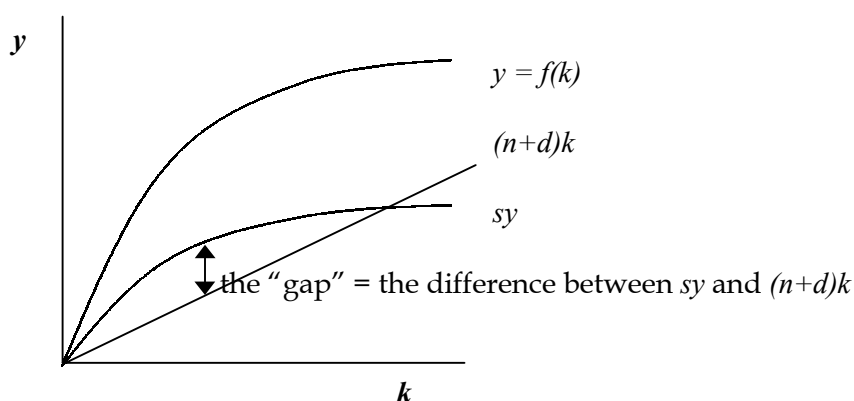
to the right of k_0 : if the capital stock is more than k_0 , then the addition to the capital stock by savings will not be enough to compensate for depreciation and population growth so the capital stock per worker will decline

thus, the amount of capital per worker will be driven to point A which is a steady-state equilibrium

- this model has some counter-intuitive aspects:
 - even though it is a growth model, the Solow model predicts a point at which no growth per capita occurs – at A the level of capital per worker is fixed at k_0 and output per worker is fixed at y_0
 - although the Solow model predicts a steady-state level of capital per worker and output per worker it is still used as a growth model under the assumption that countries' capital stocks lie to the left of k_0 (where growth occurs)
 - economies further to the left (low level of capital; low capital to labor ratio) have a longer way to go until their equilibrium steady-state than economies further to the right (high level of capital, high capital to labor ratio), so they may tend to grow more rapidly
- aggregate growth still occurs:
 - note that a steady-state does not mean the economy is static – if the population is growing then the total capital stock (K) is growing but at a rate just enough to replace depreciated capital and supply the growing labor supply with capital (capital per worker and output per worker in the steady-state remain the same)

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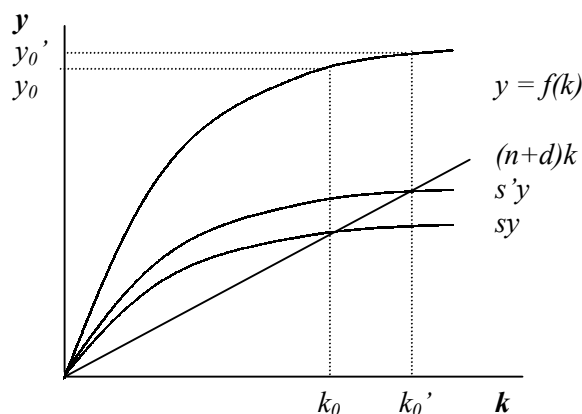
- because the population is growing at rate n , the total capital stock (K) is growing at rate n in the steady-state – however, output per worker remains fixed; population growth offsets the increase in K so that k remains constant
- predictions of the Solow growth model (assuming that countries have the same underlying factors such as technology, saving rate, etc.):
 - convergence – the Solow model predicts that countries at different levels of capital per worker and output per worker but the same saving rate (s), depreciation rate (d), and population growth rate (n) will converge to the same steady-state output per worker (y):
 - a poorer country grows more rapidly than a richer country *ceteris paribus* (all other things equal):
 - the further a country is from its steady-state, the more rapidly it will grow because there is a greater “gap” between capital formation (sY) and the amount needed to keep it constant $((n+d)k)$:



