Suburbanization and Transportation in the Monocentric Model

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Abstract

This paper presents a version of the monocentric city model that incorporates heterogeneous commuting speeds by introducing radial commuting highways. This model implies that metropolitan area population spreads out along new highways, which are positively valued by residents. Simulations of conservative specifications of the model imply that each additional highway ray causes about a 10 percent decline in central city population. Given observed central population declines and urban highway construction between 1950 and 1990, this model implies that highways can account for an important part of urban population decentralization.

1 Introduction

One standard result from the classical monocentric land use theory developed by Alonso [1], Muth [7] and Mills [6] is that an increase in transport speed reduces population density near city centers relative to suburban areas. A natural implication of this comparative static is that improvements to the transportation infrastructure may be an important explanation for falling urban population density. Indeed, despite robust population growth in metropolitan areas, urban population density has been declining rapidly. As documented in Baum-Snow [3], the aggregate population in U.S. central cities as defined by their geography in 1950 declined by 17 percent between 1950 and 1990 despite national population growth of 64 percent during this period. This paper examines the extent to which the simple mechanism of the monocentric city model can account for the post World War II declines in central city populations.

I adapt a version of the standard monocentric city model to allow for heterogeneity in commuting speeds through the introduction of radial highways. As in the standard monocentric framework, all workers commute to a unique central location to work and choose a residential location based on the trade-off between lost wages due to travel time and their preference for space. The endogenously determined land rent function distributes identical individuals over the available land such that in equilibrium, everyone's utility is equal and population density is decreasing in commuting time.

The implications of extending this model to allow for heterogenous travel speeds have been explored in several ways. Some past work (as in Leroy and Sonstelie [5]) has introduced heterogeneity in commuting speed through different travel modes. While allowing mode choice is clearly a relevant approach for a few big cities, in only 6 metropolitan areas did at least 20 percent of the population commute by public transportation in 1960 and by 1990 this had dropped to just one (New York). Moreover, most previous versions of this model have symmetric or one-dimensional space. The model developed in this paper is similar to that formulated by Anas and Moses [2]. As in Anas & Moses' model, this model incorporates heterogeneous commuting speeds through radial highways into the monocentric framework.

As such, equilibrium land use structure exhibits heterogeneity in residential density conditional on distance to the city center. While more stylized than Anas & Moses' model in some ways, the model specified in this paper generates analytical implications that apply quite generally across preference specifications and city structures. The model also generates simulation results that are quantitatively robust to a variety of metropolitan area structures.

This paper shows that even with a conservative specification for the utility function and the commuting technology, the monocentric city model implies that new highways are likely to be an important element needed to explain urban population decentralization. For a metropolitan area with half of its residents living in the central city absent any highways, simulation results indicate that the first few highway rays each cause about a 10 percent decline in central city population, with the marginal effect declining 1 or 2 percentage points for each additional ray. Simulations of the model yield effects of new highways that are quantiatively consistent with the empirical results in Baum-Snow [3]. The range of simulation estimates in this paper imply that construction of new highways can account for as much as one-third of the decline in aggregate central city U.S. population relative to national population growth. The mean 1950 geography central city saw its population decline by 28 percent and received about 2.5 highway rays between 1950 and 1990 implying that highway construction can account for nearly the full decline in the population of the average central city.

This paper proceeds as follows. Section 2 proposes the model. Section 3 proves a few relevant analytical implications of the model. Section 4 presents some facts about the empirical relationship between highways and suburbanization. Section 5 presents simulation results using two different utility functions and extends the analysis to incorporate congestion. Finally, Section 6 concludes.

2 The Model

The model laid out below is a closed city absentee landlord version of the monocentric model in which average commuting speed differs as a function of residential location. I focus on one metropolitan area that is treated as its own atomistic general equilibrium system. There is a continuum of N individuals, each of whom commutes to the central work location (referred to below as the CBD) and earns an exogenously given wage w per unit time. Individuals have direct preferences over a composite consumption good z and space s given by $U(z,s)$. U is increasing and weakly concave in each of its arguments and both goods are weakly normal. Because all individuals are identical, an equilibrium consists of a situation in which everybody has the same utility and cannot gain higher utility by moving to an alternate residential location.

The main innovation of this model is that it incorporates highways into the transportation infrastructure of a monocentric city. Highways are modelled as linear "rays" emanating from the city's core along which the travel speed is faster than on surface streets. Whiles there is a discrete number of exogenously assigned rays in the metropolitan area, a continuum of surface streets connect each point in the metropolitan area to the central work location and to the nearest highway. Define b as the inverse of speed on surface streets. Denote the speed ratio on surface streets to highways as γ. Rays are distributed evenly about the origin such that they serve the maximum number of people possible. M denotes the number of rays in the metropolitan area. I index space in polar coordinates (r, ϕ), where ϕ is the angle to the nearest highway ray and r is the distance to the work location.

In order to facilitate the analysis of equilibrium land use patterns, it is convenient to break the metropolitan area up into two regions: one in which residents do not use a highway for any part of their commutes and another whose residents commute at least partly via a highway. In the former region, commuting time is given by br. In the highway commuting region, one can imagine three reasonable assumptions about how individuals access the highway using surface streets: perpendicularly via linear streets (technology 1), around streets that form concentric circles about the origin as in Anas & Moses [2] (technology 2) or via linear streets meeting the highway at distances chosen to minimize total commuting time (technology 3). A general form for the travel time from point (r, ϕ) to the origin that incorporates all three assumptions about the commuting technology is $br\widetilde{L}(\phi)$ where[1]

$$\begin{aligned} (1) \qquad \widetilde{L}(\phi) &= \gamma \cos\phi + \sin\phi && \text{given technology (1)} \\ &= \gamma + \phi && \text{given technology (2)} \\ &= \gamma\cos\phi + \sqrt{1-\gamma^2}\sin\phi && \text{given technology (3)} \end{aligned}$$

Individuals living at each location (r, ϕ) choose the commuting route to minimize total travel times. As such, the minimum time it takes to travel to the center $(0, 0)$ from (r, ϕ) is given by:

$$(2) \qquad L(r, \phi) = \min[br, br\widetilde{L}(\phi)]$$

Define $\overline{\phi}$ as the angle separating the zone in which individuals use a highway for part of their commutes and that in which they commute directly downtown on surface streets or use an adjacent highway:

$$(3) \qquad \overline{\phi} = \min[\text{solution to } \widetilde{L}(\phi) = 1, \frac{\pi}{M}]$$

If there are sufficiently few highways such that some commute only on surface streets,

[1] The expression for $\widetilde{L}(\phi)$ given commuting technology (3) is derived as follows. Using the law of cosines, the optimal choice of CBD distance x at which to access the highway is the result of the minimization problem:

$$\min_x [b\gamma x + b\sqrt{r^2 + x^2 - 2xr\cos\phi}]$$

The first-order condition implies a quadratic equation in x that when solved implies

$$x^* = 2r\frac{(1-\gamma^2)\cos\phi \pm \sqrt{\cos^2\phi(\gamma^2-1)^2 - (1-\gamma^2)(\cos^2\phi - \gamma^2)}}{1-\gamma^2}$$

