This article was published in Economic Theory 3, (1993), 481-499.

Information Revelation in a Market with Pairwise Meetings: The One Sided Information Case *

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February 14, 2003

*We thank Douglas Gale, Jerry Green, Sergiu Hart, Andreu Mas-Colell, Eric Maskin, Thomas Söjström, and Kathy Spier for their comments. We are grateful to a referee for correcting a mistake in an earlier draft and for providing extremely helpful suggestions. The financial support of the Fulbright - MEC program (R. Serrano) and the Sloan Foundation (O. Yosha) is acknowledged.

Summary. We study a market with pairwise meetings of agents and with a one sided information asymmetry regarding the state of the world, which may be "low" or "high." We characterize the set of equilibria of the model, and study its behavior as the market becomes approximately frictionless. For any one sided information economy there is an equilibrium where trade occurs at the right price (an ex-post individually rational price). Moreover, there is an open set of economies where in all the equilibria trade occurs at the right price.

1 Introduction

Wolinsky [6] shows that in a decentralized market with asymmetric information, where transactions are concluded in pairwise meetings of agents, and prices are not called[1], the process of trade does not fully reveal the private information of informed agents to the uninformed, even when the market becomes approximately frictionless, that is when agents become almost infinitely patient. Paraphrasing Wolinsky, a non-negligible fraction of those who are uninformed when they enter the market, end up transacting at a price which they would not want to transact at, had they known the true state of the world. Such a price, which Wolinsky calls the wrong price, could not have arisen in a Walrasian economy with complete and symmetric information, nor in a fully revealing rational expectations equilibrium. In this sense, Wolinsky's decentralized economy does not resemble a centralized one even if the cost of staying in the market and learning becomes negligible.

Impatience is captured by a constant discount factor δ, so instead of saying "as agents become almost infinitely patient" we shall simply say "as $\delta \to 1$."

The information structure in Wolinsky's model is two sided - there are uninformed agents among sellers and buyers. Three forces are at work in his model. Force CL (Cost of Learning): As $\delta \to 1$ it is less costly for the uninformed to remain in the market and learn from meetings with other agents who may be informed. Force N (Noise): As $\delta \to 1$ the informative content of the pairwise meetings decreases because there are more uninformed agents on both sides of the market trying to learn. Force MI (Misrepresentation of Information): As $\delta \to 1$ the informative content of the pairwise meetings decreases because it costs less for informed agents to try and extract surplus from the uninformed (e.g. a seller who knows that the quality and production cost of the good are low, will ask for a high price in the hope that the buyer he faces is uninformed and will agree.) Force CL drives the economy towards the right price. Forces N and MI drive it towards the wrong price.

It turns out that although force MI becomes stronger as $\delta \to 1$ at the individual level, in the

[1] See Osborne and Rubinstein [5] for a survey of this literature.

aggregate it becomes negligible because as $\delta \to 1$ the agents behind force MI become a negligible proportion of the market. Thus, as $\delta \to 1$ what may prevent agents from learning is force N. Wolinsky shows that in at least one state of the world force N overcomes force CL, and trade occurs at the wrong price.

Gale [1] conjectures that Wolinsky's result depends crucially on the two sided information structure. He provides a one sided information model in which all sellers are informed whereas all buyers are not, and which departs from Wolinsky's model in several other ways.[2] Our work confirms Gale's conjecture, but it departs from Wolinsky's model only in that all sellers are informed. We assume that the sellers of a good know the state of the world (say the quality of the good), but some of the buyers do not.[3]

In the one sided information model force N is eliminated. However, the presence of force MI renders the model non-trivial. When δ increases, sellers, who are all informed, tend to stay longer in the market, but only for the reason in force MI. Uninformed buyers tend to stay longer in the market as well and learn - force CL. The interplay between these two forces determines whether trade will occur at a right or at a wrong price.

In section 2 we study the one sided information model and characterize the set of equilibria. In section 3 we study the behavior of the equilibria as the market becomes approximately frictionless. Our results are quite different from Wolinsky's result for two sided information. We find that for any one sided information economy there is an equilibrium where trade occurs at the right price. Moreover, there is an open set of economies where in all the equilibria trade occurs at the right price. In section 4 we briefly discuss a special case, where no buyers are informed. In section 5 we present the essence of Wolinsky's model. Section 6 concludes.

2 The One Sided Information Model

Time runs discretely from $-\infty$ to ∞. All periods are identical. In the beginning of each period M sellers and M buyers enter the market. Sellers want to sell one unit of an indivisible good, and buyers want to buy one unit of it. Each period sellers and buyers are randomly matched. Each meeting results in an agreement or a disagreement. Those who agree transact and exit the market, and those who disagree stay in the market to be matched anew. Thus, the number of buyers in the market is always the same as the number of sellers. All agents discount future

[2] We provide more details at the end of section 4.

[3] The reverse yields an analogous model. For example, if the state of the world is the probability of discovering oil in a certain region, the buyers of drilling rights - the oil companies - may know this probability with more precision than the government selling these rights.

payoffs by a constant factor $\delta \in (0, 1)$.

There are two states of the world. In state H buyers have a high valuation u_H for the good, and the cost of the good to the sellers c_H is also high. In state L the valuation u_L and the cost c_L are low.

When matched, agents make simultaneous announcements. Each agent can send one of two possible messages: h or l. If both agents say h they trade at price p^{hh}. If both say l they trade at price p^{ll}. If the buyer says h and the seller l they trade at price p^{hl}. Finally, if the seller says h and the buyer l there is disagreement. The payoff of perpetual disagreement is zero. It is convenient to refer to messages as "tough" (h for the seller and l for the buyer) and "soft" (l for the seller and h for the buyer). It is assumed that

$$c_L < p^{ll} < u_L < p^{hl} < c_H < p^{hh} < u_H. \tag{1}$$

In state H the price p^{hh} is defined as the right price, as at this price there are gains from trade for buyer and seller. The other two prices are defined to be wrong because the seller loses when transacting at these prices. Similarly, in state L the price p^{ll} is right and the other prices are wrong.

All sellers are informed, i.e. know the state of the world. A fraction $x_B \in (0, 1)$ of buyers who enter the market each period are also informed. The rest have a common prior belief $\alpha_H \in (0, 1)$ which is the probability that the state of the world is H. The case $x_B = 0$ will be treated in a separate section.

Note that disagreement occurs only if both buyer and seller play "tough." Thus, if an agent plays "soft" in some period, he will surely transact and leave the market. It follows that the only relevant decision variable for an agent is the number of periods during which he will play "tough." Let n_{SH} be this number for a seller who knows that the state is H. Similarly for n_{SL}, n_{BH}, and n_{BL}. Let n_B be this number for an uninformed buyer. The state of the world is either H or L, but an uninformed buyer must take into account what a seller would do if he knew that the state is H, or knew that the state is L.

Uninformed buyers extract information from the announcement of their (informed) trading partners. By playing "tough" an uninformed buyer ensures that if trade occurs, it will be at a price which is advantageous for him. If trade does not occur he will learn, updating α_H. By playing "soft" an uninformed buyer ensures trade, taking the risk of a transaction at a disadvantageous price. In a sense, he is forgoing the opportunity to learn, as in the following period he will not be able to use any information conveyed to him by his trading partner. A strategy n_B can be interpreted as a decision to keep sampling sellers for n_B periods, provided that exit has not yet occurred. The cost of sampling an additional seller is captured by δ.

Let S^h_H be the proportion of the total number of sellers in the market who in state H say h. Similarly for S^h_L, B^l_H, and B^l_L. Note that these are the proportions of agents who play "tough." Agents know the distribution of announcements amongst their trading partners in each state of the world. Uninformed buyers, though, cannot observe the prevailing distribution (S^h_L or S^h_H). Otherwise they would be able to infer the state. Let K_H and K_L be the total number of sellers (and therefore of buyers) in the market in state H and in state L. The market is said to be in steady state when these six numbers are constant through time. The analysis is performed in stationary steady state only (From now on we shall simply say steady state).

