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Growth rates

- 1) China's GDP increased from \$997.5 billion in 1999 to \$1,076.9 billion in 2000. Calculate the growth rate of China's GDP in 2000 (this data is from <http://www.worldbank.org>).
- 2) India's real GDP per capita (PPP) grew at an average annual rate of 2.00% from 1960 through 1996, increasing from \$769 to \$1,546. Assuming India's GDP per capita continues growing at this average rate from 1996 through 2046, what will India's real GDP per capita equal in 2046?
- 3) China's real GDP per capita (PPP, 1985 constant prices) increased from \$564 in 1960 to \$2,374 in 1996. Calculate the average annual growth rate of China's real GDP per capita over the period 1960-1996.

The Harrod-Domar model

- 4) From 1980 to 1990, real GDP in India grew by 5.8 percent per annum, while investment averaged 23.1 percent of GDP. What was the ICOR for India between 1980 and 1990?
- 5) In Indonesia during the 1970s the incremental capital-output ratio (ICOR) averaged 2.50.
 - a. Using the Harrod-Domar growth equation, what saving rate would have been required for Indonesia to achieve an aggregate growth rate of 8 percent per annum?
 - b. With the same ICOR, what growth target could be achieved with a saving rate of 27 percent?
 - c. If there is a large increase in the saving rate, and therefore a large increase in the amount of new capital formation, is the ICOR likely to rise, fall, or remain the same? Explain.
- 6) The government of a poor developing country fears that a political upheaval will occur unless the growth rate is at least 4 percent per annum. The ICOR and the saving rate are projected to be $v = 5.0$ and $s = 14$ percent, respectively.
 - a. Show that 4 percent growth cannot be achieved under these circumstances.
 - b. With the saving rate as given, what ICOR would be required to achieve the 4 percent growth target?

Growth Accounting

- 7) Consider an economy in which the labor force grows by 2.7 percent per annum, while the capital stock grows by 4 percent per annum. Suppose 55 percent of national income goes to labor and 45 percent to capital.
 - a. If the residual were $a = 0$, what rate of growth would the economy achieve?
 - b. The country's actual rate of growth has been 4.5 percent per annum, which is faster than the growth rate generated by the accumulation of capital and labor stocks. Calculate the value of the residual (a).
- 8) From 1970 to 1989 Singapore's growth rate averaged 8.4 percent per year. A recent growth-accounting study showed that the residual accounted for only 1.2 percent per year of Singapore's outstanding growth performance. This growth-accounting analysis used weights of 0.33 for labor and 0.67 for capital, including human capital.
 - a. Singapore's labor force grew by 2.6 percent per year during this period. What can you conclude about the annual growth rate of Singapore's capital stock (including both human and physical capital)?

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b. What fraction of the overall 8.4 percent growth rate is attributable to capital investment?

Answers

1) The expression for the growth rate in year $t+1$ is $g = \frac{Y_{t+1} - Y_t}{Y_t}$, where g represents the growth rate of GDP, Y_t represents GDP in year t , and Y_{t+1} represents GDP in year $t+1$. In this example, \$997.5 billion corresponds to Y_t and \$1,076.9 corresponds to Y_{t+1} . Thus,
$$g = \frac{\$1,076.9 - \$997.5}{\$997.5} = 0.0796 \approx 8.0\%$$
. During the year 2000, China's GDP grew at a rate of about 8.0% per year.

2) If the average growth rate over the fifty year period 1996 through 2046 equals 2.00%, then India's GDP per capita in 2046 can be expressed as: $Y_{2046} = Y_{1996}(1 + g)^{50}$, where Y_{2046} represents real GDP per capita in 2046, Y_{1996} represents real GDP per capita in 1996, and g represents the average annual growth rate. Substituting in the appropriate numbers,
 $Y_{2046} = (\$1,546)(1 + 0.0200)^{50} = \$4,161$, so real GDP per capita in 2046 (Y_{2046}) is predicted to be \$4,161.

3) Real GDP per capita in 1960, \$564, grew over a 36 year period to \$2,374. This can be expressed as: $\$2,374 = \$564(1 + g)^{36}$, where g is the average annual growth rate. To solve for g , divide both sides by \$564: $\frac{\$2,374}{\$564} = (1 + g)^{36}$. Now raise each side of the equation to $1/36$:

$\left(\frac{\$2,374}{\$564}\right)^{\frac{1}{36}} = \left((1 + g)^{36}\right)^{\frac{1}{36}} = 1 + g$. So, $g = \left(\frac{\$2,374}{\$564}\right)^{\frac{1}{36}} - 1 = 0.0407$. Thus, China's real GDP per capita grew an average of about 4.1% annually.

