JOURNAL OF ECONOMIC THEORY 67, 285–294 (1995)

A Market to Implement the Core*

ROBERTO SERRANO

Department of Economics, Box B, Brown University, Providence, Rhode Island 02912

Received July 30, 1993; revised September 5, 1994

I present a mechanism whose subgame perfect equilibrium outcomes coincide with the core of an underlying convex TU game. The mechanism resembles an asset market, in which one of the players is randomly selected as a broker who centralizes trade. In addition, the rules of the mechanism are independent of the coalitional function. Therefore, the mechanism is useful for an incompletely informed designer who wishes to implement the core of these games. The mechanism is inspired by the reduced games relevant for the consistency of the core. *Journal of Economic Literature* Classification Numbers: C71 and C72. © 1995 Academic Press, Inc.

1. INTRODUCTION

The most widely used cooperative solution in economics is the core, introduced by Edgeworth [7]. The core consists of the set of allocations that cannot be improved upon (or blocked) by any coalition of agents. Non-core allocations are hence unstable, in that at least one group of players has an incentive to block them. Nevertheless, many economists have criticized the core (and cooperative game theory in general) because it cannot explain this instability in terms of the strategic behavior of *individuals*. One way to respond to this general criticism is to develop the Nash program, which aims to design non-cooperative games whose equilibria correspond to cooperative solutions.

In the recent years, papers in the Nash program that explore non-cooperative outcomes related to the core include Selten [18], Chatterjee *et al.* [4], Selten and Wooders [19], Bloch [2], Moldovanu and Winter [13], and

* The author is grateful to J. Green, S. Hart, V. Krishna, A. Mas-Colell, E. Maskin, B. Polak, A. Salsbury, R. Vohra, M. Wooders, O. Yosha, and an anonymous referee for useful comments. Financial support from the Fulbright-MEC programm in the earlier stages of the project is acknowledged. This paper is a revised version of the author's Ph.D. thesis, "Essays on Consistency and Implementation" (Department of Economics, Harvord Univerity, 1992), Chapter II entitled "Non-Cooperative Implementation of the Core."

0022-0531/95 $12.00

Perry and Reny [17]. All of these papers present games of coalitional bargaining and their results tend to show a connection between the stationary (or stationary-like) subgame perfect equilibria of their extensive forms and the core.[1]

This paper also contributes to the Nash program. But unlike the papers mentioned above, the non-cooperative game does not model coalitional bargaining. The paper presents a non-cooperative game that resembles a brokered market. In addition, the rules of the game form do not require complete knowledge of the coalitional function. This fact is also a departure from the earlier papers and makes the mechanism helpful for an incompletely informed designer who wants to implement the core.

The game form of this paper is not the first that can implement the core. Maskin [12] shows that, in an economy with a private good, any social choice correspondence that satisfies a simple monotonicity condition is fully Nash implementable. Since the core is monotonic, Maskin's mechanism can implement it. However, since that mechanism implements *any* monotonic correspondence, it does not highlight non-cooperative attributes specific to the core. Furthermore, Maskin's mechanism is rather abstract.[2] The game form of this paper, in contrast, is simple and relates to well-known institutions (e.g., auctions, mediated bid-ask markets).[3] It phrases in a non-cooperative language the core's market dimension, long ago highlighted by the equivalence theorems between the core of large economies and their Walrasian outcomes.[4]

Following others, this paper's mechanism introduces the role of "market agent" for some players. Debreu [6] obtains the Walrasian correspondence through the Nash equilibrium outcomes of a game in which the auctioneer is an additional player. To give a non-cooperative view of the Von Neumann–Morgenstern stable sets, a game with a referee is introduced in Harsanyi [8]. Wilson [23] constructs a refereed mechanism with multiple Nash equilibria, one of which yields an allocation in the core.[5]

[1] Lagunoff [10] constructs a mechanism specific to each allocation and characterizes a class of such mechanisms related to the core. Perez Castrillo [16] constructs a game (played by "principals") related to the core of an economy of n "agents." On the other hand, Kalai *et al.* [9] implement the core in strong Nash equilibrium using a game of coalitional coordination (the Nash equilibria of their game implement the whole individual rationality set).