Let $V_B(n; \alpha_H, S^h_H, S^h_L)$ be the expected payoff of strategy n to an uninformed buyer who believes with probability α_H that the state of the world is H. Let $V_{SH}(n; B^l_H)$ be the expected payoff of strategy n to a seller in state H. Similarly for $V_{SL}(n; B^l_L)$, $V_{BH}(n; S^h_H)$, and $V_{BL}(n; S^h_L)$. Note that uninformed buyers take into account the steady state proportions of "tough" sellers in both states of the world, whereas informed agents take into account the proportions of "tough" trading partners only in the state of which they are informed.

The market is said to be in equilibrium if each agent maximizes his expected payoff and the market is in steady state. An equilibrium is fully described by eleven numbers: n_B, n_{SH}, n_{SL}, n_{BH}, n_{BL}, S^h_H, S^h_L, B^l_H, B^l_L, K_H and K_L. To write the equilibrium conditions more economically we first show

Claim 1 *In any equilibrium (a) $n_{SH} = \infty$, (b) $n_{BL} = \infty$, (c) $n_{BH} = 0$, (d) $n_B < \infty$.*

Proof. (a) A seller who knows that the state is H understands that declaring l in some period will entail immediate trade at a sure loss. Hence his best response is $n_{SH} = \infty$. (b) Similarly for a buyer who knows that the state is L. (c) A buyer who knows that the state is H understands that sellers always declare h. Thus it is optimal for such a buyer to play "soft" right away, as playing "tough" for any number of periods will simply delay the payoff $u_H - p^{hh}$. (d) Suppose $n_B = \infty$. Then as $n_{SH} = \infty$, in state H uninformed buyers never trade. Also, as $n_{BH} = 0$, the number of buyers who leave the market each period in state H is precisely $x_B M < M$, which cannot happen in a steady state.‖

Using claim 1, the following eleven conditions characterize the equilibrium.

$$M = K_H(1 - S^h_H B^l_H). \tag{2}$$

$$M = K_L(1 - S^h_L B^l_L). \tag{3}$$

$$K_L(1 - B^l_L) = M[x_B (S^h_L)^{n_{BL}} + (1 - x_B)(S^h_L)^{n_B}]. \tag{4}$$

$$K_H(1 - S^h_H) = M(B^l_H)^{n_{SH}}. \tag{5}$$

$$K_L(1 - S_L^h) = M(B_L^l)^{n_{SL}}. \tag{6}$$

$$B_H^l = \frac{(1 - x_B)n_B}{(1 - x_B)(n_B + 1) + x_B}. \tag{7}$$

$$n_{SL} \in \arg\max_n V_{SL}(n; B_L^l). \tag{8}$$

$$n_B \in \arg\max_n V_B(n; \alpha_H, S_H^h, S_L^h). \tag{9}$$

$$n_{SH} = \infty. \tag{10}$$

$$n_{BH} = 0. \tag{11}$$

$$n_{BL} = \infty. \tag{12}$$

Equations (2) and (3) are the steady state conditions for the market size in the two states of the world. Consider (2). The left hand side is the number of entering sellers (or buyers), and the right hand side is the number of sellers leaving the market, which are all the sellers in the market except for those who disagreed. The term $S_H^h B_H^l$ is the proportion of agents who disagreed, namely sellers who played "tough" and met buyers who also played "tough." Analogously for (3).

Consider (4). The left hand side is the number of buyers in state L who play "soft," i.e. those who say h. The first term on the right hand side, $Mx_B(S_L^h)^{n_{BL}}$, is the number of informed buyers who had planned to play "tough" for n_{BL} periods, and met "tough" buyers every time (the probability of this event is $(S_L^h)^{n_{BL}}$). They have now switched, as planned, to saying h. The second term, $M(1 - x_B)(S_L^h)^{n_B}$, is analogous for the uninformed buyers. The right hand side is therefore the number of buyers who have switched this period to saying h. This is also the total number of buyers who are saying h because any buyer who said h in the previous period has for sure transacted and left the market. Similarly for equations (5) and (6), where the right hand side includes only one term, as there are no uninformed sellers.

Equations (4), (5), and (6) are the stationarity conditions for the proportions of "tough" buyers in state L, and of "tough" sellers in the two states. Note that the condition for buyers in state H, $K_H(1 - B_H^l) = M[x_B(S_H^h)^{n_{BH}} + (1 - x_B)(S_H^h)^{n_B}]$, is missing. The reason is as follows. $n_{SH} = \infty$ means that $S_H^h = 1$, as all sellers are informed. Together with $n_B < \infty$ this entails $(S_H^h)^{n_B} = 1$. Also, $n_{BH} = 0$. Therefore this equation is identical to (2). The explanation is simple enough. As sellers are all "tough," trade - and therefore exit - occurs whenever a buyer switches to "soft." Thus, the number of buyers who play "soft" in a given period is also the number of buyers who trade and exit the market in this period.

Equation (7) completes the system. It is derived as follows. Buyers who play "tough" do not trade, as sellers are also playing "tough." Therefore, all the uninformed buyers who entered the market $n_B - 1$ periods ago or later are still in the market and are playing "tough." Their number is $M(1-x_B)n_B$. The uninformed buyers who entered the market n_B periods ago are also in the market but are playing "soft." Their number is $M(1-x_B)$. Uninformed buyers who entered prior to that have already left the market. Thus, the total number of uninformed buyers in the market is $M(1-x_B)+M(1-x_B)n_B$. Informed buyers remain in the market for exactly one period. Therefore the total number of buyers in the market is $M(1-x_B)+M(1-x_B)n_B+Mx_B$, and the fraction of "tough" buyers is as in (7).

We turn to the determination of n_{SL}. A seller in state L faces the following trade-off. If he plays "soft" he will trade at price p^{ll} or at price p^{hl}. If he plays "tough" he will either make a very successful trade at price p^{hh}, or will disagree with his trading partner and remain in the market. Being informed, his prior belief regarding the state of the world does not change as a result of the announcement of his trading partner. Therefore, if he remains in the market an additional period the problem he faces is identical to the one he faced in the preceding period.

The stationarity of the seller's problem implies that it can be solved by comparing the expected gain from playing "tough" for one period and then switching to "soft," to the expected gain from playing "soft" right away. The difference in expected gain is

$$\begin{aligned}\Delta V_{SL} &= V_{SL}(1;B_L^l)-V_{SL}(0;B_L^l)\\ &= (1-B_L^l)(p^{hh}-c_L)+B_L^l\delta[(1-B_L^l)(p^{hl}-c_L)+B_L^l(p^{ll}-c_L)]\\ &\quad -B_L^l[(1-B_L^l)(p^{hl}-c_L)+B_L^l(p^{ll}-c_L)]\\ &= \delta(p^{ll}-p^{hl})(B_L^l)^2+[\delta(p^{hl}-c_L)+p^{hl}-p^{hh}+c_L-p^{ll}]B_L^l\\ &\quad +p^{hh}-p^{hl}.\end{aligned} \tag{13}$$

This establishes

Claim 2 *If $\Delta V_{SL}>0$ then $n_{SL}=\infty$. If $\Delta V_{SL}<0$ then $n_{SL}=0$. If $\Delta V_{SL}=0$ then $n_{SL}\in\{0,\ldots,\infty\}$.*

We introduce the following notation. Let $B(\delta)$ be the positive root of the equation $\Delta V_{SL}=0$ (the other root is negative and is irrelevant for the model). We shall make extensive use of the fact that $\lim_{\delta\to 1}B(\delta)=1$, which follows from continuity of $B(\delta)$ and the fact that $B(1)=1$ (see (13)).

