



ELSEVIER

Journal of Mathematical Economics 24 (1995) 319–329

JOURNAL OF
Mathematical
ECONOMICS

Strategic bargaining, surplus sharing problems and the nucleolus

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Submitted October 1993; accepted May 1994

Abstract

Suppose that n agents are trying to reach an agreement on how to split an estate over which they have claims. They do so through decentralized negotiations instead of appealing to an arbitrator. In this context, I study a consistency-based multilateral bargaining game that allows for ‘partial agreements’ and ‘bilateral negotiations’. The solution to these bilateral negotiations is influenced by the outside option of going to the ‘contested garment’ court (recommended in the Talmud for bilateral problems). When the surplus to share is negative (bankruptcy), if the proposer is the highest claimant, the unique subgame perfect equilibrium (SPE) outcome is the nucleolus. If it is someone else, multiplicity of SPE outcomes might arise. When the surplus is positive, the prenucleolus is always obtained as the unique SPE outcome of the game.

Keywords: Consistency; Nash program; Nucleolus; Prenucleolus; Bargaining; Surplus sharing; Bankruptcy; Absolute priority

JEL classification: C71, C78

1. Introduction

There are three ways to understand a cooperative solution in game theory: its definition, its axiomatic foundation and its non-cooperative interpretation. When all three are developed, our understanding of the solution is enhanced and thus its relevance is increased.

The definition embodies either a pattern of social stability (like the core and the bargaining set) or a criterion of fairness (like the Shapley value and the nucleolus).

The definition is complemented by the axioms, which are essential properties unique to the solution concept. Finally, its non-cooperative interpretation often follows from describing the negotiations that lead to the solution.¹

This paper examines the non-cooperative dimension of the nucleolus.² The nucleolus is a solution concept that maximizes the ‘welfare’ of the worst-off coalitions. Although its definition emphasizes a normative principle of egalitarianism among coalitions, the bargaining game of this paper reveals a positive side of the nucleolus.

Many cooperative solutions have recently received consistency-based axiomatizations.³ A solution is consistent if its predictions are invariant to the number of players in the game. Consistency is the central axiom of the nucleolus. Using consistency, Sobolev (1975) axiomatized the prenucleolus for the class of all transferable utility (TU) games, Aumann and Maschler (1985) characterized the nucleolus for the class of bankruptcy problems, and Moulin (1985) axiomatized the prenucleolus of surplus sharing problems.⁴

However, this is not the only way that the consistency principle has been used. This paper follows several others that use consistency as a guide to design non-cooperative games that yield cooperative solutions.⁵ Serrano (1993) obtains the nucleolus of three-player superadditive games as the unique Markov perfect equilibrium outcome of a consistency-based bargaining game when the order of proposers corresponds to the power of players in the coalitional function. Serrano (1993) also provides an example that shows the impossibility of obtaining the same result for the class of n -player superadditive games. This impossibility is related to the non-monotonicity of the nucleolus in that class.

In this paper, I will restrict attention to surplus sharing problems.⁶ For the class of bankruptcy problems, Aumann and Maschler (1985) prove that the nucleolus is the unique solution consistent with the CG (contested garment) rule recommended in the Talmud for bilateral problems. Moulin (1985), on the other hand, axiomatizes a method of equal allocation of non-separable costs, which corresponds to the prenucleolus of a suitably defined coalitional game.

¹ See Nash (1953), which inaugurates the “Nash program”, a research agenda that tries to obtain cooperative solutions by means of non-cooperative games.

² The reader is referred to Gul (1989) and Hart and Mas-Colell (1992) for games that investigate the non-cooperative content of the value (the other widely used single-valued solution in coalitional games with transferable utility).

³ See Thomson (1990) for a formal definition of consistency and an excellent survey of these results.

⁴ Each bankruptcy problem generates a convex game. Thus, prenucleolus and nucleolus coincide in this class. In general, both solutions differ only in that the nucleolus requires individual rationality.

⁵ The reader is referred to Krishna and Serrano (1990), Peters et al. (1991), Serrano (1992, 1993).

⁶ Negative surplus sharing problems are also called bankruptcy problems (for a formal definition, see Section 2 below). Positive surplus sharing or simply surplus sharing is defined in Section 4.