Simplifying the expression under the square root and using the fact that $x^* < r\cos\phi$, this becomes

$$\begin{aligned} x^* &= r\frac{(1-\gamma^2)\cos\phi - \gamma\sin\phi\sqrt{1-\gamma^2}}{1-\gamma^2} \\ &= r(\cos\phi - \frac{\gamma}{\sqrt{1-\gamma^2}}\sin\phi) \end{aligned}$$

Inserting this expression into the commuting time function above and simplifying yields the claimed result.

commuters living along the ray formed by the angle $\overline{\phi}$ are indifferent between using a highway for part of their commutes and surface streets exclusively. If there are enough rays such that everybody uses a highway, $\overline{\phi}$ is the angle at which individuals are indifferent between using adjacent highways for part of their commutes. If $\gamma = 0.5$ and the commuting route uses linear surface streets perpendicular to the highway (technology 1), there exist people who do not use a highway for any part of their commutes for $M < 5$. I set the pecuniary cost of commuting to 0.[2]

Given one of these potential commuting technologies, the spatial distribution of residences is determined by utility maximization subject to time and budget constraints. I normalize individuals' total time endowment to 1 such that all time is spent either working or commuting. The price of land is given by $R(r, \phi)$ and the price of the consumption good is normalized to 1. As such, the following equation represents each individuals' resource constraint conditional on living at point (r, ϕ):

$$z + R(r, \phi)s = w[1 - L(r, \phi)] \tag{4}$$

$R(r, \phi)$ is determined endogenously as described in Section 3 below. I denote the compensated demand functions for the composite good and space as $\widetilde{z}(R, u)$ and $\widetilde{s}(R, u)$ respectively. I denote the distance from the central business district to the border between the central city and the suburbs as r_c. This variable is exogenously determined and will serve only to define boundaries within which to evaluate population decline as highway rays are added to the metropolitan area.

The bid-rent function for space is defined as the maximum an individual would be willing to pay to reside at location (r, ϕ) given utility level u. It can be derived by solving for $R(r, \phi)$ from Equation (4) and imposing that each consumer is maximizing over s and z subject to achieving utility level u.

$$\psi[L(r, \phi), u] = \max_s \left\{ \frac{w[1 - L(r, \phi)] - Z(s, u)}{s} \right\} \tag{5}$$

[2]As long as pecuniary travel cost is an increasing linear function of $L(r, \phi)$, its exclusion does not affect the qualitative urban form implied by this model. Provided the wage is high enough that pecuniary travel cost is a small fraction of income, it does not matter quantitatively either.

$Z(s,u)$ is achieved by inverting the equation $U(z,s) = u$ to solve for z. Differentiating Equation (5) reveals that bid-rent is decreasing and convex in travel time from the city center. Bid-rent functions are a convenient definition that will be used extensively in the next section to examine how equilibrium land use patterns change with the introduction of rays.

3 Equilibrium Land Use and Highways

This model retains the primary standard results of the classic monocentric city model, except that the uniform linear correspondence between travel time and distance to the center is broken. Indeed, if space is indexed in terms of travel time rather than distance to the center, this model maps exactly into the classical land use model. Since all individuals are identical, equilibrium land rent at each populated location equals bid-rent given the equilibrium level of utility. The equilibrium land use pattern in the metropolitan area is thus determined jointly by the demand function for space, a condition stating everyone has a place to live and an equation that equalizes rent at the edge of the populated area to rural land rent. I denote u^M as the equilibrium level of utility given M rays in the metropolitan area.

Metropolitan areas are large enough to accommodate the mass of N individuals with the urban fringe $r_f^M(\phi)$ endogenously determined by equating equilibrium bid-rent with the market rental rate for rural land at all angles. At angles $\phi \geq \overline{\phi}$, Equation (6) determines the distance to the fringe that I denote as $\overline{r}_f^M$:

$$\psi(b\overline{r}_f^M, u^M) = \frac{w[1 - b\overline{r}_f^M] - Z(\widetilde{s}(R_a, u^M), u^M)}{\widetilde{s}(R_a, u^M)} = R_a \tag{6}$$

Similarly, at angles $\phi \leq \overline{\phi}$, bid-rent for residential land must equal rural land rent. Because all individuals at the fringe face the same rental rate of land R_a and have the same utility u^M, it is evident that they also must have the same commuting time.

Equilibrium fringe distance is therefore given by the following function:

$$r_f^M(\phi) = \frac{\overline{r}_f^M}{\widetilde{L}(\phi)} \text{ if } 0 \leq \phi \leq \overline{\phi} \qquad (7)$$
$$= \overline{r}_f^M \quad \text{if } \phi \geq \overline{\phi}$$

This function provides the upper limit of integration in the market clearing condition for space below.

Each ray influences commuting time only in the wedge bounded by angle $\overline{\phi}$ on either side of it. In the remaining land within the angle $2\pi - 2M\overline{\phi}$, commuting time is not affected by rays. Define the function $q(\cdot)$ to solve (6) such that $\overline{r}_f^M = q(u^M)$. Substitution of the demand function and the rearranged fringe rent function into the market clearing condition for space yields:

$$N = 2M \int_0^{\overline{\phi}} \int_0^{\frac{q(u^M)}{\widetilde{L}(\phi)}} \frac{r dr d\phi}{\widetilde{s}[\psi(br\widetilde{L}(\phi), u^M), u^M]}$$
$$+(2\pi - 2M\overline{\phi}) \int_0^{q(u^M)} \frac{r dr}{\widetilde{s}[\psi(br, u^M), u^M]} \qquad (8)$$

Equation (8) determines the equilibrium utility level u^M in the city. Fujita's [4] discussion of existence and uniqueness of the equilibrium applies exactly with the replacement of distance to the center with travel time.

Figure 1 presents a schematic diagram of the spatial structure of the city and what happens to urban form when a new ray is introduced. A new highway ray represents a decline in commuting time for a sector of the metropolitan area. This elicits two effects. First, the price of land decreases because more land is accessible for each given commuting time. Holding the agricultural land rent R_a constant, this means that for $\phi > \overline{\phi}$, the fringe moves in towards the center. The decrease in land rent causes agents to increase land consumption via a price effect, inevitably pushing some people further from the center and lowering population density as a result.

Second, average net income rises, causing people to consume more land (assuming land is normal) through a wealth effect, also pushing them away from the center. Since people consume more of both space and the composite consumption good, the new highway causes the equilibrium utility level in the city to rise. In addition, the highway causes the residential land area of the metropolitan area to rise.

Proposition 1 formalizes these results. It states that an additional ray always causes (i) equilibrium utility in the metropolitan area to rise, (ii) the portion of the urban fringe in a region where nobody uses the highway for part of her commute to move inwards and (iii) the portion of the urban fringe on the new highway ray to move outward.

Proposition 1. If $M' = M + 1$ then in equilibrium

$$\text{i) } u^{M'} > u^{M} \quad \text{ii) } \overline{r}_f^{M'} < \overline{r}_f^{M} \quad \text{iii)} \frac{\overline{r}_f^{M'}}{\gamma} > \overline{r}_f^{M}$$

Proof See Appendix A.

