EC151: Problems on labor absorption, elasticity of substitution, labor migration, internal migration, discounting, and real interest rates

Labor Absorption by Industry

1. The Republic of Kita in 1996 has a labor force of 1,000 workers, of whom 150 work in the industrial sector. Because industrial policy favors growth of capital-intensive activities, on average a 10 percent increase in industrial value added brings about only 4 percent growth in industrial employment on average. In 1997, value added in the industrial sector is expected to increase by 15%. Calculate the employment growth in industry as a percentage of the initial labor force.

Elasticity of Substitution

2. Suppose a policy reform reduces the wage-rental ratio from 1.5 to 0.33. As a result, the capital-labor ratio drops from 0.8 to 0.15. What is the elasticity of substitution of capital to labor?

Internal Migration

3. Suppose the urban wage of a developing country is $1.50 per day and rural workers earn $1.00 per day. At what probability of finding a job does the Harris-Todaro model predict rural-to-urban net migration will stop? If there are 150,000 urban jobs available, what does the Harris-Todaro model predict will be the number of unemployed urban people when net migration stops?

Discounting

4. A project costs $10,000 today and $20,000 next year, but yields $500,000 10 years from now. If the discount rate is 10%, what is the net present value of this project (assume all costs/benefits are earned at the beginning of the year)? What is the benefit-cost ratio? Is the internal rate of return 30%, 35%, or 40%?

5. Suppose a child attends school for two years and works for three years, with explicit costs and earnings:

	Year 1	Year 2	Year 3	Year 4	Year 5
Costs	$200	$250	$0	$0	$0
Earnings	$0	$0	$800	$1000	$1200

A child who receives no education will have the following earnings profile (costs are zero):

	Year 1	Year 2	Year 3	Year 4	Year 5
Earnings	$100	$150	$200	$300	$400

Calculate the net present value of the two years of education with an interest rate of 9% (assume that $t = 0$ at the beginning of year 1 and all costs/benefits are earned at the beginning of the year).

Real Interest Rates

5. In 1995, the nominal interest rate in Russia was 242.4% while the inflation rate was 205.2%. Calculate the real interest rate.
6. In Zambia in mid-1992, the nominal interest rate on bank deposits was 48 percent and the inflation rate was 112 percent. Interest earned on bank deposits was subjected to a 10 percent income tax. Calculate the after-tax real interest rate (the after-tax rate of return) on bank deposits.

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Answers

1. Employment growth in industry as a percentage of the initial labor force can be determined using the equation $\Delta E_i = \eta g(V_i) S_i$, where ΔE_i = annual employment growth in industry as percent points of the labor force, η = elasticity of employment with respect to the value-added, $g(V_i)$ = growth rate of industrial value-added, and S_i = industrial employment as a fraction of total employment. η equals the percent change in industrial employment divided by the percent change in industrial value added, so $\eta = \frac{0.04}{0.10} = 0.4$. $g(V_i)$ is given as 10% and S_i equals $\frac{150}{1000} = 0.15$. Thus, $\Delta E_i = 0.4 \times 10 \times 0.15 = 0.6$. Employment growth in industry as a percentage of the initial labor force is 0.6%.

2. The elasticity of substitution is given as the ratio of the percent change in the capital-labor ratio to the percent change in the wage-rental ratio: $\sigma = \frac{\left[\frac{\Delta(K/L)}{K/L}\right]}{\left[\frac{\Delta(w/r)}{w/r}\right]}$. Substituting in the proper numbers: $\sigma = \frac{\frac{0.15-0.8}{0.8}}{\frac{0.33-1.5}{1.5}} = \frac{-81.25}{-78} = 1.04$.

3. The Harris-Todaro model predicts that net migration will stop when the rural wage (W_r) equals the expected urban wage (pW_u), $pW_u = W_r$. The probability of finding a job in the urban sector when net migration stops is, thus, $p = \frac{W_r}{W_u} = \frac{1.00}{1.50} = 2/3$.

   The equation $p = \frac{E_u}{E_u + U_u}$, where p = the probability of finding a job in the urban sector, E_u = the number of urban employed, U_u = the number of urban unemployed, can be used to predict the number of urban unemployed when rearranged to $U_u = \frac{E_u}{p} - E_u$. The number of urban jobs available is equal to the number of urban employed, so E_u = 150,000. It was determined that net migration stops when p = 2/3. Thus, the number of urban unemployed when net migration ceases is $U_u = \frac{150{,}000}{2/3} - 150{,}000 = 75{,}000$.