4) The annual growth rate (g) and investment rate are given as 5.8% and 23.1%, respectively. The Harrod-Domar model assumes that the investment rate equals the saving rate (s), so the saving rate equals 23.1%. To calculate the ICOR (v), use the Harrod-Domar equation, $g = \frac{s}{v}$, which can be rewritten as $v = \frac{s}{g}$. So, $v = \frac{23.1\%}{5.8\%} = \frac{0.231}{0.058} = 3.98 \approx 4.0$. Thus, the ICOR was about 4.0.

5)

a. Given that the ICOR (v) equals 2.50 and the targeted annual growth rate is 8% (g), the Harrod-Domar equation can be rewritten $s = gv$ in order to solve for the required saving rate (s). Thus, $s = (0.08)(2.50) = 0.20 = 20\%$. In order to achieve an 8% growth rate, Indonesia would have to have a saving rate of 20%.

b. Given the ICOR (v) equals 2.5 and the saving rate (s) equals 27%, the Harrod-Domar equation can be used to determine the expected annual growth rate (g): $g = \frac{s}{v}$. Substituting

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in the given values for s and v : $g = \frac{0.27}{2.5} = 0.108 = 10.8\%$. If the saving rate were 27%, the growth rate would be 10.8% per year.

- c. The Harrod-Domar model assumes the ICOR (v) remains constant. Thus, according to the Harrod-Domar model, an increase in the saving rate has no effect on the ICOR. One might conjecture, however, that the model's assumption is wrong and that a large increase in savings might result in some decline in the productivity of capital, hence a higher ICOR.

6)

- a. If the saving rate is 14% and the ICOR is 5.0, then the growth rate can be expected to

be $g = \frac{s}{v} = \frac{0.14}{5.0} = 0.028 = 2.8\%$. This falls short of the desired 4% growth rate.

- b. If the targeted growth rate is 4% and the saving rate is 14%, then the ICOR must equal to

$v = \frac{s}{g} = \frac{14\%}{4\%} = \frac{0.14}{0.04} = 3.5$. Note that if the ICOR were lower than 3.5, growth would be greater than 4.0% per year (assuming the saving rate remains 14%).

7)

- a. If the residual (a) equals 0, the annual labor force growth rate (g_L) equals 2.7%, the annual capital stock growth rate equals 4% (g_K), and labor's share of national income (w_L) is 55% while capital's share of national income (w_K) is 45%, then the growth rate of national income (g_Y) is: $g_Y = a + (w_K \times g_K) + (w_L \times g_L) = 0 + (0.45 \times 0.04) + (0.55 \times 0.027) = 0.03285 = 3.3\%$. Thus, the economy will grow at a 3.3% annual rate.

- b. In this case, the growth of the inputs (capital and labor) is insufficient to explain the growth of the economy. The difference between the growth rate and the sum of the weighted factor growth rates is the residual (a). This residual can be calculated using the same equation as in part a: $4.5\% = 0.045 = g_Y = a + (w_K \times g_K) + (w_L \times g_L) = a + (0.45 \times 0.04) + (0.55 \times 0.027)$. Simplifying and solving this equation for a : $a = 0.01215$. Thus, the residual equals 0.01215.

8)

- a. Given that the growth rate of income (g_Y) equals 8.4% per year, the residual (a) equals 1.2%, the labor force growth rate (g_L) is 2.6%, and the weights on the growth rates of labor (w_L) and capital (w_K) are 0.33 and 0.67, respectively, the growth accounting equation can be solved for the growth rate of capital (g_K). The growth accounting equation is $g_Y = a + (w_K \times g_K) + (w_L \times g_L)$ so $8.4\% = 1.2\% + (0.67 \times g_K) + (0.33 \times 2.6\%)$. Simplifying and solving, $g_K = 0.0947 \approx 9.5\%$. Thus, the annual growth rate of Singapore's capital stock averaged 9.5%.

- b. We are given that income grows by 8.4% annually during this period. Growth of the capital stock alone accounts for a growth rate of income of $(w_K \times g_K) = (0.67 \times 0.095) = 6.37\%$.

Thus, growth of the capital stock is responsible for $\frac{6.37\%}{8.4\%} = 0.76 = 76\%$ of the growth rate of income.