An intuitive way to see the structure of different urban equilibria with highways is through examination of bid-rent functions. Indeed, bid-rent and population density are proportional assuming several standard specifications of the utility function. Figure 2 shows how the changing shape of the bid-rent function implies the different city structures in 0, 1 and 2 ray environments. Each line in the figure is a bid-rent function conditional on the angle to the nearest ray and the number of rays in the metropolitan area. The first argument gives the travel time to the work location and the second argument gives the equilibrium utility level. The travel time of rb indicates that the bid-rent function is relevant for $\phi > \overline{\phi}$ while the travel time γrb indicates that the bid-rent function is relevant for $\phi = 0$. The superscripts on u and $\overline{r}_f$ give the number of rays in the metropolitan area.

As is a standard result from monocentric theory, an increase in the utility level in the city is associated with a decline in bid-rent for the same travel speed. This increase in utility associated with extra rays causes the bid-rent function to shift down and intersect R_a at a distance that is closer to the CBD. Given a utility level u^M, all bid-rents conditional on ϕ are the same at the center since travel time is equal at 0. As seen in Figure 2, as ϕ increases from 0 to $\overline{\phi}$, the bid-rent function gets steeper, representing the increased relative value individuals place on being near the CBD due to the lower average commuting speed. Bid-rent along the ray intersects R_a at a distance exceeding the fringe absent the ray. This reflects the fact that in order to house the full population, some extra space has to be claimed along the highway.

While the average population density in the metropolitan area declines with new highways, one area sees increased population density. Individuals move to live near the new ray and enjoy shorter commuting times. As such, an area near the urban fringe and near the highway sees increased population density as a result of the new highway. To see that population density increases near the highway but decreases elsewhere, it is instructive to compare population densities in the 0 and 1 ray equilibria at the coordinates $(\overline{r}_f^1, \overline{\phi})$ and $(\varepsilon, 0)$ as examples. To simplify notation, define $s^M(r, \phi) \equiv \widetilde{s}(\psi(L(r, \phi), u^M), u^M)$. At $(\overline{r}_f^1, \overline{\phi})$, income net of commuting cost does not change with the introduction of the ray, yet utility of the metropolitan area goes up. Therefore, consumption of space increases through a pure price effect:

$$\frac{1}{s^1(\overline{r}_f^1, \overline{\phi})} < \frac{1}{s^0(\overline{r}_f^1, \overline{\phi})}$$

In addition, note that for ε arbitrarily small,

$$\frac{1}{s^1(\varepsilon, 0)} < \frac{1}{s^0(\varepsilon, 0)} \tag{9}$$

because the general equilibrium effect of the highway reducing rents overwhelms the marginal commuting time reduction associated with using the highway for a very short commute. That is, inequality (9) holds because from Proposition 1, $u^1 > u^0$ and $\psi_u < 0$, but commuting cost declines only marginally, implying that $\psi(\varepsilon\gamma b, u^1) < \psi(\varepsilon b, u^0)$. Therefore, demand for space increases at all angles for a sufficiently small radius through a price effect. In addition, commuting time at $(\varepsilon, 0)$ falls slightly causing income net of commuting cost to rise. Demand for space near the new highway therefore also increases through an income effect. This increase in per-capita space consumption implies a decline in population density. Thus if r_c is sufficiently small, central city population is assured to decline with each new highway ray. This result makes up the main argument in the proof of Proposition 2.

Proposition 2. For $M' = M+1$ and r_c sufficiently small, population of the central city assuming M' rays is less than population of the central city assuming M rays.

Proof Central city population is given by:

$$\begin{aligned} N_c &= 2M \int_0^{\overline{\phi}} \int_0^{r_c} \frac{r dr d\phi}{\widetilde{s}[\psi(br\widetilde{L}(\phi), u^M), u^M]} \\ &+ (2\pi - 2M\overline{\phi}) \int_0^{r_c} \frac{r dr}{\widetilde{s}[\psi(br, u^M), u^M]} \end{aligned} \tag{10}$$

From Proposition 1, u is increasing in M. Using the inequalities $\psi_u < 0$, $\frac{\partial \widetilde{s}}{\partial R} < 0$ and $\frac{\partial \widetilde{s}}{\partial u} > 0$, it must be that $\frac{1}{\widetilde{s}[\psi(br, u^{M'}), u^{M'}]} < \frac{1}{\widetilde{s}[\psi(br, u^M), u^M]}$ for all $r > 0$.

Define $r^*(\phi)$ to solve $\psi(br\widetilde{L}(\phi), u^{M'}) = \psi(br, u^M)$. As in the argument made for the proof to Proposition 1 part iii, in the range $r \in [0, r^*(\phi)) X \phi \in [0, \overline{\phi})$,

$$\frac{1}{\widetilde{s}[\psi(br\widetilde{L}(\phi), u^{M'}), u^{M'}]} < \frac{1}{\widetilde{s}[\psi(br, u^M), u^M]}$$

Since at all angles from the center population density falls at some radius, population density falls in the entire region near the center of the city for an increase in rays.

4 Empirical Observations

In a companion paper, Baum-Snow [3] empirically investigates the extent to which highway construction caused suburbanization in the United States between 1950 and 1990. In that paper, I use variation across metropolitan areas in the number of highways in a 1947 plan of the national highway system as a source of plausibly exogenous variation in the number of highways actually received. I find that each ray causes approximately a 10 percent decline in central city population, ceteris paribus. When added up, highway construction empirically accounts for about one-third of the gap between metropolitan area population growth and central city population growth between 1950 and 1990.

Table 1 explores the empirical relationship between population decentralization and highway construction. Elements in Table 1 represent the average fraction of metropolitan area residents living central cities as defined by their 1950 geography in 1950 and 1990 as a function of the number of highways rays built to serve central business districts over this period. Panel A includes all metropolitan areas with at least 100,000 residents with central cities of at least 50,000 residents in 1950. Panel B limits the sample only to include metropolitan areas with central cities that are at least 20 miles from the nearest coast, lake shore or international border. I break out data for the more select sample because these metropolitan areas have structures that are most similar to that of the theoretical metropolitan area considered in this paper.

The average fraction of the population living in central cities fell from about 50 percent to about 25 percent between 1950 and 1990. Table 1 shows that highway construction is associated with this decline. As measured in shares, both panels show decreasing monotonic relationships between the decline in central city population share and new highways between 3 and 7 rays. When measured in logs, the more select sample in Panel B produces a weakly monotonic relationship between 1 and 7 rays.[3] The relationship reported in Panel B implies that the average effect of one ray is associated with a decline in log central city population share of approximately 0.07 if 0 rays are included and 0.10 if 0 rays are excluded[4]. The more thorough

[3] The anomaly at 0 rays exists primarily because cities that received 0 rays are smaller geographically than other cities. When the small size is controlled for in regressions, the anomaly largely disappears.

[4] These average rates of decline are the coeffiicients on Δray in a simple regression of $\Delta \log(\frac{N_c}{N})$ on Δray. Both coefficients are significant at the 5 percent level.

empirical analysis in Baum-Snow [3] shows that these descriptive results hold up to causal scrutiny. These numbers are similar in magnitude to the model simulation results presented in the next section.

5 Simulations

Sections 2 and 3 develop a mechanism by which new highway rays cause declines in urban population density in the context of a monocentric model. Further restrictions on the model are required to determine the magnitude of these declines. This section addresses this question through simulation. It is important to note that the model specified in this paper is developed in order to facilitate simulation. The model's parsimony means that only a few parameters need be chosen for simulation. For all simulations, I use commuting technology (1), in which commuters must access the highway using a perpendicular surface street. This technology limits the influence of new highways on central population decline more than any other technology in which travel only along straight lines is possible. As such, with an appropriate specification of preferences, these simulation results represent a near lower bound on the effect of highways on central city population as implied by this model. After presenting results from simulating the model, I extend the analysis to incorporate congestion.