4. The net present value of the education is given by the equation $NPV = \sum_{t=1}^{n} \frac{B_t}{(1+i)^t} - \sum_{t=1}^{n} \frac{C_t}{(1+i)^t}$, where NPV = net present value, C_t = the cost in year t, B_t = benefits in year t, i = the interest rate, and t = the number of years the benefits/costs in year t must be discounted to the present.

First, the stream of benefits ($\sum_{t=1}^{n} \frac{B_t}{(1+i)^t}$) is calculated: $\sum_{t=1}^{n} \frac{B_t}{(1+i)^t} = \frac{\$500{,}000}{(1+0.10)^{10}} = \$192{,}772$.

Second, the stream of costs ($\sum_{t=1}^{n} \frac{C_t}{(1+i)^t}$) is calculated:

$\sum_{t=1}^{n} \frac{C_t}{(1+i)^t} = \frac{\$10{,}000}{(1+0.10)^0} + \frac{\$20{,}000}{(1+0.10)^1} = \$28{,}182$. Thus,
$NPV = \$192{,}772 - \$28{,}182 = \$164{,}590$.

The benefit-cost ratio is given by the equation: $BCR = \frac{\sum_{t=1}^{n} \frac{B_t}{(1+i)^t}}{\sum_{t=1}^{n} \frac{C_t}{(1+i)^t}} = \frac{\$192{,}772}{\$28{,}182} = 6.8$.

The internal rate of return is the interest rate at which the project's discounted cost stream and discounted benefit stream are equal. Calculating the discounted benefit and cost streams with interest rates of 30%, 35%, and 40%:

interest rate	$\sum_{t=1}^{n} \frac{C_t}{(1+i)^t}$	$\sum_{t=1}^{n} \frac{B_t}{(1+i)^t}$
30%	\$25,384	\$36,269
35%	\$24,814	\$24,867
40%	\$24,286	\$17,286

shows that the project breaks even at an interest rate of about 35%. Thus, the internal rate of return is 35% (to the nearest 1%).

5. The net present value of the education is given by the equation
$NPV = \sum_{t=1}^{n} \frac{E_t}{(1+i)^t} - \sum_{t=1}^{n} \frac{C_t}{(1+i)^t}$, where *NPV* = net present value, C_t = the costs of additional education in year *t*, E_t = additional earnings in year *t* due to the additional education, *i* = the interest rate, and *t* = the number of years the benefits/costs in year *t* must be discounted to the present.

First, the stream of additional benefits from the education ($\sum_{t=1}^{n} \frac{E_t}{(1+i)^t}$) is calculated. The difference in earnings (E_t) must be calculated for each year:

	Year 1	Year 2	Year 3	Year 4	Year 5
Earnings with Education	\$0	\$0	\$800	\$1000	\$1200
Earnings without Education	\$100	\$150	\$200	\$300	\$400
Difference in Earnings	-\$100	-\$150	\$600	\$700	\$800

The discounted stream of additional earnings is:

$$\sum_{t=1}^{n} \frac{E_t}{(1+i)^t} = \frac{-\$100}{(1+0.09)^0} + \frac{-\$150}{(1+0.09)^1} + \frac{\$600}{(1+0.09)^2} + \frac{\$700}{(1+0.09)^3} + \frac{\$800}{(1+0.09)^4} = \$1{,}375$$

Next, the discounted stream of costs ($\sum_{t=1}^{n} \frac{C_t}{(1+i)^t}$) is calculated:

$$\sum_{t=1}^{n} \frac{C_t}{(1+i)^t} = \frac{\$200}{(1+0.09)^0} + \frac{\$250}{(1+0.09)^1} = \$429.$$

Thus, $NPV = \$1{,}375 - \$429 = \$946$.

6. The real interest rate can be calculated from the equation $r = \frac{(1+i)}{(1+p)} - 1$, where r is the real interest rate, i is the nominal interest rate, and p is the inflation rate. Thus, $r = \frac{(1+2.424)}{(1+2.052)} - 1 = 0.122 = 12.2\%$.

7. The after-tax rate of return can be calculated from the equation $r_n = \frac{1+[i(1-t)]}{(1+p)} - 1$, where r_n is the real rate of return after taxes, i is the nominal interest rate, t is the tax rate, and p is the inflation rate. Thus, $r_n = \frac{1+[0.48(1-0.10)]}{(1+1.12)} - 1 = -0.325 = -32.5\%$.