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Economics 111: Intermediate Microeconomics
Spring 2006 Answer Key
Midterm 1

You have 1 hour and 20 minutes. Only clarifying questions are allowed. Do not cheat. Do not panic. Enjoy the exam.

Questions 1 to 5 are multiple choice. Circle the correct answer. (6 points each correct answer). There were two versions of questions 1 to 5 differing on the order of the answers.

1. The utility function $U = \sqrt{x_1} + x_2$ represents:

   a. Cobb-Douglas preferences.

   b. Quasilinear preferences. ✓

   c. Semi-quartic preferences.

   d. Perfect complements.

   e. none of the above.

2. Suppose a consumer has preferences between two goods, x and y, that are perfect substitutes. Imagine that an increase in the price of good x resulted in a decrease in the consumption of that good of 4 units and no change in the consumption of good y which remained at 0. The following is true about the effect of the price change:

   a. Total effect=-4, substitution effect=-4, income effect=0

   b. Total effect=-4, substitution effect=-2, income effect=-2

   c. Total effect=-4, substitution effect=0, income effect=-4 ✓

   d. Total effect=-4, but not enough information to distinguish the other effects.

   e. Total effect=0, substitution effect=-4, income effect=4

3. The Second Welfare Theorem implies that:

a. every competitive equilibrium is efficient.

b. redistributive concerns should not lead to price manipulation. ✓

c. redistributive concerns should lead to price manipulation.

d. every efficient allocations is preferred to every inefficient allocation.

e. none of the above.

4. Alberto and Benito consume two goods. They trade only with each other and there is no production. Alberto's utility function is $U_A = 2x_{A1} + x_{A2}$ and Benito's is $U_B = x_{B1} + 2x_{B2}$. In their Edgeworth box, the set of Pareto efficient allocations is:

a. the main diagonal.

b. both diagonals.

c. the entire box.

d. the right and bottom edges of the box. ✓

e. the left and upper edges of the box.

5. In the Tale of Two Cities, the Armchair Economist argues that Grimyville's Clean Air Act will make:

a. Grimyville's tenants happier since the air will be cleaner.

b. Grimyville's landlords happier since they will be able to charge higher rents. ✓

c. Cleanstown's landlords less happy since some tenants will move to Grimyville.

d. Cleanstown's tenants happier since rents will be lower.

e. All of the above.

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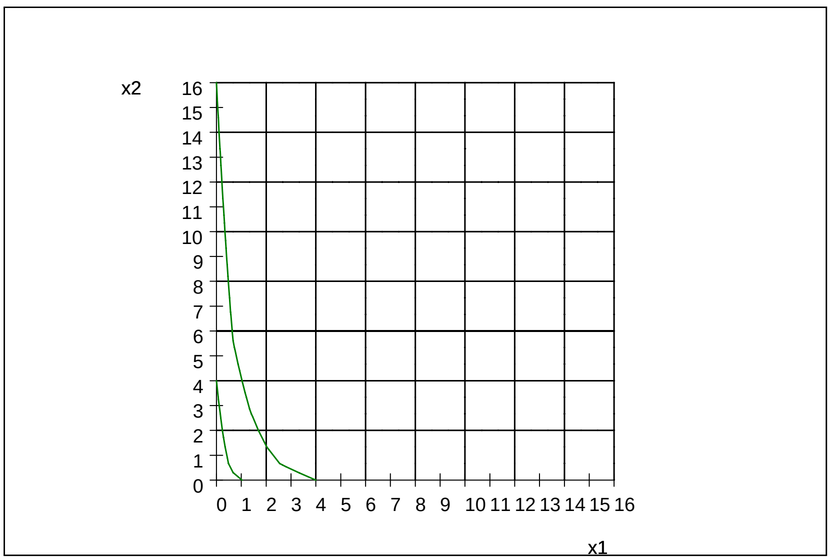
6. Consider the following utility function: $U = 2\sqrt{x_1} + \sqrt{x_2}$. (35 points)

a. For U=2 and U=4, find the function that describes each indifference curve (x_2 as a function of x_1). In the following figure graph the indifference curves for U=2 and 4. (Be sure to graph correctly where each indifference curve cuts the 45 degree line).

$x_2 = \left(U - 2\sqrt{x_1}\right)^2$ (for $U^2 \geq x_1$)

For U=2: $x_2 = \left(2 - 2\sqrt{x_1}\right)^2$ for $x_1 \leq 1$ and for U=4: $x_2 = \left(4 - 2\sqrt{x_1}\right)^2$ for $x_1 \leq 4$.



b. Find the formula for the MRS. How does it depend on x_2 and x_1? Are these preferences convex?