5.1 Basic Results

I focus primarily on simulation examples using the utility function $U = z + \alpha \ln(s)$. Quasilinear utility is a convenient preference specification for two reasons. First, the quasilinear utility function's income elasticity of demand for space of 0 limits the response of population density to faster transportation infrastructure to be driven only by a price effect. Second, quasilinear utility is convenient because results are very stable over a wide range of values for the shape parameter of the utility function, the wage, base travel speed and metropolitan area population.

Numerical calculation of equilibria requires choosing elements of the parameter vector $(N, w, b, \gamma, \alpha, R_a, M)$. Table 2 shows how several outcomes of interest change as a function of rays given metropolitan area populations of 100 thousand (Panel A) and 1 million (Panel B). Central city radius r_c is chosen such that in the 0 ray equilibrium, half of the population lives in the central city. This fraction is near the mean across metropolitan areas in the data from 1950. The other parameters are chosen to mimic conditions seen in modern U.S. metropolitan areas. A wage of 100 is chosen to represent \$10 an hour in a 10 hour day. Travel speed is chosen to be 30 mph on city streets and 60 mph on highways, implying a b of $\frac{1}{300}$ and γ of 0.5. The utility function's shape parameter α is chosen as 0.3. Finally, rural rent R_a is chosen to be 1.

Given the patterns of urban decentralization documented in Table 1, one statistic of interest is the evolution of central city population as a function of the number of rays. Solving for central city population can be reduced to two steps: solving for $\overline{r}_f^M$ from the implicit function in Equation (11) and inserting the result back into the expression for N_c, which in general is given by Equation (10).[5]

$$(11) \qquad N = \frac{R_a}{\alpha} e^{\frac{w}{\alpha} b \overline{r}_f^M} \left(\frac{wb}{\alpha}\right)^{-2} \left[\begin{array}{c} \left(\frac{wb}{\alpha}\right)^2 2M \int_0^{\overline{\phi}} \int_0^{\frac{\overline{r}_f^M}{\gamma \cos\phi + \sin\phi}} r e^{-\frac{w}{\alpha} r b \widetilde{L}(\phi)} dr d\phi \\ +(2\pi - 2M\overline{\phi})[1 - e^{-\overline{r}_f^M wb/\alpha} - \overline{r}_f^M \frac{wb}{\alpha} e^{-\overline{r}_f^M wb/\alpha}] \end{array} \right]$$

As is shown below, the evolution of central city population in response to new rays is quite stable across parameter values.

Table 2 Panels A and B track central city population, fringe distance not near a ray ($\overline{r}_f^M$), utility and rent at the origin as a function of the number of rays given small and large metropolitan area populations. Both panels show central city population, fringe distance, and central rent declining and utility increasing in the number of rays. The marginal effect of each additional ray on all of these variables declines in absolute value in the number of rays. Note that only central city population depends on the choice of r_c.

Taking central city population in the 0 ray equilibrium as given, Panel C traces out the evolution of the change in central city population as a function of the number

[5] All calculations relevant for the simulations are available upon request from the author.

of rays. Each row shows the effect on central city population of each marginal ray, holding the radius of the central city fixed. The central city radius is determined by the fraction of metropolitan area population assumed to reside in the central city listed in the left-most column. Results in Table 1 Panel C show that the first ray causes the log central city population to fall by 0.11, with the fall in central city population declining 1-2 percentage points for each additional ray. The percent decline in central city population falls with increases in central city population share in the 0 ray equilibrium and equivalently with increases in the central city radius.

Table 3 presents simulation results analogous to those in Table 2 using the Cobb-Douglas utility function. Each of the parameters is the same, with space given a 30 percent budget share. Note that since Cobb-Douglas has an income elasticity of demand for space of 1, the amount of space consumed by the metropolitan area is much greater. As such, the rent at the origin is much lower than in the quasilinear utility case reported in Table 2. Despite these differences in levels, the profile of the changes as a function of the number of rays is very similar for the two specifications of the utility function.

Table 3 Panel C presents the profile of central city population as a function of the number of rays. These results show effects within 0.04 greater in absolute value of those implied by quasilinear utility. A metropolitan area with half its population in the central city lose about 13 percent of its population for the first ray, 11 percent for the second ray and 9 percent for the third ray. This effect declines by about 1 percentage point for each additional ray thereafter.[6]

Table 4 explores the sensitivity of the central city population results to the choice of speeds on highways to city streets. Table 4 Panel A replicates the simulation results for the profile of central city population reported in Table 2 Panel C assuming speed ratios on surface streets to highways of $\frac{25}{60}$ and $\frac{35}{60}$ instead of $\frac{30}{60}$. These results show that the magnitudes of the marginal effects are decreasing in the speed ratio. Given a speed ratio of $\frac{25}{60}$, the first few rays cause 18, 14 and 11 percent central city

[6] Simulation results using the Anas-Moses [2] commuting technology in which highway users must travel around a circle centered at the CBD to access a highway give effects within 0.02 smaller in magnitude than those reported in Tables 2 and 3 Panel C.

population declines respectively. Given a speed ratio of $\frac{35}{60}$, the first few rays cause 7, 6 and 5 percent declines respectively. Panel B similarly replicates results based on Cobb-Douglas preferences, yielding marginal effects on central city population of 21, 16 and 13 percent given the low speed ratio and 8, 7 and 6 percent given the high speed ratio. Marginal effects of only the first, third and fifth rays are reported to maintain parsimony in the table.

In addition to examining the sensitivity of simulation results to travel speeds, I also explored the extent to which the results presented in Tables 2 and 3 Panel C are sensitive to the wage, metropolitan area population and the shape parameter of the utility functions. I simulated the model for all shape parameters between 0.1 and 0.9, for population levels up to 10 million and for wage rates down to 50. In no case do any numbers in Table 2 Panel C change by more than 0.03 nor do numbers in Table 3 Panel C change by more than 0.05. The largest changes for the Cobb-Douglas utility case occur when space is given a budget share of 0.9 and the metropolitan area population is at least 5 million.

5.2 Adding Congestion

The model presented in Sections 2 and 3 assumes that the transport infrastructure is not congestible. In this subsection, I extend the model to handle congestion in a simple way and present associated simulation results. I relegate the consideration of congestion to an extension of the model presented in Sections 2 and 3 for two reasons. First, the existence of congestion means that the simulation results from the previous subsection will no longer be insensitive to metropolitan area population. Second, empirically congestion did not start to become a major source of commuting time loss until the 1990s.

