Economics 147: Bargaining Theory and Applications

Spring 2005

Midterm (March 17th)

Name: Pedro__________________________

You have 1 hour and 20 minutes. Good luck!!

1. Consider the following simultaneous-moves game:

		2: A	2: B
1	A	1, 1	0, 0
	B	0, 0	3, 3

a. Find the pure strategy Nash equilibria.

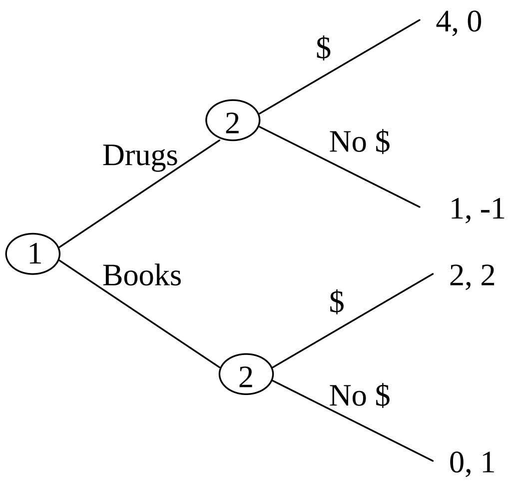
(A,A) and (B,B)

b. Find the mixed strategy Nash equilibrium.

$(\frac{3}{4}A + \frac{1}{4}B, \frac{3}{4}A + \frac{1}{4}B)$

2. The problem of nice parents:

Consider the following sequential-moves game in which player 1 (the kid) can consume either drugs or books, then player 2 (the parents) can either give him money or not. The parents would rather give money to the kid and that the kid consumed books. The moves and payoffs are as follow:



a. Find the normal form of the game.

Since the parents can condition their behavior on what the kid does, they have 4 strategies. The first letter indicates what they do if the kid buys drugs and the second letter what they do if the kid buys books.

	$$	$N	N$	NN
Drugs	4, 0	4, 0	1, -1	1, -1
Books	2, 2	0, 1	2, 2	0, 1

b. Find all the pure strategy Nash equilibria.

(D,$$), (D,$N) and (B,N$).

c. Find the sub-game perfect Nash equilibrium.

(D,$$)

The other NE involve non-credible behavior. In particular the fact that (B,N$) is a NE but not SGPE is interesting. Parents would like to convince the kid that they would punish him if the buys drugs. But if the kid knows the parents preferences, he/she knows that buying drugs will not result in a punishment. The punishment is not credible.

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3. Common-value second-price auction:

Two bidders have independent signals (s_1,s_2) distributed uniformly between 0 and 10. The value of the good for sale is equal to the sum of the signals. Each player know his or her signal but not the other signal. The bidder with the highest bid wins the object and pays the second bid. The following steps will lead you to the BNE of this game.

Assume that you are bidder 1 and you know that bidder 2's strategy is $b_2(s_2) = \alpha s_2$, where α is a constant.

a. Given player 2's strategy, what is the probability that you win if you bid b_1? (Remember that if x is a uniform random variable between 0 and 10, $P(x < a) = \frac{a}{10}$. For simplicity assume that $b_1 \leq 10\alpha$). Denote this probability $PW(b_1)$.

$PW(b_1) = P(b_1 > b_2) = P(b_1 > \alpha s_2) = P(s_2 < \frac{b_1}{\alpha}) = \frac{b_1}{10\alpha}$.

b. Given player 2's strategy, what is your expected payment to the seller if you bid b_1 and you win? (Remember that if x is a uniform random variable $E(x|x < a) = \frac{a}{2}$) Denote this expected payment as $EP(b_1)$.

$EP(b_1) = E(b_2|b_2 < b_1)$. Since $b_2 = \alpha s_2$, and s_2 is distributed uniformly, b_2 is also distributed uniformly. Then, by the formula, $EP(b_1) = \frac{b_1}{2}$.

c. Given player 2's strategy, what is your expected value of the object if your signal is s_1, you bid b_1 and you win? (Remember that if x is a uniform random variable $E(x|x < a) = \frac{a}{2}$) Denote this expected payment as $EV(b_1)$.

$EV(b_1) = E(s_1 + s_2|b_2 < b_1) = s_1 + E(s_2|s_2 < \frac{b_1}{\alpha}) = s_1 + \frac{b_1}{2\alpha}$.

d. Your expected utility from bidding b_1 can be calculated as
$EU_1(b_1, s_1) = [EV(b_1) - EP(b_1)]\, PW(b_1)$.
Find $EU_1(b_1, s_1)$ and obtain the optimal b_1 as a function of α and s_1.

$EU_1(b_1, s_1) = \left[s_1 + \frac{b_1}{2\alpha} - \frac{b_1}{2}\right] \frac{b_1}{10\alpha}$.

$\frac{dEU_1(b_1,s_1)}{db_1} = \frac{s_1}{10\alpha} + \frac{b_1}{10\alpha^2} - \frac{b_1}{10\alpha} = 0$

Solving, $b_1 = \frac{\alpha}{\alpha-1}s_1$.

e. Assume now that you are bidder 2 and that you think that bidder 1's strategy is $b_1(s_1) = \alpha s_1$. Use symmetry to find your optimal bidding strategy as as a function of α and s_2.

$b_2 = \frac{\alpha}{\alpha-1}s_2$.

f. Then, in the symmetric BNE what should α be equal to?

In a BNE, player 1's belief about player 2's strategy must be correct.

Player 1 believed that $b_2 = \alpha s_2$. But if player 2 believes the same, $b_2 = \frac{\alpha}{\alpha-1}s_2$.

Since all believes must be correct: $\alpha = \frac{\alpha}{\alpha-1}$ and, then, $\alpha = 2$.

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4. Some multiple-choice and true/false questions (there were to version of question 4):

Version 1

4.1. Every Nash Equilibrium is Pareto efficient.

true FALSE

4.2. Every Nash Equilibrium is Pareto **in**efficient.

true FALSE

4.3. Which of the following is not true:

a. every sub-game perfect equilibrium is a Nash equilibrium.

b. every Nash equilibrium is a sub-game perfect equilibrium.✓

c. Sub-game perfect equilibria do not include empty threats.

d. Backward induction gives you the sub-game perfect equilibria in finite games.

e. none of the above.

4.4. In a situation of strategic interaction having more information is always better.

true FALSE

4.5. In a situation of strategic interaction having less possible actions may be better.

TRUE false

Version 2

4.1. A Nash Equilibrium may be Pareto efficient.

TRUE false

4.2. Every Nash Equilibrium is Pareto **in**efficient.

true FALSE

4.3. Which of the following is not true:

a. every Nash equilibrium is a sub-game perfect equilibrium.✓

b. every sub-game perfect equilibrium is a Nash equilibrium.

c. Sub-game perfect equilibria do not include empty threats.

d. Backward induction gives you the sub-game perfect equilibria in finite games.

e. none of the above.

4.4. In a situation of strategic interaction having more information may be better.

TRUE false

4.5. In a situation of strategic interaction having more available actions is always better.

true FALSE