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See pages 204-207 and 432-433 in the textbook for a discussion about discounting and present value.

Problems

1. Calculate the present value of \$250 received six years from today. Assume the discount rate is 7%.
2. A private owner can sell some resource at any time according to the revenues in the following schedule:

Year	Revenue
2002 (today)	\$50,000
2003	\$55,000
2004	\$63,000
2005	\$78,000
2006	\$86,000

Assuming the discount rate is 11%, in what year should the owner sell the resource?

3. Suppose an LDC discovers a reserve of oil that it could sell for \$100 million today (in 2002) or for \$200 million in 2010.
 - a) If the discount rate were 8%, should the LDC sell the oil today or wait until 2010?
 - b) If the discount rate were 10%, should the LDC sell the oil today or wait until 2010?
 - c) At what discount rate would the LDC be indifferent about selling the oil today or in 2010?

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Answers

1. The \$250 must be discounted to determine its present value today. The present value of the \$250 can be determined using the formula:

$$PV = \frac{\$250}{(1+r)^n}$$

PV = the present value of the \$250

r = the discount rate

n = the number of years from today the \$250 will be received

The discount rate is given as 7% (so $r = 7\%$) and the \$250 will be received in 6 years (so $n = 6$). Thus, the present value of the \$250 is:

$$PV = \frac{\$250}{(1+0.07)^6} = \frac{\$250}{1.5007} = \$166.59$$

2. The owner will sell the resource in the year for which the present value of the revenue is highest. The present value of the revenue from selling the resource is given by:

$$PV = \frac{\text{revenue}}{(1+r)^n}$$

PV = the present value of the revenue

revenue = the revenue earned from selling the resource

r = the discount rate (this is given as 11% in this problem)

n = the number of years from today the resource is sold

In order to determine the year for which the revenue has the highest present value, the present value of the revenue for each year is calculated:

$$PV_{2002} = \frac{\$50,000}{(1+0.11)^0} = \$50,000$$

$$PV_{2003} = \frac{\$55,000}{(1+0.11)^1} = \$49,549.55$$

$$PV_{2004} = \frac{\$63,000}{(1+0.11)^2} = \$51,132.21$$

$$PV_{2005} = \frac{\$78,000}{(1+0.11)^3} = \$57,032.93$$

$$PV_{2006} = \frac{\$86,000}{(1+0.11)^4} = \$56,650.86$$

The highest present value of the different possible revenues is \$57,032.93 in 2005. Thus, the owner should sell the resource in 2005.

3. a) If the present value of the \$200 million from selling the oil reserve in 2010 is greater than the \$100 million the LDC would receive today, then it will sell the oil reserve in 2010. On the other hand, if the \$100 million is greater than the present value of the \$200 million, then it will sell the oil today.

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The present value of the \$200 million is calculated by the formula:

$$PV = \frac{\$200\text{million}}{(1+r)^n}$$

PV = the present value of the \$200 million

r = the discount rate

n = the number of years from today the \$200 million will be received

The discount rate (r) is given as 8%. The number of years from now the \$200 million will be received (n) is 8 (because $8 = 2010 - 2002$).

Thus, the present value of the \$200 million is:

$$PV = \frac{\$200\text{million}}{(1+0.08)^8} = \$108\text{million}$$

Because the present value of the \$200 million in 2010 is greater than the \$100 million dollars the LDC could receive today, the LDC should sell the oil reserve in 2010.

b) This is similar to part 3a, except the discount rate has changed to 10%. Thus, the present value of the \$200 million that could be earned in 2010 is:

$$PV = \frac{\$200\text{million}}{(1+0.10)^8} = \$93.3\text{million}$$

Because the present value of the revenue earned 8 years from now (\$93.3 million) is less than the revenue earned today (\$100 million), the LDC will sell the oil today.

c) The LDC will be indifferent about selling the oil today or in 2010 when the present value of the revenue that could be earned in 2010 equals the revenue that could be earned today. Thus, the present value of the revenue from 2010 (\$200 million) is set equal to the revenue today:

$$PV_{2010} = \$100\text{million}$$

The present value of the revenue from selling the oil in 2010 is:

$$PV_{2010} = \frac{\$200\text{million}}{(1+r)^8}$$

Substituting into the first equation:

$$\frac{\$200\text{million}}{(1+r)^8} = \$100\text{million}$$

Solving for r , the discount rate:

$$\frac{\$200\text{million}}{\$100\text{million}} = (1+r)^8$$

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$$\left(\frac{\$200\text{million}}{\$100\text{million}} \right)^{\frac{1}{8}} = \left((1+r)^8 \right)^{\frac{1}{8}}$$

$$\left(\frac{\$200\text{million}}{\$100\text{million}} \right)^{\frac{1}{8}} = (1+r)$$

$$\left(\frac{\$200\text{million}}{\$100\text{million}} \right)^{\frac{1}{8}} - 1 = r$$

$$r = 0.09051\dots \approx 9.05\%$$

Thus, if the discount rate equals 9.05%, the LDC is indifferent about selling the oil today or in 2010.