We will imagine that the n agents try to reach an agreement through decentralized negotiations instead of appealing to an arbitrator or a court. Hence, one advantage of our procedure will be that society can save legal fees. The concept of CG consistency inspires the following bargaining game.⁷ One of the agents is arbitrarily chosen to make a proposal, which is a starting point for the opening of bilateral negotiations. If the proposal is unanimously accepted, it is implemented. Otherwise, a ‘piecemeal agreement’ takes place, in which the acceptors are awarded their shares, and each rejector then faces the proposer in bilateral negotiations. The continuing negotiations represent a form of outside option. They can be interpreted either as a random dictator bargaining game or as an arbitrator that imposes the CG rule to solve bilateral problems.

What outcome should we expect from such negotiations? Who must be given the big role of making the proposal? The answer to these questions reveals a striking asymmetry between bankruptcy and surplus sharing problems.

In bankruptcy problems, if the proposer is the highest claimant, the game’s unique subgame perfect equilibrium (SPE) payoff is the nucleolus. Otherwise, there may be a continuum of SPE outcomes. Thus, the nucleolus is sharply obtained when we follow an ‘absolute priority’ method of choosing the proposer. A related requirement of absolute priority of claims (the fact that the biggest creditors have a more active role in the bankruptcy procedure) has been emphasized in the literature on law and economics.⁸ It is argued there that bigger claimants may possess informational advantages about the bankrupt estate and have greater incentives to solve matters efficiently.

This paper provides an additional rationale for the principle, by showing that it is a sufficient condition for the uniqueness of the equilibrium outcome. The intuition of this result relies upon the fact that the nucleolus of bankruptcy games is strictly monotonic in the share of the highest claimant and only weakly so in the others’. Therefore, the highest claimant is more vulnerable to marginal shrinkages of the estate. When a smaller claimant is the proposer, equilibrium outcomes different from the nucleolus can be supported in which higher creditors are forced to accept lower shares. When the proposer is the highest claimant, she prevents this from happening.

Unlike that of bankruptcy problems, the prenucleolus of surplus sharing problems prescribes the equal sharing of the surplus and is, thus, strictly monotonic in every component. The consequence for the strategic bargaining game is that a unique SPE outcome is obtained regardless of who is responsible for making the proposal.

⁷ For other interesting connections between bankruptcy and bargaining, see Chun and Thomson (1992), Dagan and Volij (1993) and Sonn (1992).

⁸ See Aghion et al. (1992).

2. Bankruptcy problems, CG consistency and the nucleolus

The following story, reported in Aumann and Maschler (1985), is taken from the Talmud. A man has died leaving an estate of size E . The man had three creditors when he died. Each creditor has a claim on the estate. The three claims are $d_1 = 300$, $d_2 = 200$, $d_3 = 100$. A bankruptcy problem arises when there is not enough money to pay these claims, that is, when $E < 600$. Then, it is not clear how to divide the amount E .

By using three examples, the Talmud gives some surprising recommendations. If $E = 100$, the recommended split is (33.3, 33.3, 33.3). If $E = 300$, the recommendation is (150, 100, 50). However, if $E = 200$, the Talmud recommends (75, 75, 50). Different scholars have attempted to interpret these figures. When $E = 100$, because there is very little money to go around, it is reasonable to adopt the equal division criterion. There is barely enough money to pay the smallest claim. When $E = 300$, the estate is exactly half the sum of the claims. Proportional division seems to make sense. Each creditor receives half her claim. The case of $E = 200$ is a different matter. After many centuries of scholarly attempts to understand these figures, the best ‘explanation’ prior to 1985 was an error in transcription. No general principle seemed to underlie the recommended solutions in these examples.

Let us look at another class of problems explored in the Talmud. Unlike bankruptcy problems (that can be multilateral), the ‘contested garment’ problems are bilateral. Two men argue about which of them owns a garment. One claims it all, the other claims half. To solve these problems, the principle that the Talmud uses is the equal division of the contested amount. That is, the part of the garment that each party ‘concedes’ is awarded to the other party. The rest is claimed by both parties and it must be split equally. In the above example, the second person concedes half the garment and hence, the recommended solution is (0.75, 0.25). Notice that this is different from equal (0.5, 0.5) or proportional (0.66, 0.33) division.