[2] Players' messages include entire preference profiles and integer games. The same is true in the canonical mechanisms constructed for subgame perfect implementation (Abreu and Sen [1] and Moore and Repullo [14]).

[3] See Wilson [24] for an excellent survey of these and other trading institutions.

[4] See, for example, Mas-Colell [11], chapter 7.

[5] In Wilson's work (as in this paper), the broker is one of the real players. I discuss its relation with my results at the end of Section 3. Serrano [20] also contains a different version of the game form, in which the broker is an extra player. It is shown that core outcomes are those in which the broker's payoff is maximized. This result holds for balanced TU games, for which the core is non-empty (see Bondareva [3] and Shapley [21])

This paper presents a non-cooperative game form whose subgame perfect equilibrium outcomes coincide with the core of convex transferable utility games. The mechanism is an asset market. In this market, only core allocations can be equilibrium outcomes, because non-core allocations create profit opportunities for some players. Each player in the game assumes (with some probability) the role of the broker who announces prices and centralizes trade. As a result of the monopoly power of the broker, the equilibrium outcomes in that subgame are the maximizers of the broker's payoff within the core. Providing a non-cooperative reading of a theorem of Shapley's [22] for convex games, the main result of the paper obtains the core as the union (over all probability assignments of the broker's role) of the convexifications of SPE payoffs of all brokered subgames. The core is thus implemented in an *ex ante* or expected sense.

The mechanism is inspired by a reinterpretation of the coalitional function in terms of joint returns of assets. This reinterpretation follows closely the Davis–Maschler [5] reduced game, which is relevant for the consistency property of the core.[6]

2. Core, Consistency, and Asset Returns

Let $(N; v)$ be a *transferable utility* (TU) *game in coalitional form*, where $N = \{1, 2, ..., n\}$ is a finite set of players and $v: 2^N \to \mathbb{R}$ (the coalitional function) is a map that assigns a real number $v(S)$ to every subset S of N, where $v(\{\emptyset\}) = 0$. For any $S \subset N$, $v(S)$ is called the worth of S. It is usually interpreted as the optimal value of the initial endowments of S as a group.[7]

A game $(N; v)$ is *convex* if for all $i \in N$, and for all $S, T \in 2^N$, $S \subset T$ and $i \notin T$, we have that $v(S \cup \{i\}) - v(S) \leqslant v(T \cup \{i\}) - v(T)$. A game $(N; v)$ is strictly convex if all the earlier inequalities are strict. Convexity can be interpreted as increasing returns to cooperation.

The *core* of the game $(N; v)$ is $C(N; v) = \{x \in \mathbb{R}^n$ such that $x(N) = v(N)$ and for all $S \in 2^N$, $x(S) \geqslant v(S)\}$.[8]

A cooperative solution is consistent if it is "invariant" to the number of players. Suppose we start with a game of n players. A solution of this game is consistent if, after "slicing" the game to any lower dimension (fewer number of players), the projection of the original solution to that lower

[6] The core is consistent, as shown by Peleg [15]. (See Section 2 below). Peleg also shows that, for the class of balanced TU games, the core is the *only* solution satisfying individual rationality, superadditivity, and consistency.

[7] That is, the feasible set of payoffs for coalition S is the half-space $\{x \in \mathbb{R}^s: \sum_{i \in S} x_i \leqslant v(S)\}$.

[8] If x is a payoff vector in $\mathbb{R}^n$, we use $x(S)$ to denote $\sum_{i \in S} x_i$. Also, we shall denote by x^S the vector in $\mathbb{R}^s$ which contains the components of x corresponding to members of the coalition S.

dimension is still a solution in the "sliced" game. Of course, the critical step is how we "slice" the game, or the way in which the reduced games are defined. The following is the definition of the Davis–Maschler (D–M) reduced games, which are relevant for the consistency of the core.

For any S (a non-empty proper subset of N), and for any $x \in \mathbb{R}^n$, define the *D–M reduced game* $(S; v_{xS})$ as follows: $v_{xS}(S) = v(N) - x(N \backslash S)$, $v_{xS}(\{\phi\}) = 0$, and for all other $T \subset S$, $v_{xS}(T) = \max_{Q \subseteq N \backslash S} \{v(T \cup Q) - x(Q)\}$.

This reduced game has usually been interpreted as follows: the allocation x has been proposed. The reduced game for S is defined by paying every player in $N \backslash S$ at the prices x. Clearly, the new worth of the total coalition S must be whatever is left after payments are made to $N \backslash S$. But to define the new worth of every non-empty subcoalition $T \subset S$, we allow players in T to find the best deal among the groups of players that have been left out. One interpretation is that T maximizes its return by hiring the resources of every $Q \subset N \backslash S$. T must pay for these resources at the rate x. Notice that, in this process, different coalitions might be using the resources of the same Q to get their new worth.

Let Γ be a solution, that is, a mapping from the set of TU games to subsets of the nth Euclidean space.

A solution Γ satisfies *consistency* (or the reduced game property) if, for all games $(N; v)$, for all $x \in \Gamma(N; v)$, and for all $S \subset N$, we have that $x^S \in \Gamma(S; v_{xS})$.

A solution Γ satisfies *converse consistency* (or the converse reduced game property) if we have the following. For any game $(N; v)$, let x be such that $x(N) = v(N)$. Suppose x satisfies that for all $S \subset N$, $|S| = 2$, $x^S \in \Gamma(S; v_{xS})$. Then, $x \in \Gamma(N; v)$.[9]

Next consider the following example: $N = \{1, 2, 3\}$, $v(N) = 1$, $v(\{i\}) = 0$ for all $i \in N$, $v(S) = \frac{2}{5}$ otherwise. Suppose that the allocation proposed is $x = (0, \frac{1}{3}, \frac{2}{3})$. Using the definition of the core, we know that x is not "stable," because players 1 and 2 have an incentive to cooperatively form the coalition $\{1, 2\}$ to improve upon x.

Alternatively, making use of Peleg's theorem, we know that if x is not a core allocation, there must exist a two-player reduced game whose core does not contain the projection of the given allocation. In the example, for the proposed x, the reduced game $(S; v_{xS})$ for $S = \{2, 3\}$ is such that $x^S \notin C(S; v_{xS})$ because $v_{xS}(\{2\}) = \max\{v(1, 2) - x_1, 0\} = \frac{2}{5} > \frac{1}{3} = x_2$. That is, player 2 has an incentive to buy the resources of player 1 at the rate prescribed by x ($x_1 = 0$). This leads to a reinterpretation of the coalitional function.

Suppose that the TU game $(N; v)$ represents the following underlying structure. Each player in N owns an indivisible asset and an amount of

[9] Peleg [15] shows that the core satisfies consistency and converse consistency.

a divisible good called money. We shall assume that utility functions are quasilinear. The assets are named using the players' indices, that is, player i owns asset i. The possession of a portfolio of assets S gives any player a utility of $v(S)$. If the assets owned by the players are somehow priced, a market can be created in which the agents trade assets. An allocation x is then reinterpreted as a vector of prices for the assets.

According to this interpretation, to move away from a non-core allocation, one does not require the cooperative behavior of players, but rather, such an allocation is viewed as a player's opportunity to buy underpriced assets (for some i and for some S, $i \in S$, $v_{xS}(\{i\}) > x_i$; recall the definition of v_{xS} above). On the other hand, core allocations are those in which all individual "arbitrage" opportunities have been wiped out from the market (for all i and for all S, $i \in S$, $x_i \geqslant v_{xS}(\{i\})$). Building up on these ideas, I will construct the mechanism of the next section.