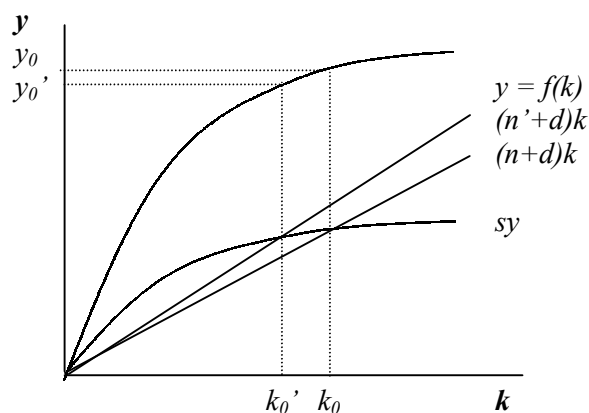
- net increments to capital per worker (k) and capital (K) decline over time (the “gap” becomes smaller) as the economy grows toward its steady-state
 - the empirical findings of this prediction are not supported if other differences among countries are not controlled for in the study (such as stability of the political systems, openness to trade, etc.); however, the empirical findings support this prediction if these differences are considered
 - conditional convergence – convergence does occur if other factors affecting growth are considered
 - thus if two countries have the same conditions in important respects, then it can be predicted that the poorer country will grow more rapidly than the richer country
- studies using regression equations:
 - the dependent variable is the rate of growth of output per worker (y) or GDP per capita
 - the independent (explanatory) variables are initial income, saving rate, etc.
 - these studies use 1 observation per country (so about 100 observations total)
 - these studies consider the rate of growth (measured by GDP per capita, etc.) over some period of time; earlier studies used the time period 1960-1985 but more recent studies use the period 1960-1995 (data before 1960 is not accurate enough or not available for enough countries)

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- all studies find that if enough independent variables are included then initial income is a good predictor of growth in the direction expected (that is, a high initial income leads to a less rapidly growing economy)
- comparative statics (changing one variable and observing the effects) and the Solow model:
 - a change in the saving rate:
 - if the saving rate is increased, it will lead to greater capital formation and quicker growth
 - here the saving rate increases from s to s' (but all the other variables stay the same) which shifts up the savings curve:



- there is a new steady-state equilibrium – the steady-state level of capital per worker increases from k_0 to k_0' and the steady-state level of output per worker increases from y_0 to y_0'
- thus, if a country increases its saving rate, there will be a higher level of steady-state equilibrium capital per worker and output per worker
- if the economy is at equilibrium at capital per worker k_0 , then an increase in the saving rate will cause a burst of growth in capital and in output per worker (y)
- if the economy is still growing (if it's not at its steady-state level of capital and output) then an increase in the saving rate will raise the growth rate
- if the country lowers its saving rate then there will be a lower steady-state level of capital per worker and output per worker – a negative growth rate is also possible
- these outcomes assume that the saving rate (s) does not change from year to year
- a change in the population growth rate:
 - the population growth rate (n) retards growth because it slows the growth of capital per worker (k)
 - n' is an increased population growth rate from the original population growth rate n :



- a higher population growth rate lowers capital per worker, lowers output per worker, and lowers the growth of output per worker
- this prediction of the Solow model supports the intuition that rapid population growth is harmful to growth and development
- the Solow growth model and environmental consequences of growth:
 - in the steady-state of the Solow model, the total national output (Y) and the total capital stock (K) can increase even if there is no increase in capital per worker or output per worker
 - the Solow model was developed in the 1950s when there was less concern for the environment – the Solow model assumes that there are no negative consequences to the growth of an economy (here the economy is said to “grow” if there is an increase in the amount of total output, even if per worker output is constant)
 - this model could be updated to adjust for the environmental carrying capacity
- technology and the Solow model:
 - because technology is improving, output per worker (y) can increase without an increase in the amount of capital per worker (k)
 - technological change can be thought of as the effective labor per worker increasing
 - the effective labor is defined as:

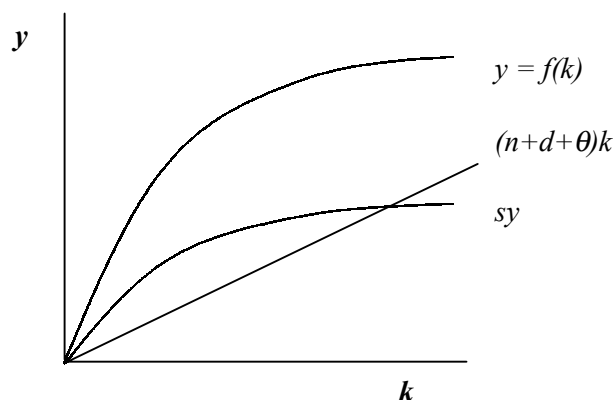
$$\text{effectivelabor} = T \times L$$

T = level of technology and efficiency of labor
 L = labor supply
 - this modifies the production function used earlier:

$$Y = F(K, L) \text{ becomes } Y = F(K, T \times L)$$
 - even if the total labor supply (L) is constant the effective labor supply ($T \times L$) can be increasing if technology is improving
 - the rate of technological change is given the symbol θ and can be considered increments to effective labor
 - k now equals the capital per effective labor unit:

$$k = \frac{K}{T \times L}$$

- the capital stock must grow to offset d , n , and θ - thus, the rate at which capital must be replaced in order for k to remain constant is now $(n+d+\theta)k$:



- at the steady-state, output per unit of effective labor is unchanging
- labor (growing at rate n) is not growing as fast as effective labor (growing at rate $n+\theta$)
- at steady-state equilibrium the total capital stock (K) grows at $(n+\theta)$ per year and the output per *effective* worker (y) is a constant (this model fixes output per effective unit of labor):

$$y = \frac{Y}{T \times L}$$

- however, capital per worker (the ratio K/L as opposed to $K/T \times L$) is growing at rate θ
- even if the effective labor unit per worker is increasing over time, there can still be a steady-state; however, in the steady-state income per worker is increasing
- the addition of technology to the Solow model moves away from the predictions of the earlier model (without technology) that there is no growth per worker at the steady-state; instead, growth can now continue indefinitely (which looks more like what's been seen in the past 150 years in industrialized countries)
- empirical evidence of convergence:
 - studies of convergence are limited - they put all countries into the same model
 - pages 69 to 71 of the text discuss some of the main findings of studies of convergence
 - level of income: conditional convergence is supported (however, if other factors are not controlled for then it does not appear that poor countries grow more rapidly)
 - life expectancy (measure of health): a higher life expectancy leads to a higher growth rate of per capita income
 - education: it is predicted that higher education levels will lead to higher growth - some studies support this
 - geography: the theory that geography affects growth is supported by Jeffrey Sachs and is highly controversial
- two important findings relating geography to economic growth:
 - access to ports is conducive to growth:
 - access to ports is measured by the percent of a country that has access to navigable waterways (a measure of ability to trade internationally)
 - Switzerland is an exception to landlocked countries that are not growing, possibly due to other mitigating factors
 - however, for Afghanistan, etc. being landlocked seems to retard growth

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- proximity to the equator (having a tropical climate) retards growth:
 - according to Jeff Sachs this may be due to the effects of climate on soil, diseases, plants, and animal husbandry; for example, in large areas in Africa it is not possible to raise cattle because of the tsetse fly
 - Sachs argues the North/South dichotomy is incorrect because it is actually a dichotomy between the poles and tropics; for example, areas far enough south of the equator in Africa, Australia, and Latin America have stronger economic growth
 - there is a concave relationship between frost-free days and growth; an empirical observation is that it is best to have an average number of frost-free days (frost kills diseases, pests, etc.)
- variables found to affect growth:
 - savings and investment work the expected way
 - trade and exchange rate policy work the way predicted by neoclassical economics
 - the exchange rate can be distorted from its equilibrium value
 - this price distortion can be measured by the black market premium of foreign currency
 - studies show price distortions hurt growth
 - countries with more natural resources (e.g. minerals, tropical crops) have had slower rates of growth – this leads to the counterintuitive notion that it is better not to have natural resources (like Taiwan, Japan, and South Korea, which relied on human capital and services for growth)
- sources of growth analysis (growth accounting analysis):
 - sources of growth analysis (growth accounting analysis) decomposes growth of output due to each factor of production and attributes the rest of growth to improved technology and efficiency
 - the production function is modified to include a term for technical efficiency:
$$Y = F(K, L, A)$$