As a function of the estate and for fixed claims, the CG rule generalizes to the following principle. For low enough estates, the solution is equal division. This is maintained until the person with the lowest claim has received half his claim ($E = 0.5$ in the example above). From that point, each additional dollar is awarded to the other claimant. This continues until the biggest claimant’s loss is exactly half the smallest creditor’s claim ($E = 1$ in the example). From then on, each additional dollar is again split equally. Since in the example $E = 1$, the solution (0.75, 0.25) arises. We will refer to this solution as the CG rule.

Remark. Notice that the CG rule is strictly monotonic (as a function of the estate) in the highest claimant’s component, and only weakly monotonic in the other.

Formally, a bankruptcy problem is a pair $(E; d)$, where $d = (d_1, d_2, \dots, d_n)$,

$0 \leq d_n \leq \dots \leq d_2 \leq d_1$, $0 \leq E \leq d_1 + d_2 + \dots + d_n$. E is the estate and d is the vector of n claims on it (there are n creditors, each of whom holds one claim).

We adopt the standard notation. Given a vector in R^n , say d , and a set of creditors, say S , we denote by $d(S)$ the sum over S of the components of d .

The set of allocations of the bankruptcy problem $(E; d)$ is the set $\{x \in R^n$ such that $x(N) = E$, $0 \leq x_i \leq d_i$ for all $i \in N\}$.

A solution to the bankruptcy problem is a function that assigns an allocation to every bankruptcy problem.

A solution $x = f(E; d)$ is CG consistent if for all bankruptcy problems $(E; d)$ and for all $i \neq j$, the division of $x_i + x_j$ prescribed by the CG rule for claims (d_i, d_j) is (x_i, x_j) . This is the notion of consistency on which the non-cooperative game of this paper is based. Notice that it is a notion of bilateral consistency, that is, it requires invariance of the solution only on bilateral reduced problems.

For any bankruptcy problem $(E; d)$, a coalitional game $(N; v)$ can be defined.⁹ In it, $N = \{1, 2, \dots, n\}$ is the set of creditors, and the coalitional function v assigns a real number $v(S)$ to each coalition $S: v(S) = \max\{0, E - d(N \setminus S)\}$. That is, the power of each coalition is defined to be what it can get without going to court (either what is left of the estate after paying the claims of the complementary coalition, or zero if that remainder is negative).

The nucleolus of a game in coalitional form is a solution concept that was introduced in Schmeidler (1969).¹⁰ It maximizes the ‘welfare’ of worst-off coalitions in a lexicographic manner. Although its definition is cumbersome, the nucleolus possesses interesting properties that include non-emptiness, uniqueness, core selection and consistency.¹¹

Returning to bankruptcy problems, it is easy to show that the game $(N; v)$ generated by the problem $(E; d)$ is convex and its core consists of all the imputations.

For the class of bankruptcy problems, the following results are known:¹²

Theorem 1. *There exists a unique CG consistent solution to a bankruptcy problem.*

The verbal description of this solution is an extension of the CG rule for n creditors.

The reader may check that the numbers of the Talmud are CG consistent.

⁹ This game was first introduced in O’Neill (1982) and subsequently used in Aumann and Maschler (1985).

¹⁰ To be precise, we should talk about the prenucleolus. In this section, I will keep using the word ‘nucleolus’ because, for the bankruptcy games defined above, the prenucleolus coincides with the nucleolus.

¹¹ I refer the reader to Maschler (1992) for the definition and properties of the nucleolus.

¹² See Aumann and Maschler (1985).

*Theorem 2. The CG consistent solution of a bankruptcy problem is the nucleolus of the corresponding game.*¹³

The next section will describe a non-cooperative negotiation procedure that leads to the nucleolus of ‘bankruptcy games’.

3. A simple bargaining game

In this section, I present a simple bargaining game that is ‘consistent’ with the CG rule for bilateral problems. It is a finite game that can be easily solved. The mechanism relaxes the assumption that all agreements must be achieved unanimously. Instead, the options of ‘partial agreements’ and ‘renegotiation’ are introduced when some responders disagree with a proposal.

The game contains subgames with bilateral negotiations where the CG rule plays a major role. Alternatively, the CG rule can be obtained as the equilibrium outcome of a random dictator bargaining game.