The uninformed buyer's problem is not stationary, for if he encounters a "tough" seller who says h, he revises α_H. The expected gain from a decision to play "tough" for n periods by a buyer with prior α_H is, recalling that $S_H^h=1$,

$$\begin{aligned}V_B(n;\alpha_H,1,S_L^h) &= \alpha_H\delta^n(u_H-p^{hh})\\ &\quad +(1-\alpha_H)(S_L^h)^n\delta^n[S_L^h(u_L-p^{hh})+(1-S_L^h)(u_L-p^{hl})]\\ &\quad +(1-\alpha_H)\textstyle\sum_{t=0}^{n-1}(S_L^h)^t(1-S_L^h)\delta^t(u_L-p^{ll}).\end{aligned} \tag{14}$$

The first term is the discounted expected gain if the state is H. Playing "tough" for n periods results in disagreement, as sellers are also playing "tough." Switching to "soft" in period $n+1$ entails trade at p^{hh}. The other two terms describe the payoff if the state is L. The first of these terms is the expected loss if the buyer is unlucky enough to encounter "tough" sellers n times, an event which happens with probability $(S_L^h)^n$. In period $n+1$, switching to "soft" results in trade with a "tough" or with a "soft" seller. In both cases the buyer loses. The last term is the discounted expected gain from finding a "soft" seller in one of the n periods in which the buyer is playing "tough." n_B maximizes the expression in (14).

Beliefs of an uninformed buyer evolve as follows. If the buyer hears h, α_H is updated to $\alpha_H/[\alpha_H + (1-\alpha_H)S_L^h]$, and so forth recursively.

The possible best responses n_B for an uninformed buyer are established in

Claim 3 *(a) If $S_L^h = 1$ then*

$$n_B \begin{cases} = & 0 \\ \in & \{0,\ldots,\infty\} \\ = & \infty \end{cases} \quad as \quad \alpha_H \begin{cases} > \\ = & \frac{p^{hh}-u_L}{u_H-u_L}, \\ < \end{cases}$$

i.e. the prior α_H determines whether V_B is strictly increasing in n, strictly decreasing, or flat.
(b) If $S_L^h < 1$ then either there is a unique $n_B \in \{0,\ldots,\infty\}$ which maximizes V_B, or there are two consecutive integers n_B, n_B+1, which maximize V_B.

Proof. Note first that the third term in (14) is equal to $(1-\alpha_H)(1-S_L^h)(u_L - p^{ll})\frac{1-(S_L^h\delta)^n}{1-S_L^h\delta}$.
(a) If $S_L^h = 1$ then, as the payoff of perpetual disagreement is zero, $V_B = 0$ for $n_B = \infty$. For $n_B < \infty$, $V_B = \delta^n[\alpha_H(u_H - p^{hh}) + (1-\alpha_H)(u_L - p^{hh})]$, which is strictly positive and strictly decreasing if $\frac{p^{hh}-u_L}{u_H-u_L} > 0$ (hence $n_B = 0$), strictly negative if $\frac{p^{hh}-u_L}{u_H-u_L} < 0$ (hence $n_B = \infty$ with $V_B = 0$), and equal to zero if $\frac{p^{hh}-u_L}{u_H-u_L} = 0$ (hence $n_B \in \{0,\ldots,\infty\}$).
(b) Consider the following continuous and differentiable function of $x \in \mathbf{R}_+$:

$$\begin{aligned} V_B(x;\alpha_H,1,S_L^h) \;=\;& \alpha_H\delta^x(u_H - p^{hh}) \\ &+ (1-\alpha_H)(S_L^h)^x\delta^x[S_L^h(u_L - p^{hh}) + (1-S_L^h)(u_L - p^{hl})] \\ &+ (1-\alpha_H)(1-S_L^h)(u_L - p^{ll})\frac{1-(S_L^h\delta)^x}{1-S_L^h\delta}. \end{aligned} \tag{15}$$

Its first derivative is

$$\begin{aligned} \frac{\partial}{\partial x}V_B(x;\alpha_H,1,S_L^h) \;=\;& \alpha_H\delta^x(\log\delta)(u_H - p^{hh}) \\ &+ (1-\alpha_H)(S_L^h\delta)^x\log(S_L^h\delta)[S_L^h(u_L - p^{hh}) + (1-S_L^h)(u_L - p^{hl})] \\ &- (1-\alpha_H)\frac{1-S_L^h}{1-S_L^h\delta}(u_L - p^{ll})(S_L^h\delta)^x\log(S_L^h\delta). \end{aligned} \tag{16}$$

One of the following is true: (I) $\frac{\partial}{\partial x} V_B(x) > 0$ for all x, (II) $\frac{\partial}{\partial x} V_B(x) < 0$ for all x, or (III) $\frac{\partial}{\partial x} V_B(x^*) = 0$ for some x^*. If (I) is true, then $V_B(n)$ is strictly increasing in n, and $n_B = \infty$. Similarly, if (II) is true, then $n_B = 0$. If (III) is true, then note that $\frac{\partial^2}{\partial x^2} V_B(x^*) < 0$, implying that x^* is a unique global maximum of $V_B(x)$. Thus, $V_B(n)$ is maximized at n_B, at $n_B + 1$, or at both, where n_B is the integer part of x^*.‖

The system of equations which characterizes the equilibrium must be modified whenever best responses are not singletons. Suppose that uninformed buyers are indifferent between n_B and $n_B + 1$. Define $g_B \in [0, 1]$ as the fraction of uninformed buyers who adopt strategy n_B, while the remaining uninformed buyers adopt strategy $n_B + 1$. Then equations (4) and (7) become

$$K_L(1 - B^l_L) = M\{x_B(S^h_L)^{n_{BL}} + (1 - x_B)[g_B(S^h_L)^{n_B} + (1 - g_B)(S^h_L)^{(n_B+1)}]\}. \tag{17}$$

$$B^l_H = \frac{(1 - x_B)[g_B n_B + (1 - g_B)(n_B + 1)]}{(1 - x_B)[g_B n_B + (1 - g_B)(n_B + 1) + 1] + x_B}. \tag{18}$$

Suppose that uninformed buyers are indifferent between any $n_B \in \{0, \ldots, \infty\}$. Define $r_B \in [0, 1]$ as the fraction of uninformed buyers who adopt strategy $n_B = 0$, while the remaining uninformed buyers adopt strategy $n_B = \infty$. Then equations (4) and (7) become[4]

$$K_L(1 - B^l_L) = M[x_B(S^h_L)^{n_{BL}} + (1 - x_B)r_B]. \tag{19}$$

$$B^l_H = (1 - x_B)(1 - r_B). \tag{20}$$

If sellers in state L are indifferent between any $n_{SL} \in \{0, \ldots, \infty\}$, we define $r_S \in [0, 1]$ in an analogous manner, replacing equation (6) by

$$K_L(1 - S^h_L) = M r_S. \tag{21}$$

Existence. Thus far, we have specified the equations which determine the equilibrium, and have established agents' best responses. We turn to the issue of existence.

Proposition 1 *For any configuration of the parameters of the model satisfying (1) an equilibrium exists. More specifically,*

(a) In any equilibrium $S^h_L < 1$, $n_{SL} < \infty$, *and* $\Delta V_{SL} \leq 0$.

(b) There is an equilibrium, denoted E1, where $\Delta V_{SL} < 0$, $n_{SL} = 0$, $n_B = 1$, *and* $B^l_L = 1$, *if and only if*

$$\alpha_H \leq \frac{p^{hl} - p^{ll}}{(1 - \delta)(u_H - p^{hh}) + p^{hl} - p^{ll}}. \tag{22}$$

[4]As uninformed buyers are indifferent between any $n_B \in \{0, \ldots, \infty\}$, there may be equilibria where these agents choose strategies other than $n_B = 0$ and $n_B = \infty$. Any such equilibrium is equivalent to an equilibrium where the only strategies which are chosen are $n_B = 0$ and $n_B = \infty$, for some value of $r_B \in [0, 1]$. See Wolinsky [6], p. 10, footnote 2.