3. A Mechanism with a Broker

In this section, a mechanism is introduced in which one of the players is chosen at random to perform the role of a broker who centralizes trade.

Let N be a set of at least two players. The TU coalitional function $(N; v)$ describes the underlying "joint returns" of any portfolio formed with the n assets.

For each $q = (q_1, q_2, ..., q_n) \in \Delta^{n-1}$ (the $(n-1)$st unit simplex), the mechanism G_q is described as follows. At the outset, nature chooses player i to be the broker with probability q_i $(i = 1, 2, ..., n)$. The subgame where player i $(i = 1, ..., n)$ is the broker has two stages. In stage 0, player i announces an allocation x $[x(N) = v(N)]$. The vector x is then a price vector in the efficient hyperplane of the game $(N; v)$.

In stage 1, players in $N \backslash \{i\}$ decide sequentially according to a fixed and arbitrary protocol P whether to accept or reject x. Formally, the action space for the players in $N \backslash \{i\}$ (in the subgame where i is the broker) is the set $\{$"yes;" "no, B_j," $j \notin B_j\}$. If player j says "yes" to x, player i buys player j's asset at price x_j and j stops playing. If all accept x, the game ends. If player j says "no, B_j," he bids for the portfolio B_j; he would buy (if available from the broker at j's turn according to P) the assets B_j at the prices x. Otherwise, he will walk away with his asset j and not buy any other.

Formally, denote the m rejectors by j_r, $r = 1, 2, ..., m$. The order of the rejectors is determined by the protocol P: player j_r moved before player j_{r+1}, for $r = 1, 2, ..., m-1$.

Denote by R_0 the set $R_0 = N \backslash \{j_1, j_2, ..., j_m\}$. The assets R_0 are held by the broker i at the end of the responses.

Finally, the trading with the rejectors takes place. This stage is not formally part of the game (there are no decision nodes in it). Player j_1 will then buy assets B_1 if $B_1 \subseteq R_0$ and $\{\emptyset\}$ otherwise. Denote by A_1 the portfolio of assets that player j_1 buys. For $r = 1, 2, ..., m$, define the set $R_r = R_{r-1} \backslash A_r$. Then, following the order assigned to the rejectors, player j_r can buy assets B_r if $B_r \subseteq R_{r-1}$ at the prices x, or $\{\emptyset\}$ otherwise. After player j_m has traded, player i has the portfolio of assets R_m and the final asset allocation results.

Given a game $(N; v)$ payoffs can be assigned to the outcome function described. In the subgame brokered by i, player i's payoff is $x_i + v(R_m) - x(R_m)$ if asset i was traded, and $v(R_m) - x(R_m \backslash \{i\})$ otherwise. For all $j \in N \backslash \{i\}$, j's payoff is x_j if he sells his asset to i, or $v(A_j \cup \{j\}) - x(A_j)$ otherwise. Of course, the payoff to any player in the game G_q is the expected sum over the subgames with different brokers, where the subgame with broker i has a weight of q_i.

Assumption A. In what follows, we assume that a player accepts the announced prices whenever he is indifferent between accepting them and receiving the payoff after rejection prescribed in the continuation.[10]

Let E_q be the *set of subgame perfect equilibrium payoffs* of G_q.
We can now state our main result.

THEOREM 1. *Let the underlying game $(N; v)$ be (strictly) convex. Then, under Assumption* A, $y \in C(N; v)$ *if and only if* $y \in \bigcup_{q \in \Delta^{n-1}} E_q$.[11]

Proof. First, denote by M_i the set of core allocations where player i receives his maximum payoff. That is, $M_i = \{x \in C(N; v)$ such that for all $y \in C(N; v)$, $y_i \leqslant x_i\}$. Note that, by Shapley's theorem for convex games, $M_i \neq \{\emptyset\}$ and if $x \in M_i$, $x_i = v(N) - v(N \backslash \{i\})$[12] To prove the theorem, we proceed in two steps.

Step I. $E_q \subset C(N; v)$. To prove this step, we first show that in the subgame where player i is the broker, z is a subgame perfect equilibrium outcome if and only if $z \in M_i$. This will be proved by the following two claims.

[10] Alternatively, the same results are obtained if we assume that a player who wants to buy assets must pay a fee to participate in the trading stage and we take the limit as the fee goes to 0, or if we make use of "trembling hand" arguments.

[11] In fact, strict convexity is much stronger than the condition needed, which is that for all $i \in N$, $v(N) - v(N \backslash \{i\}) > v(S) - v(S \backslash \{i\})$ for all other S containing i.

[12] Shapley's theorem [22] says that $C(N; v)$ is the convex hull of all the vectors x_π of marginal contributions if and only if $(N; v)$ is convex. For an arbitrary protocol $\pi = (\pi_1, ..., \pi_n)$ of the players, x_π's ith component is $v(\{\pi_1, ..., \pi_{i-1}, \pi_i\}) - v(\{\pi_i, ..., \pi_{i-1}\})$.

CLAIM 1. *Let player i be the broker. Consider any subgame that begins immediately after player i has announced a vector x with* $x_i \geqslant v(\{i\})$. *Then, in any SPE under Assumption* A, $x \in C(N; v)$ *if and only if every player in* $N\backslash\{i\}$ *accepts x.*

Proof of Claim 1. If $x \in C(N; v)$, Peleg's theorem implies that for all $j \in N$, $x_j \geqslant v_{xS}(\{j\})$ for all S containing j (there are no profit opportunities in the market). This fact and Assumption A imply that all players in $N\backslash\{i\}$ will accept x (by backward induction reasoning on the responders).

If $x \notin C(N; v)$, we argue by contradiction. That is, suppose every player accepts the prices x. Denote by k the last (according to P) member of the last blocking coalition (say, S). Note that $k \neq i$ because $x_i \geqslant v(\{i\})$. Again by Peleg's theorem, for k and for some $T, k \in T$, $v_{xT}(\{k\}) \geqslant v(S) - x(S\backslash\{k\}) > x_k$. Clearly, player k has an incentive to announce "no, $S\backslash\{k\}$," while (by backward induction and Assumption A the players moving after him will continue to accept. Hence, player k will profit from the rejection. Therefore, at least one player must reject x. ∎

CLAIM 2. *Let player i be the broker. Consider any subgame that starts immediately after player i has announced x. Then, in any SPE under Assumption* A, *if x is rejected by at least one player in* $N\backslash\{i\}$, *player i's payoff from announcing x is strictly less than* $v(N) - v(N\backslash\{i\})$.

Proof of Claim 2. Denote by $R_m \neq N$ the portfolio held by player i at the end of the game. Player i's payoff would be $x_i + v(R_m) - x(R_m)$ if asset i was traded, or $v(R_m) - x(R_m\backslash\{i\})$ if it was not.

If asset i was traded, we claim that $x_i \leqslant v(N) - v(N\backslash\{i\})$. Suppose not and let player j be the one who purchased asset i. Player j receives a payoff of $v(B_j \cup \{j\}) - x(B_j)$, $i \in B_j$. However, player j could profit by announcing "no, $B_j\backslash\{i\}$" instead of "no, B_j." To see this, note that $v(B_j \cup \{j\}) - x(B_j) = v([B_j\backslash\{i\}] \cup \{j\}) - x(B_j\backslash\{i\}) + [v(B_j \cup \{j\}) - v([B_j\backslash\{i\}] \cup \{j\}) - x_i] < v([B_j\backslash\{i\}] \cup \{j\}) - x(B_j\backslash\{i\})$.