$MRS = 2\frac{\sqrt{x_2}}{\sqrt{x_1}}$, which is increasing in x_2 and decreasing in x_1. The preferences are convex.

c. Find the MRS for $x_1 = 4$ and $x_2 = 0$ and for $x_1 = 0$ and $x_2 = 16$. Is there any combination of prices and incomes that would make the agent buy any of these two bundles?

$MRS(4,0) = 0$ and $MRS(0,16) = \infty$. The indifference curves touch the horizontal axis with zero slope and the vertical axis with infinite slope. Therefore, you would need $p_2 = 0$ to have MRS=$\frac{p_1}{p_2}$, but then the agent would not consume a finite amount of good 2. There is no combination of prices and incomes that would make the agent consume a bundle on the axis.

d. Find the demand functions for general p_1, p_2 and m. (Some of the algebra may get a little difficult/ugly! Do not panic.)

Since preferences are convex and smooth, we have that in the optimum $MRS = 2\frac{\sqrt{x_2}}{\sqrt{x_1}} = \frac{p_1}{p_2}$ and $p_1 x_1 + p_2 x_2 = m$.

From the first equation: $x_2 = \frac{1}{4}\left(\frac{p_1}{p_2}\right)^2 x_1$. Substituting in the second equation we have that: $p_1 x_1 + \frac{1}{4}\frac{p_1^2}{p_2} x_1 = m$. Solving, $x_1^* = \frac{m}{p_1 + \frac{1}{4}\frac{p_1^2}{p_2}}$ which can be simplified to $x_1^* = \frac{m}{p_1}\frac{p_2}{p_2 + \frac{1}{4}p_1}$. Substituting in the budget line and doing some algebra we get: $x_2^* = \frac{m}{p_2}\frac{p_1}{4p_2 + p_1}$.

d. How are these goods?: Normal or inferior? Giffen or ordinary? Complements or substitutes?

These goods are normal ($\frac{dx_1^*}{dm} > 0, \frac{dx_2^*}{dm} > 0$), ordinary ($\frac{dx_1^*}{dp_1} < 0, \frac{dx_2^*}{dp_2} < 0$) and substitutes ($\frac{dx_1^*}{dp_2} = \frac{4m}{(4p_2+p_1)^2} > 0, \frac{dx_2^*}{dp_1} > 0$).

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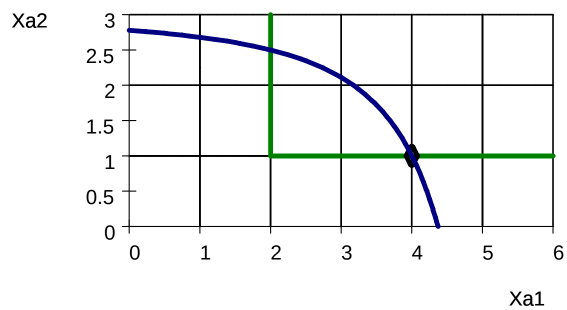
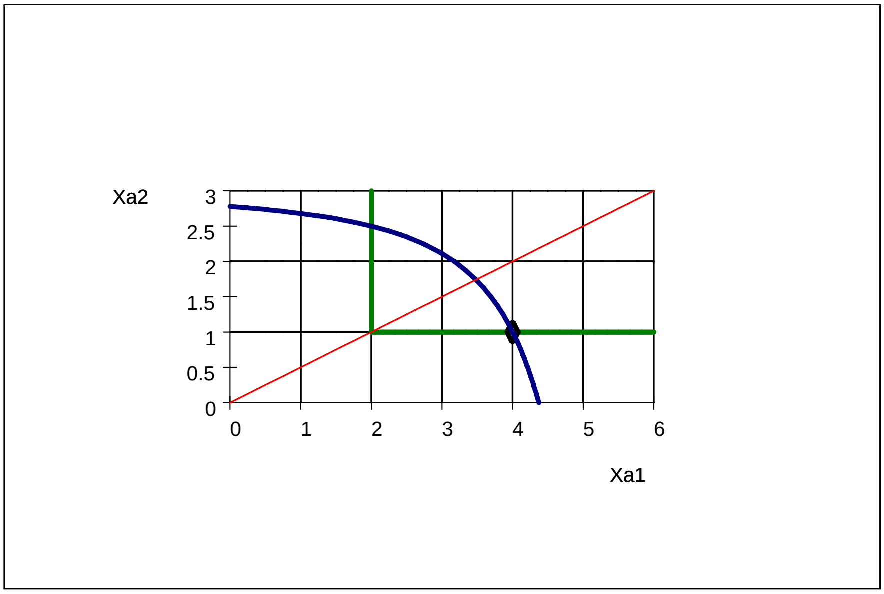
7. Consider a world with two agents: A and B. The utility of A is $U_A = \min\{\frac{x_{A1}}{2}, x_{A2}\}$ and the utility of B is $U_B = x_{B1}^2 x_{B2}$. The initial endowments are $\omega_A = (4, 1)$ and $\omega_B = (2, 2)$. (35 points)

a. In the following figure draw the Edgeworth Box labeling all axis and draw the initial endowment and the indifference curves that go through it.



b. Find the Contract Curve. Draw the Contract Curve in the Edgeworth Box of point a.