(c) There is an equilibrium, denoted E2, where $\Delta V_{SL} < 0$, $n_{SL} = 0$, $n_B = 0$, and $B_L^l = x_B$ if and only if

$$\alpha_H \geq \frac{p^{hl} - p^{ll}}{(1-\delta)(u_H - p^{hh}) + p^{hl} - p^{ll}} \tag{23}$$

and $x_B > B(\delta)$.

(d) There is an equilibrium where $\Delta V_{SL} < 0$, $n_{SL} = 0$, $x_B < B_L^l < 1$, and where a fraction $g_B \in (0,1)$ of the uninformed buyers choose $n_B = 0$, while the rest of the uninformed buyers choose $n_B = 1$, if and only if $\alpha_H = \frac{p^{hl} - p^{ll}}{(1-\delta)(u_H - p^{hh}) + p^{hl} - p^{ll}}$.

(e) For any $\alpha_H \in (0,1)$, if $x_B < B(\delta)$ then there are equilibria where $\Delta V_{SL} = 0$, $n_{SL} \in \{0, \ldots, \infty\}$, n_B is determined according to claim 3 part (b), and $B_L^l = B(\delta)$. Any equilibrium where $\Delta V_{SL} = 0$ is denoted an E3 equilibrium.

The model can only have equilibria of the types described in proposition 1. When $\Delta V_{SL} = 0$, by definition, any equilibrium is of type E3. When $\Delta V_{SL} < 0$, $n_{SL} = 0$ implying $S_L^h = 0$, which in turn implies that the function $V_B(n; \alpha_H, 1, 0)$ is maximized at $n_B = 0$ (E1), at $n_B = 1$ (E2), or at both. Equilibria with $\Delta V_{SL} > 0$ are impossible.

Proof of proposition 1. We prove first parts (a) through (e).

(a) Suppose $S_L^h = 1$. Then only "soft" buyers trade. As $n_{BL} = \infty$ (claim 1), informed buyers never trade. Therefore the number of buyers who leave the market each period is at most $(1 - x_B)M < M$, which cannot happen in a steady state. Hence $S_L^h < 1$. It follows that $n_{SL} < \infty$, otherwise $S_L^h = 1$. Also, $\Delta V_{SL} \leq 0$, otherwise $n_{SL} = \infty$.

(b) *Step 1.* We show first that if (22) holds then the configuration in part (b) is an equilibrium. Suppose $\Delta V_{SL} < 0$ then $n_{SL} = 0$ and hence $S_L^h = 0$, which implies that for $n \geq 1$, $V_B(n; \alpha_H, 1, 0) = \alpha_H \delta^n (u_H - p^{hh}) + (1 - \alpha_H)(u_L - p^{ll})$, and for $n = 0$, $V_B(n; \alpha_H, 1, 0) = \alpha_H(u_H - p^{hh}) + (1 - \alpha_H)(u_L - p^{hl})$. Note that for $n \geq 1$, $V_B(n; \alpha_H, 1, 0)$ is strictly decreasing in n. The intuition is that the sellers' behavior is fully revealing. Thus, after one period of disagreement a buyer learns that the state is H and therefore that sellers are "tough." Insisting on l for more periods only delays the payoff $u_H - p^{hh}$.

The trade-off which the buyer faces is between $n_B = 0$, i.e. playing "soft" right away, and $n_B = 1$, i.e. hoping to trade at p^{ll} in the first period, or to learn that the state is H, switch to h, and trade at p^{hh} in the second period.

Step 2. Condition (22) implies $V_B(1; \alpha_H, 1, 0) \geq V_B(0; \alpha_H, 1, 0)$, and thus $n_B = 1$ is a best response. As $S_L^h = 0$, all buyers exit after one period in the market. Thus the uninformed buyers do not get a chance to switch from l to h, and therefore (as $n_{BL} = \infty$) $B_L^l = 1$, yielding $\Delta V_{SL} = (\delta - 1)(p^{ll} - c_L) < 0$, which verifies that indeed $\Delta V_{SL} < 0$.

Step 3. The necessity of (22) for an E1 equilibrium follows from the fact that in this equilibrium the condition $V_B(1; \alpha_H, 1, 0) \geq V_B(0; \alpha_H, 1, 0)$ is equivalent to (22).

(c) *Step 1.* Identical to step 1 in the proof of part (b).

Step 2. Condition (23) implies $V_B(1; \alpha_H, 1, 0) \leq V_B(0; \alpha_H, 1, 0)$, and thus $n_B = 0$ is a best response. As $S_L^h = 0$, all buyers exit after one period in the market. The informed buyers exit after one period of "tough" play, whereas the uninformed exit after one period of "soft" play. Thus $B_L^l = x_B$. If $x_B > B(\delta)$, then indeed $\Delta V_{SL} < 0$.

Step 3. Analogous to step 3 in the proof of part (b).

(d) When $\Delta V_{SL} < 0$ the condition $\alpha_H = \frac{p^{hl} - p^{ll}}{(1-\delta)(u_H - p^{hh}) + p^{hl} - p^{ll}}$ is equivalent to the condition $V_B(1; \alpha_H, 1, 0) = V_B(0; \alpha_H, 1, 0)$. Hence uninformed buyers are indifferent between choosing $n_B = 0$ and $n_B = 1$. In order to sustain $\Delta V_{SL} < 0$ we must have $B_L^l = x_B + (1 - g_B)(1 - x_B) > B(\delta)$. If $x_B > B(\delta)$ then for any $g_B \in (0, 1)$ we have $x_B + (1 - g_B)(1 - x_B) > B(\delta)$. If $x_B < B(\delta)$ then there is a $\hat{g}_B \in (0, 1)$ such that for all $g_B \in (0, \hat{g}_B)$ we have $x_B + (1 - g_B)(1 - x_B) > B(\delta)$.

(e) Consider the equilibrium system when best responses are singletons. As $n_{SH} = \infty$, $S_H^h = 1$. As $B_H^l < 1$ (see (7)), (5) becomes an identity. By simple substitutions, using $S_L^h < 1$ (part (a) above) and $n_{BL} = \infty$ (claim 1), the system can be reduced to equations (2), (7), and the equations

$$1 - (1 - S_L^h B_L^l)(B_L^l)^{n_{SL}(B_L^l)} = S_L^h, \tag{24}$$

$$1 - (1 - S_L^h B_L^l)(1 - x_B)(S_L^h)^{n_B(S_L^h, \alpha_H)} = B_L^l, \tag{25}$$

where $n_{SL}(B_L^l)$ and $n_B(S_L^h, \alpha_H)$ are agents' best responses.

When uninformed buyers are indifferent between n_B and $n_B + 1$, we use equations (17) and (18), and equation (25) is replaced by

$$1 - (1 - S_L^h B_L^l)(1 - x_B)[g_B (S_L^h)^{n_B} + (1 - g_B)(S_L^h)^{(n_B+1)}] = B_L^l. \tag{26}$$

When uninformed buyers are indifferent between any $n_B \in \{0, \ldots, \infty\}$, we use equations (19) and (20), and equation (25) is replaced by

$$1 - (1 - S_L^h B_L^l)(1 - x_B) r_B = B_L^l. \tag{27}$$

When sellers are indifferent between any $n_{SL} \in \{0, \ldots, \infty\}$, we use equation (21), and equation (24) is replaced by

$$1 - (1 - S_L^h B_L^l) r_S = S_L^h. \tag{28}$$

The fractions g_B, r_B, and r_S can take any value in the interval $[0,1]$.

Equations (24) and (25) (and their respective replacements) define a correspondence from $[0,1]^2$ to itself. It suffices to focus on the fixed points $(\hat{B_L^l}, \hat{S_L^h})$ of this correspondence. Then B_H^l will be determined by (7), (18), or (20), which in turn will determine K_H using (2).