In addition, we claim that $v(R_m) - x(R_m) < 0$. This follows from convexity (and hence, superadditivity) of the game $(N; v)$ and from Assumption A. Denoting by A_r, $r = 1, ..., m$, the assets purchased by j_r, by Assumption A we have that for all r, $v(j_r \cup A_r) - x(A_r) > x(j_r)$. By superadditivity of v, we can write $v(j_1 \cup A_1) + \cdots + v(j_m \cup A_m) + v(R_m) \leqslant v(N)$.[13] Hence, $[v(j_1 \cup A_1) - x(j_1 \cup A_1)] + \cdots + [v(j_m \cup A_m) - x(j_m \cup A_m)] + [v(R_m) - x(R_m)] \leqslant v(N) - x(N) = 0$. Since the first m brackets are positive, the last one is negative. Hence, i's payoff from announcing x is strictly less than $v(N) - v(N\backslash\{i\})$.

If asset i was not traded, we know that $v(R_m\backslash\{i\}) - x(R_m\backslash\{i\}) \leqslant 0$, since otherwise any player in $R_m\backslash\{i\}$ (say, player k) could profitably change his

[13] A game $(N; v)$ is superadditive if for all $S, T \in 2^N$, $S \cap T = \{\varnothing\}$, $v(S) + v(T) \leqslant v(S \cup T)$.

announcement from "yes" to "no, $R_m \setminus \{i, k\}$." Hence, i's payoff is no greater than $v(R_m) - v(R_m \setminus \{i\}) < v(N) - v(N \setminus \{i\})$ by strict convexity. ▌

Claims 1 and 2 imply that if z is a SPE payoff in the subgame where i is the broker, every player accepts the announced prices z and $z \in M_i$.[14]

Thus we have shown that, in the subgame where player i is the broker, z is a subgame perfect equilibrium payoff if and only if $z \in M_i$ (the announcements of other core vectors will be accepted; on the other hand, Claim 2 applies following non-core vectors). Since q is fixed in G_q and the TU core is a convex set, any outcome $y \in E_q$ must be in the core.

Step II. $C(N; v) \subset \bigcup_q E_q$. By Shapley's theorem, $C(N; v)$ is the convex hull of all the points x_π for all π if $(N; v)$ is convex. Every x_π belongs to at least one M_i. Then, given $y \in C(N; v)$, there exists q such that $y \in E_q$. ▌

This result is related to Wilson's work [23], who presents a competitive exchange game with non-transferable utility and l commodities. There are $n+1$ traders in the economy, out of whom n are bidders and the other is a referee (player 0). The game is as follows: each bidder submits a sealed envelope to player 0 with a finite set of proposed feasible trades (belonging to his consumption set). Player 0 then selects at most one trade from each bidder. If all of a bidder's proposed trades are rejected, he receives his initial endowments. Otherwise, he participates in trade. Player 0 receives the negative of the trade in the economy.

Wilson shows the existence of what he calls a "principal Nash equilibrium." This yields an allocation in the core in which player 0 gets his highest possible utility in the core. In this equilibrium, the bidders face perfect competition, in the sense that if any player were to propose a better trade, player 0 would exclude him from trade altogether. Player 0 is the only one not facing perfect competition, although his monopoly power is countered by his passive position in the bidding stage.

In the game presented in this section, the broker has monopoly power in that he alone announces the vector of prices. The other traders face the "competitive pressure" of having to sell their assets for the broker's asking prices or bidding for the right portfolio (hoping on its availability, since other traders could bid for the same assets). As in Wilson's game, the referee's monopoly power enables him to achieve his best core payoff in that subgame. This results since, in equilibrium, the rest of the players are willing to accept the broker's asking prices (all profit opportunities in the

[14] We shall not write explicitly the strategies off the equilibrium path. Since the game is one of finite horizon, they could be easily written by backward induction excluding complicated history dependence.

market have been wiped out when the vector of prices is in the core). The implementation of the core is then obtained in an expected sense thanks to the special structure of convex TU games. Theorem 1 uses that, for these games, the core coincides with the convex hull of maximizers of players' payoffs within the core, thereby providing a "non-cooperative reading" of Shapley's theorem [22].

