Economics 147: Bargaining Theory and Applications
Spring 2004
Midterm (March 8th)

Name: _____

You have 1 hour and 20 minutes. Good luck!!

1. Consider the following simultaneous-moves game:

		2: A	2: B
1	A	0, 0	1, 2
	B	2, 1	0, 0

a. Find the pure strategy Nash equilibria.

(A,B) and (B,A)

b. Find the mixed strategy Nash equilibrium.

$(\frac{1}{3}A + \frac{2}{3}B,\ \frac{1}{3}A + \frac{2}{3}B)$

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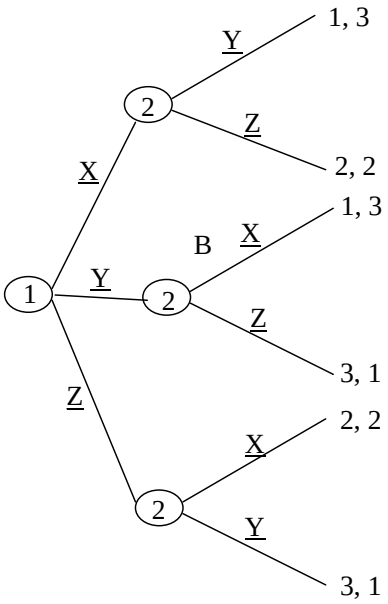
2. Voting by alternative veto:

Two people select a policy that affects them by alternately vetoing policies until one remains. There are three policies: X, Y and Z. These policies give, respectively, a utility of 3, 2, 1 for player 1 and 1, 2, 3 for player 2. First, player 1 vetoes a policy and then player 2 vetoes one of the remaining policies. The remaining policy is adopted.

a. Complete the payoffs in the following extensive form game (the underlined letters denote the vetoed policies).



b. Find the normal form of the game.

	YXX	YZY	YXY	YZX	ZXX	ZZY	ZXY	ZZX
X	1, 3	1, 3	1, 3	1, 3	2, 2	2, 2	2, 2	2, 2
Y	1, 3	3, 1	1, 3	3, 1	1, 3	3, 1	1, 3	3, 1
Z	2, 2	3, 1	3, 1	2, 2	2, 2	3, 1	3, 1	2, 2

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c. Find all the pure strategy Nash equilibria.

(Z,YXX) and (Z,ZXX)

d. Find the sub-game perfect Nash equilibrium.

(Z,YXX)

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3. Cost externality:

Two fishing companies sell fish in a perfect competitive market with price P. They fish in the same area and therefore their costs are related: $C_i(q_i) = Qq_i$, for $i = 1, 2$ and $Q = q_1 + q_2$.

a. Find the Nash equilibrium produced quantities and profits.

Making P=MC for both firms taking the quantity of the other firm as given and solving the system: $q_1^{NE} = q_2^{NE} = \frac{P}{3}$. $\pi_1^{NE} = \pi_2^{NE} = \frac{P^2}{9}$.

b. Assume now that both firms merge. What would be the quantity of production and profits now?

$Q^M = \frac{P}{2}$ and $\pi^M = \frac{P^2}{4}$.

c. Why are total profits higher when the firms merge?

Once they merge the firms internalize the negative externalities on the cost of production.

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d. Assume now that instead of two firms there are N firms. Find the Nash equilibrium produced quantities and profits. What happens to profits as N increases?

$q_1^{NE} = ... = q_N^{NE} = \frac{P}{N+1}$ and $\pi_1^{NE} = ... = \pi_N^{NE} = \frac{P^2}{(N+1)^2}$.

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4. Duopoly with unknown costs:

Two firms sell water in a market with inverse demand function $P = 7 - Q_i$, with $Q = q_1 + q_2$. Firm 1 can have one of two costs functions: either $C_1(q_1) = q_1$ (high cost) or $C_1(q_1) = 0$ (low cost). Firm 1 knows which cost she has while firm 2 believes that both costs are equally likely. Firm 2 has a know cost function $C_2(q_2) = q_2$.

a. What is the best response of firm 1 to a given q_2 if $C_1(q_1) = 0$?

$q_1^L = 3.5 - \frac{1}{2}q_2$

b. What is the best response of firm 1 to a given q_2 if $C_1(q_1) = q_1$?

$q_1^H = 3 - \frac{1}{2}q_2$

c. Write the expected profits of firm 2 if firm 1 produces q_1^H when its costs are high and q_1^L when its costs are low.

$E\pi_2 = \frac{1}{2}\left[\left(7 - q_1^L - q_2\right)q_2 - q_2\right] + \frac{1}{2}\left[\left(7 - q_1^H - q_2\right)q_2 - q_2\right] = 6q_2 - q_2^2 - \frac{1}{2}\left(q_1^H + q_1^L\right)$

d. Use your answer to point c to find the best response of firm 2 if firm 1 produces q_1^H when its costs are high and q_1^L when its costs are low.

$q_2 = 3 - \frac{1}{4}\left(q_1^H + q_1^L\right)$

d. Find the Bayes Nash equilibrium production quantities: q_1^H, q_1^L and q_2.

Plugging player's 1 best responses into the last equation:

$q_2 = 1.833$

$q_1^L = 2.583$

$q_1^H = 2.083$