Fix $x_B \in (0,1)$, and choose $\hat{\delta} < 1$ such that for all $\delta \geq \hat{\delta}$, $x_B < B(\delta)$. This can be done because $\lim_{\delta \to 1} B(\delta) = 1$. We are looking for fixed points of a map $F : X \times I \to X$, where $X = [0,1]^2$, and $I = [0,1]$ is the set of parameter values α_H. Clearly, X is compact and convex. It is straightforward to see that $n_{SL}(B_L^l)$ is u.h.c. Using the implicit correspondence theorem[5], $n_B(S_L^h, \alpha_H)$ is shown to be u.h.c. as well. Both correspondences are convex valued as best responses are singletons or intervals. Hence, the correspondence defined by the system is u.h.c. and convex valued.

We invoke a theorem of Mas-Colell which states that the graph of fixed points of a correspondence like F contains a connected set which projects onto the set I.[6] Recalling that $\lim_{\delta \to 1} B(\delta) = 1$, we can choose δ sufficiently close to 1 such that equilibrium E2 (part (c) of the proposition) and the equilibrium described in part (d) of the proposition do not exist. Equilibrium E1 is a fixed point of F, with $(\hat{B_L^l}, \hat{S_L^h}) = (1,0)$ and α_H satisfying (22). As the graph of E1 equilibria does not project onto the domain of α_H, the graph of fixed points must contain E3 equilibria. Moreover, when $\Delta V_{SL} = 0$, by definition of $B(\delta)$ we have that $B_L^l = B(\delta) < 1$. Thus the graph of E3 equilibria cannot connect with the graph of E1 equilibria, and must itself project onto the domain of α_H. Hence, for any $x_B \in (0,1)$ there is a $\hat{\delta} < 1$ such that for all $\delta \geq \hat{\delta}$ an E3 equilibrium exists for any $\alpha_H \in (0,1)$.

It remains to verify that for any configuration of the parameters of the model satisfying (1) an equilibrium exists. When $x_B < B(\delta)$ there is an E3 equilibrium. When $x_B > B(\delta)$ there is an E1 equilibrium for values of α_H satisfying (22), and an E2 equilibrium for values of α_H satisfying (23). When $\alpha_H = \frac{p^{hl} - p^{ll}}{(1-\delta)(u_H - p^{hh}) + p^{hl} - p^{ll}}$ we can find g_B sufficiently close to 0 so that $x_B + (1-g_B)(1-x_B) > B(\delta)$, and hence the equilibrium described in part (d) of the proposition exists.

When $x_B = B(\delta)$ then for values of α_H satisfying (22) there is an E1 equilibrium. When $\alpha_H > \frac{p^{hl} - p^{ll}}{(1-\delta)(u_H - p^{hh}) + p^{hl} - p^{ll}}$ then if $n_{SL} = S_L^h = 0$ then $n_B = 0$ is a best response for uninformed buyers, entailing $B_L^l = B(\delta) = x_B$ and thus $\Delta V_{SL} = 0$. Hence $n_{SL} = 0$ is a best response for

[5] See e.g. Mas-Colell [4], p.49.

[6] The theorem is an extension of a theorem of F. Browder. In fact, the theorem is stated for X open, but can be extended to closed sets as follows. Let X be closed. Define $G : R^n \times I \to X$ such that $G(x, \alpha_H) = F(y, \alpha_H)$ where $y \in X$ is the foot of x in X. Mas-Colell's theorem applies to G. As F and G have the same graph of fixed points, it also applies to F.

sellers. This is an E3 equilibrium.‖

Remark. We have not shown existence of E3 equilibria when $x_B > B(\delta)$, although they may well exist. As we shall be concerned with the behavior of sequences of equilibria as $\delta \to 1$, this is not a material omission.

In equilibrium E1 uninformed buyers enter the market playing "tough." The condition in (22) says that they will find it optimal to do so as long as their prior belief regarding the state being H is not too strong. The right hand side of (22) increases with δ. The intuition is that when buyers become more patient, their inclination to play "tough" at the risk of staying in the market for one period and learn rises, so they will do so despite a stronger belief that the state is H. When $\delta \to 1$, the right hand side of (22) approaches 1. Thus, for any $\alpha_H \in (0,1)$, there is a $\hat{\delta} < 1$ such that for all $\delta \geq \hat{\delta}$ an E1 equilibrium exists.

In equilibrium E1 trade occurs at the right price, and sellers are truth-telling, i.e. force MI is not active. Recall that force N is not present in the one sided information model. It is therefore not surprising that trade occurs at the right price, and that as $\delta \to 1$ agents become more inclined to learn - force CL. More precisely, uninformed buyers play "tough" for a larger region of the prior α_H.

In equilibrium E2 sellers are truth-telling, as in equilibrium E1. In state L informed buyers trade at the right price, but uninformed buyers, believing that the probability of the state being H is big, play "soft" and pay p^{hl} for the good, a price at which they would not have agreed to trade had they known that the state is L. The restriction on x_B guarantees that there are enough "tough" buyers around, to discourage sellers from misrepresenting their information. Note that for sufficiently big δ condition (23) and the condition $x_B > B(\delta)$ do not hold, so this type of equilibrium ceases to exist.

3 The Revelation of Information

The question we now ask is whether trade at the wrong price is possible even when the market becomes approximately frictionless. As in state H sellers are always "tough," trade occurs at the right price p^{hh}. Also, as informed buyers in state L are always "tough," they trade at the right price p^{ll}. Thus, the only transactions which might occur at the wrong price are those involving uninformed buyers in state L. Let the fraction of transacting uninformed buyers who in state L end up trading at the wrong price be

$$f_B = \frac{K_L(1 - B_L^l)}{M(1 - x_B)}. \tag{29}$$

In the following propositions we characterize the behavior of equilibrium sequences for which limits exist as $\delta \to 1$.

Proposition 2 *If* $(1-x_B)\frac{\alpha_H}{1-\alpha_H}(u_H - p^{hh}) \neq p^{ll} - c_L$ *then along any sequence of equilibria for which limits exist, as* $\delta \to 1$*, the following statements are equivalent:*

(I) $\lim_{\delta\to 1} f_B = 0$.
(II) $\lim_{\delta\to 1} S_L^h < 1$.
(III) $\lim_{\delta\to 1} n_{SL} < \infty$.

The condition on the parameters in proposition 2 is almost always satisfied, in the sense that it takes a rare coincidence for it to be violated.

Proof of proposition 2.

Step 1. Along any sequence of E1 equilibria $S_L^h = 0$, $n_{SL} = 0$, and $f_B = 0$. Note that for δ sufficiently big E2 equilibria do not exist.

Step 2. We turn to sequences of E3 equilibria. Although sellers are indifferent between any $n_{SL} \in \{0, \ldots, \infty\}$, we shall assume throughout the proof that they all adopt the same n_{SL}. Without this assumption the proof is essentially the same, with $r_S \in [0,1]$ replacing $(B_L^l)^{n_{SL}}$. It may also happen that buyers are indifferent between any $n_B \in \{0, \ldots, \infty\}$ or between n_B and $n_B + 1$. In the former case replace $(S_L^h)^{n_B}$ by $r_B \in [0,1]$. In the latter case replace $(S_L^h)^{n_B}$ by $g_B(S_L^h)^{n_B} + (1-g_B)(S_L^h)^{n_B+1}$.
We now derive some facts which will be used in the following steps.
(a) $f_B = \frac{1-B_L^l}{(1-x_B)(1-B_L^l S_L^h)}$. This is obtained by substituting for $\frac{K_L}{M}$ in (29) using (3).
(b) $f_B = (S_L^h)^{n_B}$. Noting that $n_{BL} = \infty$ and that in equilibrium $S_L^h < 1$, this follows from (4) and (29).
(c) $(1-x_B)f_B = 1 - (B_L^l)^{n_{SL}+1}$. Multiply both sides by M. The left hand side is the number of uninformed buyers who trade at the wrong price. This must be equal to the number of sellers who trade at the wrong price, namely all the sellers except those who switched to "soft" after having met "tough" buyers for n_{SL} periods, and then met a "tough" buyer one more time, trading at the right price p^{ll}.
(d) $S_L^h = \frac{1-(B_L^l)^{n_{SL}}}{1-(B_L^l)^{n_{SL}+1}}$. Write (3) as $M = K_L[(1-B_L^l) + B_L^l(1-S_L^h)]$. Eliminate $\frac{M}{K_L}$ using (6) and solve for S_L^h.