A straightforward way of extending the analysis to incorporate congestion, similar to that employed by Anas and Moses [2], is to make γ an increasing function of the population affected by a highway:

$$(12)\qquad \gamma = f\left(\int_0^{\overline{\phi}} \int_0^{\frac{\overline{r}_f^M}{\widetilde{L}(\phi)}} \frac{r dr d\phi}{\widetilde{s}[\psi(br\widetilde{L}(\phi), u^M), u^M]}\right)$$
$$f' \geq 0$$

Denote γ_M to be the equilibrium value of γ given M rays. Congestion represents another force pushing population density up near the center of the city in response to an increase in N because $\frac{dN_c}{d\gamma} > 0$ and $\frac{df}{dN} > 0$. Similarly, this formulation of congestion implies that *ceteris paribus,* more rays lead to a weak decline in the equilibrium value of γ because they induce some highway users to move from using other rays, thereby reducing the population around each ray. Thus holding the number of lanes constant, the effect of the first ray on central city population is smaller than it would be without congestion. The effect of the second ray is larger than it would be if γ were fixed at γ_1 because the next ray causes γ to fall to γ_2. The parameterization of the function f and the profiles of the price and wealth effects of demand for space as a function of γ determine whether the response of central city population to the second ray in a world with congestion is less or greater than in a world without congestion.[7]

I evaluate the potential importance of congestion by simulating the model allowing γ to be determined by the equilibrium population using the highway. I take the formula for speed on congested highways from the Texas Transportation Institute's (TTI) 2004 Mobility Report [8] and adapt it slightly to fit this context.[8] The formula is estimated by the TTI based on data taken from sampled portions of the highway network. It is piecewise linear and weakly monotonic in average daily traffic. Equilibrium highway travel speed is 60 mph at traffic levels below 13,260 vehicles per lane

[7] Vickrey [9] proposes a more complicated formulation which in the context of this model would manifest itself as a "flow congestion" term in the travel time function: $L(r,\phi) = \min[br, br(\gamma\cos\phi + \sin\phi) + \lambda\int_0^{r\cos\phi}\left(\frac{N(v)}{t(v)}\right)^k dv]$ where $N(v)$ is the number of commuters using the highway between v and the edge of the MSA and t is the throughput of the highway.

[8] I alter the TTI's congestion function in order to make it monotonic. Details are in Appendix Table 1.

per day, with the equilibrium speed dropping to 30 mph by 72,000 vehicles per lane per day. The formula used is reported in Appendix Table 1. For the simulations, I assume that each new highway is 4 lanes in each direction and that 2 individuals commute together in each vehicle. Given that only about half the U.S. population commutes to work outside the home, the 2 person per car assumption allows the model to better capture commuting patterns for observed population levels.

Unlike the results presented in the previous subsection, simulation results incorporating congestion are sensitive to the population of the metropolitan area. Table 5 presents simulation results with γ endogenized to account for the number of road users according to the formula detailed in Appendix Table 1. Numbers in Table 5 are calculated using the same parameter values as are used for Table 2 except population. With this formulation of the congestion function and transportation infrastructure, congestion starts to reduce travel speeds when the metropolitan area reaches about 500,000 people. As such, Panel A reports simulation results for a metropolitan area of this size. Panel A shows that congestion reduces the reduction in central city population caused by the first ray by up to 4 percentage points. Five rays is enough infrastructure to eliminate all congestion in this case.

Table 5 Panel B reports results for a metropolitan area of 1 million inhabitants. A metropolitan area of 1 million inhabitants with half its population in the central city in the 0 ray equilibrium sees central city population drop by 3 percent for the first ray, 4 percent for the second, third and fourth rays and 5 percent for the fifth ray. At 10 rays, the spatial distribution of the population looks the same as in the uncongested case because there is enough transportation infrastructure to bring γ back to its uncongested value.

Data collected by the Texas Transportation Institute indicates that congestion is not likely to be a major force mitigating highways' influence on changing residential land use patterns since 1950. Among the 139 largest metropolitan areas in 1950, the average ratio of free-flow to congested traffic speeds on limited access highways was 1.16 in 1990. Therefore, it appears that communities built up their highway infrastructure almost sufficiently to fulfill the increased travel demand associated with their rising and decentralizing populations.

6 Conclusions

This paper proposes a land use and commuting model that incorporates radial highways. I show that new highways affect urban form by causing the population to spread out along the highways. In addition, holding the population of the metropolitan area constant, the urban fringe in areas not near the highway moves inwards. This simple model implies that highway construction may have contributed markedly to the dramatic change in urban land use patterns observed between 1950 and 1990 in the United States. Results from simulating the model imply that indeed new highways are likely to have had a sizable impact on central city populations. Applying the simulation results in Table 2 Panel C to observed average highway construction of about 2.5 rays per metropolitan area, counterfactual central city population estimates imply that nearly the full decline of 28 percent in average central city population can be explained by highway construction. Descriptive evidence using data from 139 metropolitan areas reveals changes in observed central city population as a function of highway construction that is similar to that implied by the simulation results.

While the mechanism proposed here produces qualitative and quantitative results that are quite robust, the model used is highly stylized. Indeed, casual empirical observation reveals that employment decentralization has occurred apace with residential decentralization, a phenomenon that is ignored by the model in this paper. While others have proposed models that endogenize employment location, these models have few general analytical comparative static implications. As such, it is valuable to understand the extent to which a simple tractable model featuring highway construction can explain suburbanization.

A Proof of Proposition 1

If $M' = M + 1$ then in equilibrium

i) $u^{M'} > u^M$: Compare a city with 0 rays and 1 ray. The market clearing condition for space (Equation 8 in the text) implies that

$$0 = 2\int_0^{\overline{\phi}} \int_0^{\frac{q(u^1)}{\widetilde{L}(\phi)}} \frac{rdrd\phi}{\widetilde{s}[\psi(br\widetilde{L}(\phi), u^1), u^1]}$$

$$+(2\pi - 2\overline{\phi}) \int_0^{q(u^1)} \frac{rdr}{\widetilde{s}[\psi(br, u^1), u^1]} - 2\pi \int_0^{q(u^0)} \frac{rdr}{\widetilde{s}[\psi(br, u^0), u^0]} \tag{13}$$

Recall that $q(\cdot)$ gives the equilibrium fringe distance far from highways through solving the fringe rent condition (Equation 6 in the text) for $\overline{r}_f^M$. Suppose that $u^0 > u^1$. I prove by contradiction that utility cannot be decreasing in M. Using Equation (6), the Implicit Function Theorem and the Envelope Theorem, $\frac{d\overline{r}_f}{du} = -\frac{-\frac{\partial z(s,u)}{\partial u}}{-wb} < 0$ so $q(u^1) > q(u^0)$. Using the fact that space is a normal good, $\widetilde{s}[\psi(L(r,\phi), u^1), u^1] < \widetilde{s}[\psi(L(r,\phi), u^0), u^0]$. These two conditions imply that

$$2\pi \left[\int_0^{q(u^1)} \frac{rdr}{\widetilde{s}[\psi(br, u^1), u^1]} - \int_0^{q(u^0)} \frac{rdr}{\widetilde{s}[\psi(br, u^0), u^0]} \right] > 0 \tag{14}$$

and applying Equation (14) to (13) we have

$$\int_0^{\overline{\phi}} \int_0^{\frac{q(u^1)}{\widetilde{L}(\phi)}} \frac{rdrd\phi}{\widetilde{s}[\psi(br\widetilde{L}(\phi), u^1), u^1]} < \overline{\phi} \int_0^{q(u^1)} \frac{rdr}{\widetilde{s}[\psi(br, u^1), u^1]} \tag{15}$$