Namely, given the bilateral problem $(E; (d_1, d_2))$, the following game yields the CG rule. One of the creditors is chosen arbitrarily to make a proposal x , $x_1 + x_2 = E$, $0 \leq x_i \leq d_i$. If accepted, it is implemented; if rejected, a fair coin is tossed appointing the dictator. The outcome of this game for creditor i ($i = 1, 2$) is $1/2 \min\{E, d_i\} + 1/2 \max\{0, E - d_j\} = \max\{0, E - d_j\} + 1/2 \min\{E, d_i\} - 1/2 \max\{0, E - d_j\} = \max\{0, E - d_j\} + 1/2[E - \max\{0, E - d_j\} - \max\{0, E - d_j\}]$, that is, claimant i gets what j concedes plus half of the contested amount. This is precisely the solution recommended in the Talmud.¹⁴ To save notation in the game that follows, we will use the CG rule in the bilateral subgames, but the reader should keep in mind that in its place we could have the above non-cooperative game.

The mechanism g_i is a finite horizon game. There are $n \geq 2$ creditors and the underlying structure is a bankruptcy problem. Creditor i makes a proposal x , $x(N) = E$, $0 \leq x \leq d$. Responders reply sequentially according to an arbitrary protocol.¹⁵ If all agree with x , x is implemented and the game ends. Otherwise, the acceptors receive their components of x and each rejector j renegotiates her share with the proposer i following the protocol and using (as an outside option)

¹³ CG consistency is weaker than the consistency of the prenucleolus for general TU games (Sobolev, 1975) because it imposes requirements only on two-player reduced games.

¹⁴ The recommendation in the Talmud appears then naturally as awarding a creditor the average of the best and the worst possible outcome in a bankruptcy problem.

¹⁵ Responses can be simultaneous and the same results are obtained.

the CG rule over the ‘bilateral pie’ defined below. Hence, the final shares are z with

$$\begin{aligned} z_j &= x_j \text{ for every acceptor } j; \\ z_j &= \text{CG}_j(y^k + x_j; (d_i, d_j)) \text{ for every rejector } j; \text{ and} \\ z_i &= E - z(N \setminus \{i\}), \end{aligned}$$

where y^k is defined recursively. It is the share that i is left with after renegotiating with k creditors. Of course, before the first renegotiation, $y^0 = x_i$. Then, the shares z are awarded and the game ends.

The game g_i should be interpreted in the following context. Creditor i is the administrator of the estate. She proposes to solve the problem through decentralized negotiations, in which the CG bilateral court stands as an outside option for the creditors. While she has enough power to award shares, she must be receptive to complaints that might otherwise end up in court.

The negotiations have the feature that the problem gets broken into smaller pieces, which means that the individual veto from a claimant cannot take all the creditors to court.

The following is our main result:

Theorem 3. Let $(E; d)$ be a bankruptcy problem such that $d_1 \geq d_2 \geq \dots \geq d_n$. The game g_1 has a unique subgame perfect equilibrium outcome x^* which is the nucleolus of the game $(N; v)$ associated with the bankruptcy problem.

Proof. First we show uniqueness in three steps.

Step 1. Creditor 1 can guarantee a payoff at least equal to x_1^* . She can do so by offering x_j^* to every $j \neq 1$. An easy backwards induction argument shows that, regardless of the profile of responses, creditor 1’s payoff is x_1^* .

Step 2. Creditor 1’s payoff in any SPE must be x_1^* . We begin by showing a preliminary step. Let σ be a SPE with outcome z . Let x be the equilibrium offer and suppose that r creditors reject it according to σ .

It must be the case that for all $j \neq 1$, $z_j \geq \text{CG}_j(z_1 + z_j; (d_1, d_j))$. To see this, notice that in equilibrium $x_1 = y^0 \geq y^1 \geq \dots \geq y^r = z_1$ and consider the following cases:

Case 1. Creditor j accepts x according to σ . Suppose that there are k rejectors of x before j in the protocol. Then, $z_j = x_j$ and $x_j \geq \text{CG}_j(y^k + x_j; (d_1, d_j)) \geq \text{CG}_j(z_1 + x_j; (d_1, d_j))$, where the first inequality follows from j ’s acceptance and the second from the monotonicity of the CG rule.

