

Economics 147: Bargaining Theory and Applications

Spring 2006

Midterm (March 16th)

Name: _____

You have 1 hour and 20 minutes. Good luck!!

1. Consider the following simultaneous-moves game:

		2	
		A	B
1	A	1, 2	0, 0
	B	0, 0	2, 1

a. Find the pure strategy Nash equilibria.

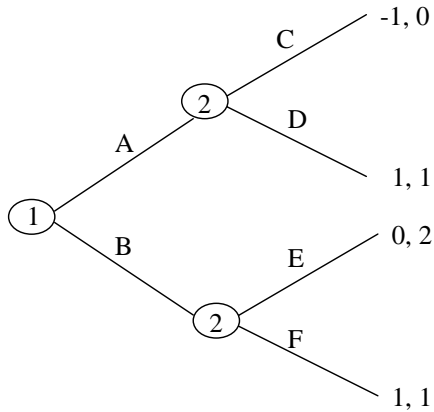
(A,A) and (B,B)

b. Find the mixed strategy Nash equilibrium.

$(\frac{1}{3}A + \frac{2}{3}B, \frac{2}{3}A + \frac{1}{3}B)$

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2. Consider the following game:



a. Find the normal form of the game.

		Player 2			
		CE	CF	DE	DF
player 1	A	-1, 0	-1, 0	1, 1	1, 1
	B	0, 2	1, 1	0, 2	1, 1

b. Find all the pure strategy Nash equilibria.

Best responses in italics:

		Player 2			
		CE	CF	DE	DF
player 1	A	-1, 0	-1, 0	<i>1, 1</i>	<i>1, 1</i>
	B	<i>0, 2</i>	<i>1, 1</i>	0, 2	1, 1

NE: (A, DE); (A,DF) and (B,CE).

c. Find all the pure strategy subgame perfect Nash equilibria.

SGPE: (A, DE).

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3. Private-value first-price auction:

Two bidders have independent valuations (v_1, v_2) distributed uniformly between 0 and 10. Each player knows his or her valuation but not the other bidder's valuation. The bidder with the highest bid wins the object and pays his or her bid. The following steps will lead you to the BNE of this game.

Assume that you are bidder 1 and you know that bidder 2's strategy is $b_2(v_2) = \alpha v_2$, where α is a constant.

a. Given player 2's strategy, what is the probability that you win if you bid b_1 ? (Remember that if x is a uniform random variable between 0 and 10, $P(x < a) = \frac{a}{10}$. For simplicity assume that $b_1 \leq 10\alpha$). Denote this probability $PW(b_1)$.

$$PW(b_1) = \frac{b_1}{10\alpha}$$

b. Given player 2's strategy, what is your payment to the seller if you bid b_1 and you win? Denote this payment as $P(b_1)$. (Hint: it may be simpler than you think).

$$P(b_1) = b_1$$

c. Write your expected utility from bidding b_1 if your valuation is v_1 as a function of $v_1, PW(b_1)$ and $P(b_1)$.

$$EU_1(b_1 | v_1) = (v_1 - P(b_1)) PW(b_1)$$

d. Using your answers to points a to c find the your expected utility from bidding b_1 if your valuation is v_1 as a function of v_1 and α .

$$EU_1(b_1 | v_1) = (v_1 - b_1) \frac{b_1}{10\alpha}$$

d. Using point d find your optimal bidding function given bidder 2 bidding function.

$$\frac{dEU_1(b_1|v_1)}{db_1} = \frac{1}{10\alpha} (v_1 - 2b_1) = 0$$

Solving, $b_1 = \frac{v_1}{2}$

e. Assume now that you are bidder 2 and that you think that bidder 1's strategy is $b_1(v_1) = \alpha v_1$. Use symmetry to find your optimal bidding strategy.

By symmetry, $b_2 = \frac{v_2}{2}$ is the best response to $b_1(v_1) = \alpha v_1$.

f. Then, in the symmetric BNE what should α be equal to?

In the symmetric BNE $\alpha = \frac{1}{2}$.

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4. Some multiple-choice and true/false questions:

4.1. Every Nash Equilibrium is Pareto efficient.

True

False ✓

If true, explain. If false, provide a counter example.

Prisoner's dilemma game.

4.2. A subgame perfect equilibrium is never a Nash equilibrium.

True

False ✓

If true, explain. If false, provide a counter example.

The SGPE in problem number 2.

4.3. In a second price private value auction it is an equilibrium to bid:

a) more than your true value.

b) less than your true value.

c) exactly your true value. ✓

d) all of the above.

e) none of the above.

4.4. In a situation of strategic interaction having more actions is always better.

True

False ✓

If true, explain. If false, provide a counter example.

An example of this is the Entrant game we saw in class. If the incumbent has only the choice of Fight then he is better off than when he also can choose to Accommodate. Commitment is important in games.

4.5. Compare the expected revenue for the seller in first price and second price auctions with private values with **risk averse** bidders:

- a) first price auction results in lower expected revenue than second price auction.
- b) first price auction results in higher expected revenue than second price auction. ✓
- c) first price auction results in the same expected revenue than second price auction.
- d) depends on the number of bidders.
- e) none of the above.