But

$$
\begin{aligned}
\overline{\phi} \int_0^{q(u^1)} \frac{rdr}{\widetilde{s}[\psi(br,u^1),u^1]} &= \int_0^{\overline{\phi}} \int_0^{q(u^1)} \frac{rdrd\phi}{\widetilde{s}[\psi(br,u^1),u^1]} \\
&< \int_0^{\overline{\phi}} \int_0^{\frac{q(u^1)}{\widetilde{L}(\phi)}} \frac{rdrd\phi}{\widetilde{s}[\psi(br,u^1),u^1]} \\
&< \int_0^{\overline{\phi}} \int_0^{\frac{q(u^1)}{\widetilde{L}(\phi)}} \frac{rdrd\phi}{\widetilde{s}[\psi(br\widetilde{L}(\phi),u^1),u^1]}
\end{aligned}
$$

which contradicts (15). Thus, it must be that $u^1 > u^0$. An analogous argument follows for all $M > 0$.

ii) $\overline{r}_f^{M'} < \overline{r}_f^M$: To understand how equilibrium land use changes with M, we must first understand how the equilibrium land rent function changes with M. We can express land rent in terms of the bid-rent function:

$$
\psi(L(r,\phi),u) = \max_s \left\{ \frac{w[1-L(r,\phi)] - Z(s,u)}{s} \right\} \tag{16}
$$

Using the envelope theorem, $\frac{\partial \psi}{\partial u} < 0$. Thus given result i, areas of the city with no change in travel times see land rents fall with M. Therefore, since R_a does not change, fringe distance in these same areas also falls with M.

iii) $\frac{\overline{r}_f^{M'}}{\gamma} > \overline{r}_f^M$: Once again, consider the case of moving from a regime with 0 rays to a regime with 1 ray. Result i states that utility rises with M and ii) shows that equilibrium land rent falls with M for $\phi > \overline{\phi}$. Thus since space is a normal good, $\widetilde{s}[\psi(L(r,\phi),u^1),u^1] > \widetilde{s}[\psi(L(r,\phi),u^0),u^0]$ in the region $\phi > \overline{\phi}$. Also note from result ii that $\psi(0,u^1) < \psi(0,u^0)$.

To examine the $\phi < \overline{\phi}$ region, it is instructive to think about the shape of the bid-rent function for land in the 0-ray equilibrium compared to that in the 1-ray equilibrium at $\phi \geq \overline{\phi}$ and $\phi = 0$. The derivative of the rent function as a function of

r is:

$$\psi_r = -\frac{w\min[b, b\widetilde{L}(\phi)]}{\widetilde{s}[\psi(L(r,\phi),u),u]} \tag{17}$$

The rent function is thus less steep in the region $\phi < \overline{\phi}$ than in the remainder of the metropolitan area. Further, the rent function at all points is less steep in the 1-ray equilibrium than the 0-ray equilibrium. Define $r^*(\phi)$ to solve $\psi(rb\widetilde{L}(\phi), u^1) = \psi(rb, u^0)$ for the region $\psi(rb, u^0) > R_a$. Given the normality of land and the fact that the fringe distance is the furthest from the center at $\phi = 0$, it must also be true that for $r \leq r^*(\phi)$, $s^1(r,\phi) > s^0(r,\phi)$. Using the market clearing condition for space and the result that $\overline{r}_f^1 < \overline{r}_f^0$:

$$\begin{aligned} & N - (2\pi - 2\overline{\phi})\int_0^{\overline{r}_f^0} \frac{rdr}{\widetilde{s}[\psi(br,u^0),u^0]} - 2\int_0^{\overline{\phi}}\int_0^{r^*(\phi)} \frac{rdrd\phi}{\widetilde{s}[\psi(br,u^0),u^0]} \\ > \; & N - (2\pi - 2\overline{\phi})\int_0^{\overline{r}_f^1} \frac{rdr}{\widetilde{s}[\psi(br,u^1),u^1]} - 2\int_0^{\overline{\phi}}\int_0^{r^*(\phi)} \frac{rdrd\phi}{\widetilde{s}[\psi(br\widetilde{L}(\phi),u^1),u^1]} \end{aligned}$$

or

$$2\int_0^{\overline{\phi}}\int_{r^*(\phi)}^{\frac{\overline{r}_f^1}{\widetilde{L}(\phi)}} \frac{rdrd\phi}{\widetilde{s}[\psi(br\widetilde{L}(\phi),u^1),u^1]} > 2\int_0^{\overline{\phi}}\int_{r^*(\phi)}^{\overline{r}_f^0} \frac{rdrd\phi}{\widetilde{s}[\psi(br,u^0),u^0]}$$

That is, there remain more people to be housed in the region $r \in [r^*(\phi), \infty)$ X $\phi \in (0, \overline{\phi})$ in the 1-ray equilibrium than the 0-ray equilibrium. $r^*(\phi)$ must exist for some ϕ, otherwise not everybody could be housed in the 1-ray equilibrium. By definition of r^* and the fact that $\psi_r(b\gamma r, u^1) > \psi_r(br, u^0)$, rent must be greater in the 1-ray equilibrium than the 0-ray equilibrium in the region $r > r^*(\phi)$. Because R_a is the same in both equilibria, the extent of the city at $\phi = 0$ must be greater in the 1-ray equilibrium than the 0-ray equilibrium. The same argument follows for all $M > 0$. *Q.E.D.*

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Figure 1: The Effect on Urban Form of a New Ray