Case 2. Creditor j rejects x according to σ . Again let k be the number of renegotiating creditors before j . Then,

$$\begin{aligned} z_j &= \text{CG}_j(y^k + x_j; (d_1, d_j)) = \text{CG}_j(y^{k+1} + z_j; (d_1, d_j)) \\ &\geq \text{CG}_j(y^r + z_j; (d_1, d_j)). \end{aligned}$$

Thus, as claimed, we have shown that

$$\text{for all } j \neq 1, z_j \geq \text{CG}_j(z_1 + z_j; (d_1, d_j)) \quad (1)$$

Next we show that $z_1 \leq x_1^*$. Suppose not: $z_1 > x_1^*$. Then, there exists j such that $z_j < x_j^*$. This implies that either

(I) $z_1 + z_j \geq x_1^* + x_j^*$, which in turn implies (by consistency and monotonicity of the CG rule) that $\text{CG}_j(z_1 + z_j; (d_1, d_j)) \geq \text{CG}_j(x_1^* + x_j^*; (d_1, d_j)) = x_j^*$, which together with (1) yields a contradiction, or

(II) $z_1 + z_j < x_1^* + x_j^*$. In this case, by consistency and monotonicity of the CG rule, we have $\text{CG}_j(z_1 + z_j; (d_1, d_j)) \leq \text{CG}_j(x_1^* + x_j^*; (d_1, d_j)) = x_j^* < z_j$, which (by efficiency of the CG allocation) contradicts (1).

The previous arguments, coupled with Step 1, prove Step 2.

Step 3. If z is an SPE outcome, $z = x^*$. Suppose not. Then there exists j such that $z_j < x_j^*$. Recall that $z_1 = x_1^*$. Then, by the strict monotonicity of the CG rule in the share of the highest claimant, it follows that $z_1 = \text{CG}_1(x_1^* + x_j^*; (d_1, d_j)) > \text{CG}_1(z_1 + z_j; (d_1, d_j))$, which (by efficiency of the CG rule) would violate (1). Therefore, $z_j \geq x_j^*$ for all $j \neq 1$. Since $z(N) = E = x^*(N)$, $z = x^*$.

On the other hand, existence is easily shown. We claim that the following strategies form an SPE: creditor 1 proposes x^* . For any proposal x , we can write the responses recursively (starting with the first responder) as follows: every responder j accepts x if and only if $x_j \geq \text{CG}_j(y^k + x_j; (d_1, d_j))$ where y^k is the amount defined above when k creditors have rejected x before creditor j . Clearly, the responders are at a best response. And so is creditor 1 because after any offer x and given these responses, the final split z satisfies (1) and, therefore, by the last argument in Step 2, $z_1 \leq x_1^*$. \square

If $n > 2$ and $i \neq 1$, although it can be shown that $z_i = x_i^*$ in every SPE of g_i (Steps 1 and 2), the game g_j may have a continuum of SPE outcomes (Step 3 need not go through), as shown by the following example.

Example. Let $(E; d) = (350; (300, 200, 100))$. For this problem, $x^* = (200, 100, 50)$. With the same strategies as those in the proof of Theorem 3, it can be checked that

- (1) x^* is the unique SPE outcome of g_1 ;
- (2) $(a, 100, 250 - a)$ for $150 \leq a \leq 200$ are SPE outcomes of g_2 ; and
- (3) $(a, 300 - a, 50)$ for $100 \leq a \leq 250$ are SPE outcomes of g_3 .

Note that if the proposer is a smaller creditor, the 'size' of the set of SPE increases. This is because higher claimants than the proposer are among the

responders, and they are ‘vulnerable’ in the bilateral negotiations with respect to the CG allocation. For example, all the SPE outcomes of g_2 (except x^*) are such that creditor 1 is punished with respect to the CG allocation; in g_3 , on the other hand, there are equilibria in which either 1 or 2 are punished. Therefore, ‘absolute priority’ of claimants in the choice of the proposer is essential for uniqueness of the equilibrium outcome. In contrast, notice that the result is completely insensitive to the protocol of responses.

4. Surplus sharing problems

Thus far we have studied bankruptcy (or negative surplus sharing) problems. This section is devoted to positive surplus sharing (or simply surplus sharing) problems. Suppose there are n agents engaged in an investment project of total return E . Each of them contributed an amount d_i to the project.

Formally, a surplus sharing problem is a pair $(E; d)$, where $d = (d_1, d_2, \dots, d_n)$ is the vector of contributions to the project with return E , such that $0 \leq d_n \leq \dots \leq d_2 \leq d_1$, $0 \leq d_1 + d_2 + \dots + d_n \leq E$.

The set of allocations of the surplus sharing problem $(E; d)$ is $\{x \in R^n$ such that $x(N) = E$, $d_i \leq x_i$ for all $i \in N\}$.