The contract curve includes all the points in the kink of A's indifference curves: $x_{A2} = \frac{x_{A1}}{2}$.



c. Find the demand functions of A for general p_1, p_2 and m_A.

The optimal consumption bundle must satisfy $x_{A2} = \frac{x_{A1}}{2}$ (kink of the indifference curve) and the budget line. Solving we get: $x^*_{A1} = \frac{2m_A}{2p_1+p_2}$ and $x^*_{A2} = \frac{m_A}{2p_1+p_2}$.

d. Find the demand functions of B for general p_1, p_2 and m_B.

Solving in the usual way for Cobb-Douglas preferences: $x^*_{B1} = \frac{2}{3}\frac{m_B}{p_1}$ and $x^*_{B2} = \frac{1}{3}\frac{m_B}{p_2}$.

d. Find the competitive equilibrium price p^*_1 (assume that $p_2 = 1$). Find the equilibrium allocation ($x^*_{A1}, x^*_{A2}, x^*_{B1}$ and x^*_{B2}).

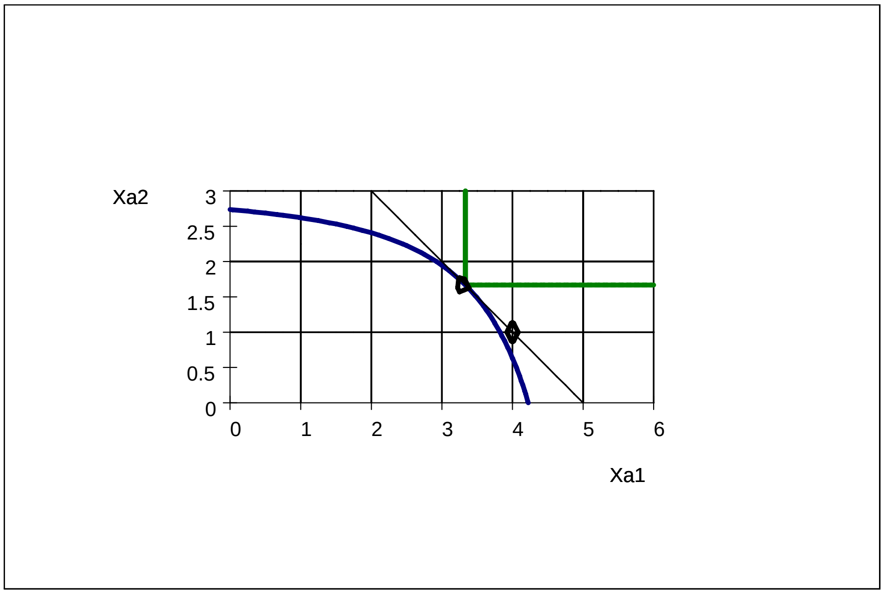
In this case there are two ways of solving this problem:

1. Since the Contract Curve is $x_{A2} = \frac{x_{A1}}{2}$ and $MRS_B = 2\frac{x_{B2}}{x_{B1}} = 2\frac{3-x_{A2}}{6-x_{A1}}$ we have that at any point in the contract curve $MRS_B = 2\frac{3-\frac{x_{A1}}{2}}{6-x_{A1}} = 1$. Then by the First Welfare Theorem and $MRS_B = \frac{p_1}{p2} = 1$.

2. Find p^*_1 that clears market 1: $x^*_{A1} + x^*_{B1} = 6$.
$\frac{2(4p_1+1)}{2p_1+1} + \frac{2}{3}\frac{(2p_1+2)}{p_1} = 6$. Solving algebraically: $p^*_1 = 1$.

After finding the equilibrium price we obtain the quantities by plugging the price into the demand functions: $x^*_{A1} = \frac{10}{3}, x^*_{A2} = \frac{5}{3}, x^*_{B1} = \frac{8}{3}$ and $x^*_{B2} = \frac{4}{3}$.

f. In the following figure draw the initial endowments, the equilibrium budget lines and the equilibrium consumption bundles. (Label each of them in the graph).



Unfortunately I cannot label the graph nicely with this software...