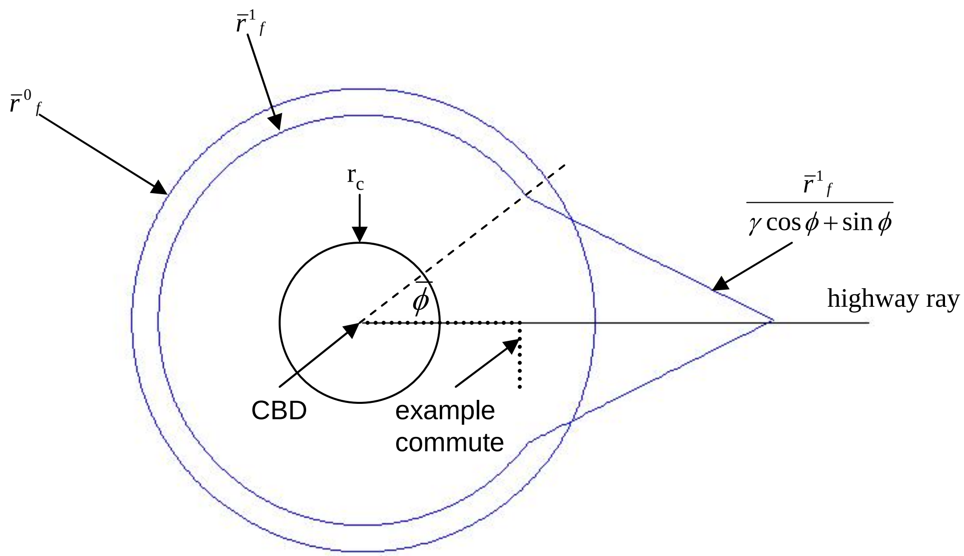

Figure 2
Graphical Depiction of Rent Functions in 0, 1 and 2 Ray Equilibria

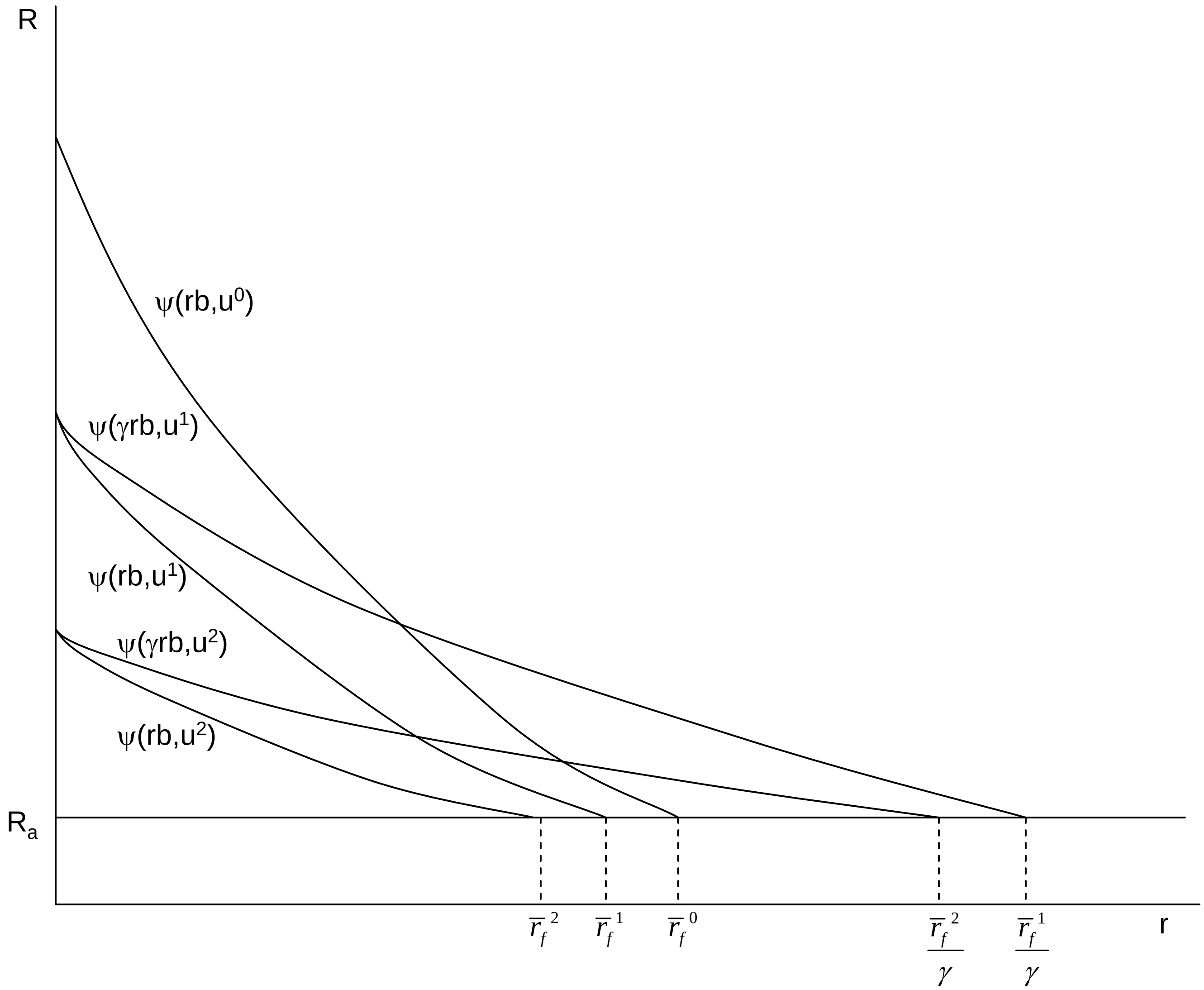

Table 1: Suburbanization and Highway Construction

Fraction of Metropolitan Area Population in the Central City

Panel A: Large Metropolitan Areas

	Change in the Number of Rays							
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
1950	0.44	0.48	0.48	0.41	0.47	0.48	0.54	0.64
	(0.14)	(0.18)	(0.17)	(0.15)	(0.14)	(0.07)	(0.17)	(0.01)
1990	0.23	0.25	0.23	0.18	0.22	0.18	0.18	0.27
	(0.10)	(0.12)	(0.13)	(0.09)	(0.11)	(0.07)	(0.09)	(0.11)
Change	-0.21	-0.23	-0.25	-0.23	-0.25	-0.30	-0.36	-0.37
	(0.17)	(0.22)	(0.21)	(0.17)	(0.18)	(0.10)	(0.19)	(0.11)
Δ log Fraction	-0.74	-0.71	-0.79	-0.92	-0.86	-1.04	-1.19	-0.90
N	16	21	36	22	27	11	4	2

Panel B: Large Inland Metropolitan Areas

	Change in the Number of Rays							
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
1950	0.43	0.47	0.47	0.39	0.45	0.48	0.54	0.63
	(0.13)	(0.21)	(0.19)	(0.12)	(0.16)	(0.07)	(0.17)	
1990	0.22	0.25	0.23	0.17	0.19	0.18	0.18	0.19
	(0.10)	(0.13)	(0.14)	(0.08)	(0.09)	(0.07)	(0.09)	
Change	-0.21	-0.22	-0.24	-0.22	-0.26	-0.30	-0.36	-0.44
	(0.16)	(0.25)	(0.24)	(0.14)	(0.18)	(0.10)	(0.19)	
Δ log Fraction	-0.76	-0.65	-0.77	-0.90	-0.94	-1.04	-1.19	-1.19
N	15	13	27	13	17	10	4	1

Notes: Entries are the average fraction of metropolitan area population living in central cities as defined by their 1950 borders. Year 2000 geography is used for metropolitan areas. Standard deviations are in parentheses. "Δ log fraction" gives the change in the average log fractions for the group. The sample in Panel A includes each metropolitan area with at least 100,000 people and a central city of at least 50,000 people in 1950. Panel B restricts the sample to include only metropolitan areas with central cities located at least 20 miles from a coast, lake shore or international border.

Table 2: Simulations Using Quasilinear Utility

Panel A: Outcomes Given Metropolitan Area Population of 100,000

Fraction in CC with 0 Rays	Number of Rays 0	1	2	3	4	5	6	7	8	9
CC Pop (,000)	50	45	41	38	35	34	32	31	30	29
Fringe Distance	10.2	10.0	9.9	9.7	9.6	9.4	9.3	9.2	9.2	9.1
Utility	66.95	67.00	67.04	67.07	67.11	67.14	67.16	67.18	67.20	67.21
Rent at Origin	2897	2463	2141	1895	1699	1540	1419	1329	1260	1205

Panel B: Outcomes Given Metropolitan Area Population of 1,000,000

Fraction in CC with 0 Rays	Number of Rays 0	1	2	3	4	5	6	7	8	9
CC Pop (,000)	500	447	408	378	354	335	320	308	298	290
Fringe Distance	13.2	13.0	12.8	12.7	12.5	12.4	12.3	12.2	12.1	12.1
Utility	66.26	66.31	66.35	66.39	66.42	66.45	66.47	66.49	66.51	66.52
Rent at Origin	28,895	24,548	21,339	18,872	16,916	15,329	14,121	13,221	12,528	11,977

Panel C: Δ log (Central City Population Fraction) / Δ Ray

Fraction in CC with 0 Rays	Number of Rays 0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
0.10	-0.15	-0.12	-0.11	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03
0.25	-0.13	-0.11	-0.10	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.03
0.50	-0.11	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.03	-0.02
0.75	-0.08	-0.07	-0.05	-0.05	-0.04	-0.03	-0.03	-0.02	-0.02	-0.02

Notes: CC stands for central city. The utility function used is U = z+.3ln(s). These results assume 30 mph on surface streets and 60 mph on highways. All simulations use w=100, b=1/300, gamma=.5 and Ra=1. This implies a 10 hour day. The results in Panel C use a metropolitan area population of 1 million.