Y = total output
 K = total capital stock
 L = total labor supply
 A = a measure of technical efficiency
 - the production function could also be modified to include other factors of production, such as measures of arable land, natural resources, unskilled and skilled labor, etc.
 - the growth of output will be attributed to the growth of factors of production and the efficiency with which they are used – the problem is to determine what proportion of growth to attribute to the growth of each factor of production
 - we typically use a constant returns to scale production function:
$$Y = AK^\alpha L^{1-\alpha}$$

Y = total output
 A = efficiency with which factors of production are used (total factor productivity)
 K = capital supply
 L = labor supply

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- the growth accounting analysis begins by taking the natural logarithm of the production function:

$$\ln Y = \ln A + \alpha \ln K + (1 - \alpha) \ln L$$

α = elasticity of output of capital (or capital's share of total output)

$1 - \alpha$ = elasticity of output of labor (or labor's share of total output)

α , and $(1 - \alpha)$ can be determined econometrically (empirically); to determine α empirically, note that theoretically, the wage is equal to the marginal productivity of labor:

$$w = MPL = \frac{\partial Y}{\partial L} = (1 - \alpha) \frac{Y}{L}$$

w = wage rate

MPL = marginal productivity of labor

because the wage rate times the total amount of labor equals labor's total share of (earnings from) output, the share of output due to labor can be written as:

$$\text{labor's share of output} = \frac{w \times L}{Y} = 1 - \alpha$$

thus, $1 - \alpha$ represents labor's share of total output; similarly α represents capital's share of total output

- an equation derived from the production function relates the growth rate of output and the weighted growth rates of inputs (factors of production):

$$g_Y = a + (w_K \times g_K) + (w_L \times g_L)$$

g_Y = growth rate of output (Y)

a = growth rate of productivity

w_K = capital's share of national income (α) or capital's estimated elasticity

g_K = growth rate of capital (K)

w_L = labor's share of national income ($1 - \alpha$) or labor's estimated elasticity

g_L = growth rate of labor (L)

other factors could be added into this equation by adding the growth rate of each factor multiplied by its weight

- if the growth rate of output (g_Y) exactly equals the sum of the weighted growth rates of the inputs (here only labor and capital) then productivity did not grow; however, if there is a difference between the growth rate of output and the sum of the weighted growth rates of inputs then the increase in the amount of the factors of production is insufficient to explain the growth of output
- an example where productivity does not grow: if the weights of labor and capital are each $\frac{1}{2}$ ($\frac{1}{2} = \alpha = 1 - \alpha$), $g_K = 6\%$, $g_L = 4\%$, and $g_Y = 5\%$, then the growth rate of output is fully explained by the growth rates of the factors of production
- an example where productivity grows: if the weights of labor and capital are each $\frac{1}{2}$ ($\frac{1}{2} = \alpha = 1 - \alpha$), $g_K = 6\%$, $g_L = 4\%$, and $g_Y = 6\%$, then the growth of factors of production does not entirely explain the growth of output – the difference between the weighted

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growth rate of the inputs and the growth rate of output can be attributed to the rate of productivity growth (a)