Step 3. Along a sequence of E3 equilibria, whenever $(1-x_B)\frac{\alpha_H}{1-\alpha_H}(u_H - p^{hh}) \neq p^{ll} - c_L$, (I) implies (II). We establish this by showing that $\lim_{\delta\to 1} S_L^h = 1$ implies $\lim_{\delta\to 1} f_B \neq 0$.

Suppose $\lim_{\delta\to 1} S_L^h = 1$. From fact (a), noting that $\lim_{\delta\to 1} B_L^l = 1$, and using the assumption $\lim_{\delta\to 1} S_L^h = 1$, we can use l'Hôpital's rule to obtain $\lim_{\delta\to 1} f_B = \frac{\lim_{\delta\to 1} B'}{(1-x_B)(\lim_{\delta\to 1} S' + \lim_{\delta\to 1} B')}$, where B' and S' are the derivatives of B_L^l and S_L^h with respect to δ along a sequence of E3 equilibria.

From $\Delta V_{SL} = 0$ (see (13)) we obtain $\lim_{\delta\to 1} B' = -\lim_{\delta\to 1} \frac{\partial \Delta V_{SL}/\partial \delta}{\partial \Delta V_{SL}/\partial B_L^l} = \frac{p^{ll} - c_L}{p^{hh} - p^{ll}} > 0$.

Consider the equation $\frac{\partial}{\partial x} V_B(x; \alpha_H, 1, S_L^h) = 0$ (see (16)). Substitute f_B for $(S_L^h)^{n_B}$ (fact (b)) and solve for f_B. Taking limits, recalling the assumption $\lim_{\delta\to 1} S_L^h = 1$, and using l'Hôpital's rule to compute $\lim_{\delta\to 1} \frac{\log \delta}{\log(\delta S_L^h)}$ and $\lim_{\delta\to 1} \frac{1 - S_L^h}{1 - \delta S_L^h}$, we obtain an equation involving $\lim_{\delta\to 1} f_B$ and $\lim_{\delta\to 1} S'$. Using the expressions for $\lim_{\delta\to 1} f_B$ and $\lim_{\delta\to 1} B'$ derived above, we obtain

$$\lim_{\delta\to 1} f_B = \frac{(1 - x_B)\frac{\alpha_H}{1-\alpha_H}(u_H - p^{hh}) - (p^{ll} - c_L)}{(1 - x_B)[(p^{hh} - u_L) - (p^{ll} - c_L)]}, \tag{30}$$

which establishes step 3.

Step 4. Along a sequence of E3 equilibria, where $\delta \to 1$, (II) implies (III). Consider the expression in fact (d). $\lim_{\delta\to 1} S_L^h < 1$ implies that $\lim_{\delta\to 1} (B_L^l)^{n_{SL}} = 1$. Otherwise, as $\lim_{\delta\to 1} B_L^l = 1$, $\lim_{\delta\to 1} S_L^h = 1$.

Dividing the numerator and the denominator of the right hand side by $1 - B_L^l$ and taking limits we get $\lim_{\delta\to 1} S_L^h = \lim_{\delta\to 1} \frac{1 + B_L^l + (B_L^l)^2 + \cdots + (B_L^l)^{n_{SL}-1}}{1 + B_L^l + (B_L^l)^2 + \cdots + (B_L^l)^{n_{SL}}} = \lim_{\delta\to 1} \frac{n_{SL}}{n_{SL}+1}$, which establishes step 4.

Step 5. Along a sequence of E3 equilibria, where $\delta \to 1$, (III) implies (I). As $\lim_{\delta\to 1} B_L^l = 1$ and $\lim_{\delta\to 1} n_{SL} < \infty$, we have $\lim_{\delta\to 1} (B_L^l)^{n_{SL}} = 1$. Using fact (c) the result follows.‖

Corollary *Under the assumption on parameters in proposition 2 and the assumption* $(1 - x_B)\frac{\alpha_H}{1-\alpha_H}(u_H - p^{hh}) - (p^{ll} - c_L) \neq (1 - x_B)[(p^{hh} - u_L) - (p^{ll} - c_L)]$, *along any sequence of E3 equilibria for which limits exist, as* $\delta \to 1$, *(a) If* $\lim_{\delta\to 1} S_L^h > 0$ *then* $\lim_{\delta\to 1} n_B = \infty$, *and (b) If* $\lim_{\delta\to 1} S_L^h = 0$ *then* $\lim_{\delta\to 1} n_B = 1$ *and moreover, there is a* $\hat{\delta} < 1$ *such that for all* $\delta \geq \hat{\delta}$, $n_B = 1$.

The additional assumption on parameters is almost always satisfied as well.

Proof. Suppose $\lim_{\delta\to 1} S_L^h > 0$. If $\lim_{\delta\to 1} f_B = 0$, then by proposition 2 $\lim_{\delta\to 1} S_L^h < 1$. From fact (b) in proposition 2 it must be that $\lim_{\delta\to 1} n_B = \infty$. If $\lim_{\delta\to 1} f_B \neq 0$, then by proposition 2 $\lim_{\delta\to 1} S_L^h = 1$. Suppose $\lim_{\delta\to 1} n_B < \infty$. From fact (b) in proposition 2 it must be that $\lim_{\delta\to 1} f_B = 1$. But by (30) $\lim_{\delta\to 1} f_B \neq 1$.

Suppose $\lim_{\delta\to 1} S_L^h = 0$. Then the sequence of functions $V_B(n; \alpha_H, 1, S_L^h)$ approaches the function $V_B(n; \alpha_H, 1, 0)$, which for δ sufficiently close to 1 is maximized at $n_B = 1$ (See the proof of part (b) of proposition 1).‖

Remark. Whenever trade occurs at the wrong price the fraction of wrong price trades is a well determined number given by (30).

Proposition 3 *(a) For any cost, valuation, and price configuration satisfying (1), there is an open region of the parameters α_H and x_B for which, along any sequence of equilibria such that limits exist, as $\delta \to 1$, trade always occurs at the right price.*

(b) For any cost, valuation, and price configuration satisfying (1), and any $\alpha_H \in (0,1)$ and $x_B \in (0,1)$, there exist a sequence of E1 equilibria and a sequence of E3 equilibria along which, as $\delta \to 1$, trade occurs at the right price.

Proof. (a) Let $x_B \in (0,1)$. Consider figure 1. There are two cases: (i) $(p^{hh} - u_L) - (p^{ll} - c_L) > 0$, in which case in point A $\alpha_H = \frac{p^{hh}-u_L}{u_H-u_L}$ and in point B $\alpha_H = \frac{p^{ll}-c_L}{u_H-p^{hh}+p^{ll}-c_L}$, and (ii) $(p^{hh} - u_L) - (p^{ll} - c_L) < 0$, in which case in point B $\alpha_H = \frac{p^{hh}-u_L}{u_H-u_L}$ and in point A $\alpha_H = \frac{p^{ll}-c_L}{u_H-p^{hh}+p^{ll}-c_L}$. In both cases the shaded areas in the figure are such that the expression in (30) is strictly negative or strictly greater than one. By definition of f_B this is not possible. Therefore, in the shaded regions sequences of E3 equilibria such that $\lim_{\delta\to1} S_L^h = 1$ are not possible, and thus along any sequence of equilibria such that limits exist, as $\delta \to 1$, $\lim_{\delta\to1} S_L^h < 1$. By proposition 2, $\lim_{\delta\to1} f_B = 0$.