We can associate the following coalitional game to every surplus sharing problem $(E; d)$: the game $(N; v)$ is defined by $v(S) = \max\{0, E - d(N \setminus S)\} = E - d(N \setminus S)$ for every coalition S . The worth of a coalition is what its members can get as dictators, paying to the complement only their contributions.

The game $(N; v)$ is subadditive and its core is empty. Moreover, it can be easily checked that its prenucleolus awards to player i the amount $x_i = d_i + [E - d(N)]/n$. This extends the Talmudic principle to these problems: the prenucleolus awards the equal split of the contested amount, which in this case is the surplus. Notice that it is not individually rational [$\text{Pnc}_i(N; v) < v(\{i\})$].

Of course, the prenucleolus satisfies consistency and in particular bilateral consistency. That is, if we call CG’ to the bilateral rule that splits the surplus equally, the prenucleolus is CG’-consistent. However, notice that unlike in bankruptcy, the prenucleolus of surplus sharing games is strictly monotonic in all its components. Therefore, with the same game g_i defined in Section 3, we have our next result.

Theorem 4. Let $(E; d)$ be a surplus sharing problem. Then, for any $i \in N$, the game g_i has a unique subgame perfect equilibrium outcome x^* , which is the prenucleolus of the game $(N; v)$ associated with the surplus sharing problem.

Proof. It is identical to the proof of Theorem 3, after realizing that the CG’ rule is strictly monotonic in all its components. \square

5. Concluding remarks

The results of this paper have once more shown the relevance of consistency and monotonicity of axiomatic solutions to find connections between the two counterparts of game theory. The prenucleolus is not an easy solution to understand. In particular, it is not monotonic in the general class of TU games.

Monotonicity is satisfied by the prenucleolus of a surplus sharing game and is, coupled with consistency, at the heart of its non-cooperative interpretation.

The prenucleolus seems to be more difficult to implement than the Shapley value. In Gul (1989) and Hart and Mas-Colell (1992), the proposer can be chosen at random to obtain the Shapley value. In Serrano (1993), only one protocol (the one that orders players according to their power in the coalitional function) yields the prenucleolus. A similar conclusion has been reached in this paper: in general, only the game where the most powerful player is the proposer yields the prenucleolus. Strikingly enough, the choice of the proposer does not matter when prenucleolus and Shapley value coincide (as is the case in surplus sharing problems and in bankruptcy when the estate is very small [$E < nd_n/2$]).

The rules of the mechanism presented can be readily made independent of the claims and thus help an incompletely informed court. Suppose that each claim is backed by a body of evidence (say, a number of witnesses that can show up to testify). In this case, the only possible way to misrepresent the claim is by under-reporting but never by over-reporting it (a creditor may choose to call a subset of the witnesses, but she cannot fabricate new ones). If things end up in court (bilateral subgames), and since the prenucleolus payoff is strictly monotonic in a creditor's own claim, no creditor will choose to under-report it.

I wish to emphasize that this exercise represents a considerable departure from most of the implementation literature, which has concentrated on the case where feasible sets are known and preference profiles are not. In a surplus sharing problem, we study the opposite case in which the feasible set of allocations is unknown. Our closest precursor in this respect is Hurwicz et al. (1980), who study the implementation problem in exchange economies in which preferences are known, but initial endowments are not. Like here, the only possible misrepresentation is the under-reporting of the initial endowments.

However, unlike in Hurwicz et al. (1980), an agent's private information (the claim) is one-dimensional. Therefore, if verification of the claims is possible (as assumed in the previous paragraphs), the strict claim-monotonicity of the prenucleolus of these problems makes it implementable in dominant strategies. This is the reason why we have chosen to emphasize the bargaining dimension of the mechanism, rather than the implementation itself. On the other hand, it would be desirable to implement the prenucleolus of these problems when verification of the claims is not possible. But then, the very notion of mechanism is a problematic one. How is the planner supposed to check whether an outcome is feasible or not when the set of allocations will never become known to her?

Acknowledgments

I am grateful to Nir Dagan, Sergiu Hart, Harold Houba, Vijay Krishna, Andreu Mas-Colell, Michael Maschler, Eric Maskin, Amy Salsbury, Rajiv Vohra, Oscar Volij, Oved Yosha and Iñigo Zapater for helpful conversations.

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