- an example where productivity grows (a is positive): suppose $g_K = g_L = 0.04$ and $g_Y = 0.05$, $w_K = w_L = 0.5$ (each factor earns 50% of national income):

$$g_Y = a + (0.5)(0.04) + (0.5)(0.04)$$

$$g_Y = 0.05 = a + 0.04$$

$$a = 0.01$$

thus, factor productivity (a) grows at 1% per year; so 4/5 of the growth of national output is explained by the growth of factors of production and 1/5 of the growth is explained by the growth of productivity

- the growth of output and the growth of productivity – what part of g_Y is explained by a ?
 - studies found that the growth of the capital stock and growth of the labor supply did not explain as much of the growth of income as expected; this “unexplained residual” came to be known as technological change
 - this technological change can be attributed to: an increase in allocative efficiency, an increase in technical efficiency (x-efficiency), technological change, and adoption of improved techniques
 - allocative efficiency:
 - allocative efficiency is the efficiency with which factors are drawn into production
 - allocative efficiency may differ between planned and protectionist economies and market economies
 - in a planned or protectionist economy a firm might be using more factors of production than necessary, or might be using factors in economically inappropriate proportions, but because the entrepreneur is making sufficient profit, he does not have an incentive to reduce the amount of factors of production he is using – this is an inefficiency in the economy because the extra factors of production he is using could be more productive elsewhere
 - technical efficiency (x-efficiency):
 - managers and workers work harder, which is usually attributed to improved motivation or incentives
 - for example, competition will pressure inefficient monopolies or protected firms to maximize efficiency and will improve the motivation of managers
 - technological change:
 - technological change is an improvement in techniques due to innovation
 - technological change enables a greater amount of output to be produced with a given amount of inputs
 - adoption of improved techniques:
 - for example, the Green Revolution led to increased yields; as more and more farmers adopted new techniques, output moved closer to the new technology frontier
- research on technological change (discussion on pages 76-78 in text):
 - research focusing on technological change in developed and developing countries in the early 1990s found that much of growth in industrialized countries was due to

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technological change; however, when the same techniques were applied in developing countries the contribution of technological change to growth was smaller

- Alwin Young's study of the growth of the "four tigers":
 - market economists argue that this growth was due to a movement toward market forces; others argue that this was due to appropriate government intervention following the Japanese model; in either case, growth was attributed to institutions
 - Young found that the growth was mostly due to a high growth rate of capital due to high saving rates, not to increasing efficiency due to institutions
- study by Collins and Bosworth in 1996:
 - showed that the four tigers grew over the past 25 years due to factor accumulation but also due to total factor productivity growth
 - page 77, table 2-7 shows that the productivity growth of the four tigers was higher than that of the earlier industrialized countries
- total factor productivity (TFP):
 - total factor productivity differs from the increase in productivity of only one factor; for example, labor productivity (productivity of a single factor) would be defined as output per worker:

$$y = \frac{Y}{L}$$

Y = total output

L = total number of workers

y = labor productivity (output per worker)

labor productivity would grow if each worker has more capital to work with or if technology or incentives improve

similarly, if labor or capital is added to land, the productivity of the land will increase

- total factor productivity combines the productivity of the factors of production:

$$\frac{Y}{\text{combined factors}} = \frac{Y}{w_K K + w_L L}$$

w_K = capital's share of national income (α) or capital's estimated elasticity

K = capital stock

w_L = labor's share of national income ($1-\alpha$) or labor's estimated elasticity

L = labor supply

in the denominator, each of the factors (machines, workers, land, etc.) is multiplied by its weight when they are added together for a measure of combined inputs

- endogenous growth models:
 - other models modify the Solow model; for example, by assuming increasing returns to scale (the Solow model assumes diminishing return to scale)

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- increasing returns to scale might be due to positive externalities and economies of agglomeration; for example, there are positive spillover effects to the education of another scientist
- if an economy has low levels of education, poor infrastructure for communication and transportation, and low levels of innovation, then the addition of a small amount of educated workers will have a small effect due to fewer positive externalities and fewer complementary factors