(b) For any cost, valuation, and price configuration satisfying (1), and any $\alpha_H \in (0,1)$ and $x_B \in (0,1)$, there exists a $\hat{\delta} < 1$ such that for all $\delta \geq \hat{\delta}$ equation (22) holds, and therefore an E1 equilibrium exists.

We turn to sequences of E3 equilibria. We construct a sequence of E3 equilibria as follows. By the corollary above there is a $\hat{\delta} < 1$ such that for all $\delta \geq \hat{\delta}$, $n_B = 1$. Take some $\delta \geq \hat{\delta}$, and compute $B(\delta)$. Substitute $B_L^l = B(\delta)$ and $n_B = 1$ into equation (24), which becomes $B(\delta)(S_L^h)^2 - S_L^h + \frac{1-B(\delta)}{1-x_B} = 0$, with two strictly positive roots, one of which indeed approaches 0 as $\delta \to 1$ (The second root approaches 1. By the corollary, this implies that $\lim_{\delta\to1} n_B = \infty$. As, by construction, $n_B = 1$, this root is not part of an E3 equilibrium). Using the first root we solve (28) for $r_S = \frac{1-B(\delta)}{1-B(\delta)S_L^h}$. We now have an E3 equilibrium. The construction can be repeated for any $\delta \geq \hat{\delta}$, and can be done for any $\alpha_H \in (0,1)$.$\|$

Remark. For parameter configurations in the non-shaded area in figure 1, we cannot rule out sequences of E3 equilibria such that in the limit trade takes place at the wrong price, although we have not found any example of such a sequence. Curiously, if trade in the limit does occur at the wrong price, the limiting fraction of wrong price trades is a precise number determined by equation (30).

Discussion. In equilibrium E1 sellers play "soft" ($\Delta V_{SL} < 0$), i.e. force MI (Misrepresentation of Information) is not active. Force CL (Cost of Learning) induces uninformed buyers to play "tough," and trade occurs at the right price. In equilibrium E2 sellers also play "soft," but α_H is sufficiently high to induce uninformed buyers to play "soft," so they trade at the wrong price. Force CL is not strong enough to overcome the prior belief regarding the state of the world. When $\delta \to 1$ force CL becomes stronger, breaking this equilibrium.

Along a sequence of E3 equilibria as $\delta \to 1$, sellers have, other things equal, a greater incentive to misrepresent their information and play "tough" - force MI. But as $\delta \to 1$ buyers are also becoming very "tough" ($\lim_{\delta\to 1} B_L^l = 1$) - force CL. By construction of an E3 equilibrium this exactly offsets the incentive of sellers to be "tougher," rendering them indifferent between playing "tough" or "soft" ($\Delta V_{SL} = 0$ and $n_{SL} \in \{0, \ldots, \infty\}$).

The question is what happens to the "toughness" of sellers in the limit, i.e. to $\lim_{\delta\to 1} S_L^h$. If $\lim_{\delta\to 1} S_L^h = 0$ then one period of "tough" play on the part of uninformed buyers ($n_B = 1$) is sufficient to guarantee trade at the right price. If $\lim_{\delta\to 1} S_L^h > 0$, then if n_B approaches infinity very fast, the only way to maintain a steady state is for some sellers to switch from "tough" to "soft," so that $\lim_{\delta\to 1} S_L^h < 1$ (implying that $\lim_{\delta\to 1} n_{SL} < \infty$). This can be interpreted as force CL overcoming, in the limit, force MI, with trade occurring at the right price. When the speed at which n_B approaches infinity is not sufficiently big, S_L^h may approach 1 (with $\lim_{\delta\to 1} n_{SL} = \infty$) along a sequence of (steady state) equilibria, and trade might occur at the wrong price.

We provide some intuition for the shaded regions in figure 1, where trade always occurs at the right price. When x_B is big most buyers are informed, so in state L sellers face a population which is predominantly "tough." This induces sellers to switch from "tough" to "soft," as the probability of extracting surplus from an uninformed buyer is low. Hence $\lim_{\delta\to 1} S_L^h < 1$. This is *a fortiori* true when α_H is low, which contributes to greater "toughness" on the part of the uninformed buyers. This explains the shaded region in the lower right hand side of the figure.

The other region may be explained as follows. When x_B is small, most of the buyers are uninformed. If S_L^h were to approach 1 uninformed buyers would find it optimal to switch from "tough" to "soft" after a very short stay in the market. But then it would not be possible to sustain a steady state with B_L^l approaching 1, which must hold in any E3 equilibrium by construction. This is *a fortiori* true when α_H is high, which contributes to greater "softness" on the part of the uninformed buyers. This explains the shaded region in the upper left hand side of the figure.

4 Extreme One Sided Information

Suppose all buyers are uninformed: $x_B = 0$. This case is special in two important ways. First, there is no E2 equilibrium for any value of δ. Technically speaking this happens because when $x_B = 0$ the condition $x_B > B(\delta)$ in proposition 1, part (c), cannot be met. Another way to see this is by noting that if $n_B = 0$, then $B_L^l = 0$ and $\Delta V_{SL} = p^{hh} - p^{hl} > 0$, so $n_{SL} = 0$ cannot be sustained. Intuitively, the E2 equilibrium disappears due to the disappearance of informed buyers, who in state L always play "tough." If sellers are sufficiently impatient, the presence of these buyers may deter sellers from misrepresenting their information, but with no such buyers around, truth-telling is not a best response for the sellers.

The second special feature of this case is the appearance of another equilibrium, denoted E4, where trade occurs at the wrong price. In this equilibrium we have $\Delta V_{SL} > 0$ and thus $n_{SL} = \infty$, implying that $S_L^h = 1$. Therefore $n_B < \infty$, otherwise there would be no trade and no steady state equilibrium. Then $V_B(n; \alpha_H, 1, 1) = \delta^n[\alpha_H(u_H - p^{hh}) + (1 - \alpha_H)(u_L - p^{hh})]$. As we want $n_B < \infty$, it must be that $[\alpha_H(u_H - p^{hh}) + (1 - \alpha_H)(u_L - p^{hh})] \geq 0$, or

$$\alpha_H \geq \frac{p^{hh} - u_L}{u_H - u_L}. \tag{31}$$

When this inequality is strict, $n_B = 0$ implying that $B_L^l = 0$. Then indeed we have $\Delta V_{SL} = p^{hh} - c_L > 0$: When all buyers are "soft" it is optimal for sellers to play "tough." Given that sellers are always "tough," buyers enter the market playing "soft." Trade occurs immediately at the wrong price. What drives this equilibrium is the fact that buyers attribute a high probability to the state being H. When the state is in fact L these beliefs are "very wrong." Sellers take advantage of this, misrepresenting their information (force MI).[7] Note that in this equilibrium there is no learning, as sellers behave in the same way in both states.[8]

Except for these two differences the extreme one sided information model is analogous to the general one sided information model (namely, there are E1 equilibria for α_H satisfying (22), E3 equilibria for δ big enough, and propositions 2 and 3 hold).

Remark. Equilibria E1, E4, and E3 are reminiscent of the types of equilibria that one finds in signaling models. The E1 equilibrium can be thought of as a separating equilibrium, where the

[7] For example, when the differences between adjacent magnitudes in (1) are all equal, (31) becomes $\alpha_H \geq 3/4$.

[8] When the inequality in (31) is not strict there are more equilibria of this type, where n_B is not zero. They are computed by solving the quadratic inequality $\Delta V_{SL} > 0$. For each value of B_L^l satisfying this inequality there is a corresponding value of n_B determined by (7). In these equilibria buyers enter the market playing "tough" for several periods despite the fact that sellers play "tough" in both states. The reason is that a buyer's expected payoff from any transaction is exactly zero. This expectation does not change through time as in this equilibrium the pairwise meetings convey no information. The buyer is completely indifferent between receiving zero now or later. Trade occurs at the wrong price with a delay.

behavior of informed agents fully reveals their private information. The E4 equilibrium can be viewed as a pooling equilibrium, where informed agents behave in the same way regardless of their private information. E3 equilibria are similar to semi-separating equilibria, where informed agents are indifferent between revealing and not revealing their private information.