References

1. D. Abreu and A. Sen, Subgame perfect implementation: A necessary and almost sufficient condition, *J. Econ. Theory* **50** (1990), 285–299.
2. F. Bloch, Sequential formation of coalitions with fixed payoff division, Working Paper 92-5, Department of Economics, Brown University, 1990.
3. O. N. Bondareva, Some applications of linear programming methods to the theory of cooperative games, *Probl. Kibern.* **10** (1963), 119–139. [In Russian]
4. K. Chatterjee, B. Dutta, D. Ray, and K. Sangupta, A non-cooperative theory of coalitional bargaining, *Rev. Econ. Stud.* **60** (1993), 463–477.
5. M. Davis and M. Maschler, The kernel of a cooperative game, *Nav. Res. Logist. Quart.* **12** (1965), 223–259.
6. G. Debreu, A social equilibrium existence theorem, *Proc. Nat. Acad. Sci. USA* **38** (1952), 886–893.
7. F. Y. Edgeworth, "Mathematical Psychics," Kegan Paul, London, 1881.
8. J. C. Harsanyi, An equilibrium point interpretation of stable sets and a proposed alternative definition, *Manage. Sci.* **20** (1974), 1472–1495.
9. E. Kalai, A. Postlewaite, and J. Roberts, A group incentive compatible mechanism yielding core allocations, *J. Econ. Theory* **20** (1979), 13–22.
10. R. Lagunoff, A simple non-cooperative core story, *Games Econ. Behav.* **7** (1994), 54–61.
11. A. Mas-Colell, "The Theory of General Economic Equilibrium: a Differentiable Approach," Cambridge Univ. Press, Cambridge, 1985.
12. E. Maskin, Nash equilibrium and welfare optimality, mimeo, MIT, 1977.
13. B. Moldovanu and E. Winter, Order independent equilibria, mimeo, University of Bonn, 1991.
14. J. Moore and R. Repullo, Subgame, perfect implementation, *Econometrica* **56** (1988), 1191–1220.
15. B. Peleg, On the reduced game property and its converse, *Int. J. Game Theory* **15** (1986), 187–200.
16. D. Perez Castrillo, Cooperative outcomes through non-cooperative games, *Games Econ. Behav.*, forthcoming.
17. M. Perry and P. Reny, A non-cooperative view of coalition formation and the core, *Econometrica* **62** (1994), 795–817.
18. R. Selten, A non-cooperative model of characteristic function bargaining, *in* "Essays in Game Theory and Mathematical Economics in Honor of Oskar Morgenstern" (R. Aumunn *et al*, Eds.), pp. 131–151, Bibliographisches Institut Mannheim, Mannheim, 1981.
19. R. Selten and M. Wooders, A game equilibrium model of thin markets, *in* "Game Equilibrium Models III: Strategic Bargaining" (R. Selten Ed.), Springer-Verlag, Berlin, 1991.
20. R. Serrano, Essays on Consistency and Implementation, Ph.D. thesis, Department of Economics, Harvard University, 1992.

21. L. S. SHAPLEY, On balanced sets and cores, *Nav. Res. Logist. Quart.* **14** (1967), 453–460.
22. L. S. SHAPLEY, Cores of convex games, *Int. J. Game Theory* **1** (1971), 11–26.
23. R. WILSON, Competitive exchange, *Econometrica* **46** (1978), 577–585.
24. R. WILSON, Game theoretic analysis of trading processes, *in* "Advances in Economic Theory (Fifth World congress of the Econometric Society)" (T. Bewley, Ed.), Cambridge Univ. Press, Cambridge, 1987.