Table 3: Simulations Using Cobb-Douglas Utility

Panel A: Outcomes Given Metropolitan Area Population of 100,000

Fraction in CC with 0 Rays	Number of Rays 0	1	2	3	4	5	6	7	8	9
CC Pop (,000)	50	44	40	37	34	32	31	29	28	27
Fringe Distance	211	207	203	199	196	194	191	189	188	186
Utility	0.89	2.47	2.51	2.54	2.57	2.60	2.62	2.64	2.66	2.67
Rent at Origin	57	49	43	38	35	32	29	28	26	25

Panel B: Outcomes Given Metropolitan Area Population of 1,000,000

Fraction in CC with 0 Rays	Number of Rays 0	1	2	3	4	5	6	7	8	9
CC Pop (,000)	500	440	395	361	333	311	294	281	270	261
Fringe Distance	255	252	250	248	247	245	244	243	242	241
Utility	0.56	1.80	1.84	1.88	1.91	1.94	1.96	1.98	2.00	2.01
Rent at Origin	540	459	400	354	318	288	266	249	236	226

Panel C: Δ log (Central City Population Fraction) / Δ Ray

Fraction in CC with 0 Rays	Number of Rays 0-1	1-2	2-3	3-4	2-3	5-6	6-7	7-8	8-9	9-10
0.10	-0.15	-0.13	-0.11	-0.10	-0.09	-0.07	-0.06	-0.05	-0.04	-0.03
0.25	-0.14	-0.12	-0.10	-0.09	-0.08	-0.07	-0.05	-0.04	-0.04	-0.03
0.50	-0.13	-0.11	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.03
0.75	-0.11	-0.09	-0.07	-0.06	-0.05	-0.05	-0.04	-0.03	-0.03	-0.02

Notes: CC stands for central city. The utility function used is $U = .7\ln(z)+.3\ln(s)$. These results assume 30 mph on surface streets and 60 mph on highways. All simulations use w=100, b=1/300, gamma=.5 and Ra=1. The results in Panel C use a metropolitan area population of 1 million.

Table 4: Sensitivity of Simulation Results

Panel A: Sensitivity of Quasilinear Results to gamma

gamma	CC Frac. With 0 Rays	0-1	2-3	4-5
0.42	0.10	-0.23	-0.15	-0.10
0.42	0.25	-0.21	-0.13	-0.09
0.42	0.50	-0.18	-0.11	-0.07
0.42	0.75	-0.14	-0.08	-0.05
0.58	0.10	-0.09	-0.07	-0.06
0.58	0.25	-0.08	-0.06	-0.05
0.58	0.50	-0.07	-0.05	-0.04
0.58	0.75	-0.05	-0.04	-0.03

Panel B: Sensitivity of Cobb-Douglas Results to gamma

gamma	CC Frac. With 0 Rays	0-1	2-3	4-5
0.42	0.10	-0.24	-0.15	-0.11
0.42	0.25	-0.23	-0.14	-0.10
0.42	0.50	-0.21	-0.13	-0.08
0.42	0.75	-0.18	-0.10	-0.07
0.58	0.10	-0.09	-0.08	-0.06
0.58	0.25	-0.09	-0.07	-0.06
0.58	0.50	-0.08	-0.06	-0.05
0.58	0.75	-0.06	-0.05	-0.04

Notes: CC stands for central city. Panel A shows results of marginal rays on central city population analogous to those reported in Table 2 Panel C given a surface street speed of 25 mph and a highway speed of 60 mph (gamma=0.42) and 35 mph and 60 mph (gamma=0.58) respectively. Panel B shows similar results analogous to those reported in Table 3 Panel C.

Table 5: Simulations With Congestion
Quasilinear Utility

Panel A: Population of 500,000

Fraction in CC	Number of Rays									
with 0 Rays	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
0.10	-0.11	-0.12	-0.12	-0.11	-0.10	-0.07	-0.06	-0.05	-0.04	-0.03
0.25	-0.10	-0.10	-0.10	-0.10	-0.08	-0.06	-0.05	-0.04	-0.03	-0.03
0.50	-0.09	-0.09	-0.08	-0.08	-0.07	-0.05	-0.04	-0.03	-0.03	-0.02
0.75	-0.06	-0.06	-0.06	-0.06	-0.05	-0.03	-0.03	-0.02	-0.02	-0.02
gamma	0.543	0.529	0.517	0.505	0.500	0.500	0.500	0.500	0.500	0.500

Panel B: Population of 1 million

Fraction in CC	Number of Rays									
with 0 Rays	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
0.10	-0.05	-0.05	-0.06	-0.06	-0.07	-0.08	-0.18	-0.12	-0.08	-0.05
0.25	-0.04	-0.05	-0.05	-0.06	-0.06	-0.07	-0.16	-0.10	-0.07	-0.04
0.50	-0.03	-0.04	-0.04	-0.04	-0.05	-0.05	-0.13	-0.09	-0.06	-0.04
0.75	-0.02	-0.03	-0.03	-0.03	-0.03	-0.04	-0.09	-0.06	-0.05	-0.03
gamma	0.675	0.664	0.653	0.640	0.626	0.611	0.551	0.523	0.506	0.500

Notes: The utility function used is $U = z + .3\ln(s)$. Each panel shows simulation results assuming that the speed on surface streets is 30 mph. Speed reductions due to congestion only occur on highways according to the function given in Appendix Table 1. The values given for gamma apply to the larger number of rays listed in the column headers. Parameter values are the same as those used for Table 2 Panel C. Magnitudes are within .01 for other reasonable coefficients on ln(s).

Appendix Table 1: The Congestion Function

TTI Congestion Function		Congestion Function Used for Simulations	
Annual Average Daily Traffic *(thousands)*	Travel Speed On Highway *(miles per hour)*	Annual Average Daily Traffic *(thousands)*	Travel Speed On Highway *(miles per hour)*
Less than 15	60	Less than 13.26	60
15 - 17.5	74.45-1.09*aadt	13.26 - 17.57	74.45-1.09*aadt
17.5 - 20	109.76-3.1*aadt	17.57 - 20.59	109.76-3.1*aadt
20 - 25	135.08-4.33*aadt	20.59 - 24.44	135.08-4.33*aadt
Greater than 25	72.03-1.75*aadt	Greater than 24.44	72.03-1.75*aadt

Notes: The TTI congestion formula is found in Schrank & Lomax [8]. It is altered for the purposes of the simulations because near the kink points it is not monotonic. The only difference between the two functions is the set of points at which the linear functions being evaluated changes.