The relation to Gale's work. Gale studies a one sided information model where all sellers are informed and all buyers are uninformed. His model differs from our model in that (a) $p^{hl} = p^{ll}$, and (b) A fixed fraction $\pi < 1$ of sellers has high costs in state L.[9] The first difference contributes to trade at the right price, as in our model a meeting between two "soft" agents results in trade at price p^{hl} which is always wrong, whereas in Gale's model it results in trade at price p^{ll} which in state L (the only state in which wrong price trade is possible) is the right price. The second difference, though, contributes to trade at the wrong price, as the fixed fraction of sellers who always have high costs play "tough" also in state L, increasing the likelihood of disagreement or of wrong price trade.

Gale studies a particular equilibrium where sellers with low costs always call the low price (for sellers with high costs it is a dominant strategy to call the high price). Then the proportion of sellers who call the high price in state L is $\pi < 1$. Gale shows that the equilibrium becomes fully revealing as $\delta \to 1$.

This result is perfectly compatible with our analysis. In the particular equilibrium which Gale studies sellers are truth-telling, i.e. force MI is not active. The proportion of "tough" sellers is $S_L^h = \pi < 1$, for all $\delta < 1$, and therefore, in the spirit of proposition 2, $\lim_{\delta \to 1} f_B = 0$.

5 Wolinsky's Two Sided Information Model

Let $x_S \in (0, 1)$ be the fraction of informed sellers who enter the market each period. The rest of the sellers are uninformed and have the same prior α_H as uninformed buyers. There is an additional variable in the model, n_S, the strategy of an uninformed seller. The equilibrium system when best responses are singletons is as follows.[10]

$$M = K_H(1 - S_H^h B_H^l). \tag{32}$$

$$M = K_L(1 - S_L^h B_L^l). \tag{33}$$

[9] Another difference is that Gale fixes market size exogenously. The agents who trade are replaced by the same number of agents with the same characteristics as those who traded. This turns out to be equivalent to our formulation, although it raises a difficulty at the level of interpretation: As $\delta \to 1$ the number of transactions becomes negligible in absolute terms (not only relatively to market size).

[10] We present the simplest version of the two sided information model.

$$K_H(1 - S^h_H) = M[x_S(B^l_H)^{n_{SH}} + (1 - x_S)(B^l_H)^{n_S}]. \quad (34)$$

$$K_H(1 - B^l_H) = M[x_B(S^h_H)^{n_{BH}} + (1 - x_B)(S^h_H)^{n_B}]. \quad (35)$$

$$K_L(1 - S^h_L) = M[x_S(B^l_L)^{n_{SL}} + (1 - x_S)(B^l_L)^{n_S}]. \quad (36)$$

$$K_L(1 - B^l_L) = M[x_B(S^h_L)^{n_{BL}} + (1 - x_B)(S^h_L)^{n_B}]. \quad (37)$$

Best responses maximize the respective value functions. Let the fraction of transacting uninformed sellers who in state H end up trading at the wrong price be

$$f_S = \frac{K_H(1 - S^h_H)}{M(1 - x_S)}. \quad (38)$$

Similarly for buyers in state L, as in equation (29). Wolinsky's main result is that along a sequence of equilibria where $\delta \to 1$, and such that $\lim_{\delta\to 1} f_S$ and $\lim_{\delta\to 1} f_B$ exist, at least one of these limits is positive.

Wolinsky shows that (a) In state H the fraction of informed buyers in the market and (b) In state L the fraction of informed sellers in the market approach zero as $\delta \to 1$. Let us focus on state L. Wolinsky shows that informed sellers are "less tough" than uninformed sellers ($n_{SL} \leq n_S$): Both informed and uninformed sellers are becoming more patient as $\delta \to 1$, but for a seller who knows that the state is L switching from "tough" to "soft" never entails a loss, whereas an uninformed seller who switches trades at a loss with positive probability (Note that perpetual disagreement guarantees a payoff of zero). As $\delta \to 1$, $n_S \to \infty$ due to the lower cost of learning.

On the buyers' side of the market $n_{BL} = \infty$ for all δ, and $n_B \to \infty$ for the same reason that $n_S \to \infty$. Steady state requires that n_{SL} either not approach infinity, or at least do so at a slower speed than n_S. As a result, the population of sellers who exit the market each period becomes predominantly composed of informed sellers, implying that most sellers in the market are uninformed. Symmetrically for buyers in state H: As $\delta \to 1$ the market becomes predominantly composed of uninformed buyers.

Suppose $f_S \approx 0$ and $f_B \approx 0$. Wolinsky shows that $f_S \approx 0$ implies $\lim_{\delta\to 1} \frac{n_S}{n_B} = \infty$ and that $f_B \approx 0$ implies $\lim_{\delta\to 1} \frac{n_S}{n_B} = 0$. But n_S and n_B are the same numbers in both states of the world (as they are chosen by uninformed agents), so their ratio cannot approach infinity and zero at the same time. Thus, either $\lim_{\delta\to 1} f_S > 0$ or $\lim_{\delta\to 1} f_B > 0$, i.e. in at least one state of the world some agents don't learn even when the market becomes approximately frictionless.

The following reasoning may shed some light on this result. Suppose the state of the world is H and $f_S \approx 0$. Therefore, sellers are not switching from h to l, as any seller who switches trades at price p^{hl} or p^{ll}, which are both wrong. As the market is in steady state we know that trade is occurring, and since all sellers are playing "tough" transactions are being executed at price p^{hh}. This means that some buyers must be switching from l to h. Recalling that almost all of these buyers are uninformed, this means that although both n_S and n_B are increasing and approaching infinity, n_S must be increasing faster, so sellers exit the market before they get a chance to switch from "tough" to "soft." Analogously for state L.

Remark. A particular case of Wolinsky's model occurs when $x_S \in (0, 1)$ and $x_B = 0$. This is a two sided information model as there are uninformed agents on both sides of the market. The intuitive argument for Wolinsky's result goes through. In fact, it is even simpler, as in state H the population behind force MI (informed buyers) is not present. The case $x_S = 0$ and $x_B \in (0, 1)$ is symmetric.

6 Concluding Remarks

In the two sided information model the agents behind force MI (informed sellers in state L and informed buyers in state H) become a negligible fraction of the market as $\delta \to 1$. This is true even if the entering fractions of informed agents (x_S and x_B) are very close to 1. As long as these fractions are not 1, the steady state requirement forces these agents, as $\delta \to 1$, out of the market. What drives the economy towards the wrong price is the Noise force, namely the fact that, as $\delta \to 1$, most sellers in state L and most buyers in state H are uninformed.

In the one sided information model the agents behind force MI cannot become a negligible fraction of the market, by construction. The case $x_S = 1$ exhibits a discontinuity with respect to the two sided information model. For any δ all sellers in state L are informed. Hence, the potential for learning by the uninformed buyers to occur is there. Whether the economy is driven to a wrong or to a right price depends on the relative strength of the incentive to misrepresent information on the part of sellers (Misrepresentation of Information force) and the willingness to stay in the market and learn by uninformed buyers (Cost of Learning force).

Thus, the *reason* for wrong price trades is not the same in the two models. In the two sided information model wrong price trades occur due to the low informational content of the pairwise meetings. Uninformed agents do not learn from informed agents because the informed agents disappear from the market before the uninformed learn. In the one sided information model, whenever there is trade at the wrong price, this happens because informed agents prefer to misrepresent their information rather than play the role of teachers. Propositions 2 and 3 show that "very often" this is *not* the case, with the Cost of Learning force overcoming the Misrepresentation of Information force, driving the economy to an ex-post individually rational price.

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