

Economics 147: Bargaining Theory and Applications

Spring 2003

Final (May 9th)

Name: _____

You have 2 hours. Good luck!!

1. Small town vs. big city I:

Each of the N citizens of a town have an income of \$10. Each must decide what to do with his or her money: they can expend it on personal consumption or contribute to a public good. Citizen's i utility is $u_i = (10 - c_i) + \alpha \sum_{j=1}^N c_j$, where c_i is the contribution to the public good of player j and the parameter $\alpha \in (\frac{1}{N}, 1)$.

a. Find the efficient outcome in which all citizens contribute the same amount.

b. Find the Nash equilibrium contributions and utilities.

Assume now that they play this game every year and their discount factor is δ .

c. What would be each citizen's discounted utility if they all contribute the efficient amount from point a?

d. For what values of δ can they support the efficient contribution in a subgame perfect equilibrium? (Assume that after a deviation they play for ever the NE from point b).

e. Where are you more likely to observe contributions to the public good? In a small town or in a big city?

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2. Small town vs. big city II:

A number of years ago, a young woman (Kitty Genovese) was stabbed to death in the central courtyard of her apartment building - while many neighbors watched and no one called the police. How could this happen? Hundreds of articles were written about the inhumanity of modern society - but there is a perfectly rational explanation if calling the police has a cost for the caller. Suppose there are N neighbors. Each has two possible actions: call the police (C) or don't call (D). If no one calls, Kitty dies and the all the neighbors get a payoff of 0. If anyone calls, Kitty lives and all the neighbors who called get a payoff of $1/2$ and the neighbors who didn't call get a pay off of 1.

a. Find the utility of C and D for one neighbor if each of the other neighbors independently calls with probability p . (Hint: if each of the other neighbors calls with probability p , then the probability that at least one of them calls is $1 - (1 - p)^{N-1}$)

b. Find the symmetric mixed strategy Nash Equilibrium.

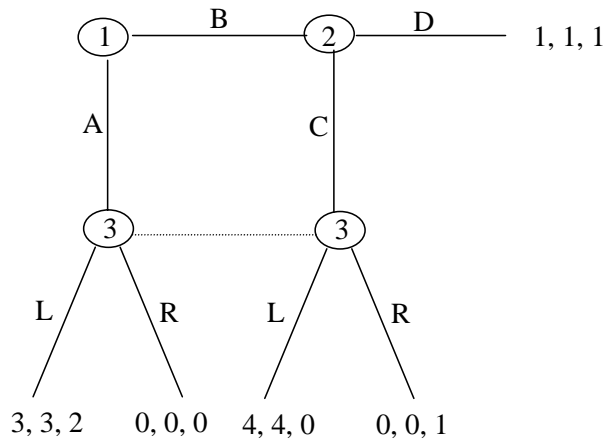
c. Find the probability $\pi(N)$ that *no one* calls the police.

d. How does $\pi(N)$ change with N .

e. Does the fact that people are less helpful in big cities mean that they care less about others?

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3. Consider the following game:



a. Find the normal form of the game (player 1 chooses rows, player 2 chooses columns and player 3 chooses matrixes).

b. Find all the pure strategy NE.

c. Find all the pure strategy subgame perfect NE.

d. Find all the pure strategy perfect Bayes NE. (Remember to specify the beliefs of player 3).

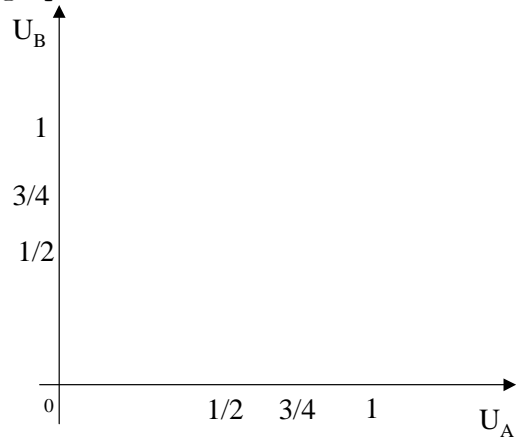
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4. Risk aversion and bargaining power:

Player A and B are bargaining on how to split a dollar. Their utility functions are $U_A = x_A^\alpha$ with $\alpha \in (0, 1]$ and $U_B = x_B$, where x_A and x_B are their shares of the dollar. If they do not agree both get zero.

a. Find the set of possible agreement utilities (Ω). (Write U_B as a function of U_A)

b. Draw Ω (for $\alpha = \frac{1}{2}$ and $\alpha = 1$) and the disagreement payoff d in the following graph.



c. Find the Nash bargaining solution utilities (U_A^{NBS}, U_B^{NBS}) .

d. Find the Nash bargaining solution division of the dollar (x_A^{NBS}, x_B^{NBS}) .

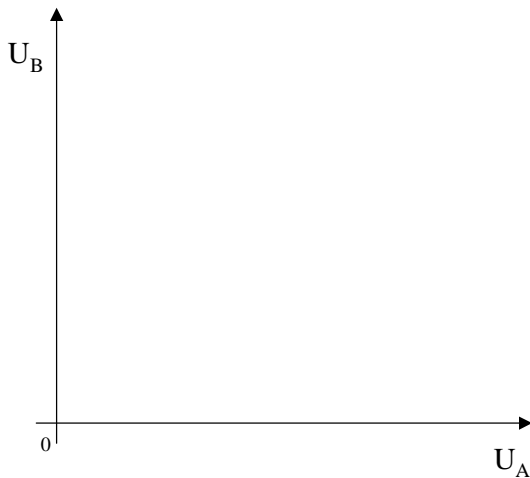
e. How does the division of the dollar change if player A becomes more risk averse?

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5. Bargaining in the household (may be difficult):

A and B are happily married and must decide how to expend their income I . Unfortunately they have different preferences: $U_A = 2c + e$ while $U_B = c + 2e$ where c and e denote the amount of income expended by the family in cars and the education of the children respectively. If they are in good terms, each of them also earns an extra utility of $\frac{S}{2}$. If they are not in good terms, each partner must decide separately how to allocate their own income. Assume that A has an income of αI , and B has an income of $(1 - \alpha)I$ and that consumption of car is private while the consumption of education is public when they are in bad terms. (For example, if they are in bad terms and A expends all his income in the car and B all her income in education their utilities are $U_A = 2\alpha I + (1 - \alpha)I$ and $U_B = 2(1 - \alpha)I$).

- a. Find the set of possible agreement utilities (Ω) and draw it in the following graph.



b. Find the disagreement point.

c. Show that both partners can profit from some agreement.

d. Find the Nash bargaining solution utilities (U_A^{NBS}, U_B^{NBS}) .

e. Find the Nash bargaining solution division of expenditures (c^{NBS}, e^{NBS}) .

f. How does a change in α affect the amount of education received by their children?

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6. Bargaining with nay sayers:

Consider a Rubinstein infinite alternating offers model with discount factor δ and a pie of size 1. Player A gets to make the first offer. Players A and B both enjoy rejecting offers. Every time player A says NO, she gets an utility of a . Every time player B says NO, he gets an utility of b . Assume that $1 > \frac{a}{1-\delta} + \frac{b}{1-\delta}$.

a. Show that the sum of utilities is greater if they reach an agreement in period 0 than if they stay saying No to each other for ever.

b. Players A and B follow these strategies:

$$s_A = \begin{cases} \text{always offer } x_A^* \\ \text{always accept } x_B \leq x_B^* \\ \text{reject otherwise} \end{cases} \quad s_B = \begin{cases} \text{always offer } x_B^* \\ \text{always accept } x_A \leq x_A^* \\ \text{reject otherwise} \end{cases} \quad . \text{ Find } x_A^* \text{ and } x_B^*.$$

c. How does x_A^* change if a increases? Provide intuition.

d. How much pie will player B get in equilibrium?

e. Find the value of τ that makes x_A^* coincide with the share that player A would get from an asymmetric Nash bargaining solution with $d = \left(\frac{a}{1-\delta}, \frac{b}{1-\delta}\right)$ and parameter of asymmetry τ . (Hint: rearrange the result from point b. to look like the solution from an ANBS)

Scratch paper: