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Welfare Analysis of a Market with Pairwise Meetings and Asymmetric Information [*]

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Summary

We study the welfare properties of a market with pairwise meetings and asymmetric information, establishing an equivalence between asymptotically ex-post individually rational and asymptotically (ex-ante, interim and ex-post) efficient equilibrium sequences.

1 Introduction

Consider a decentralized market without an auctioneer, in which prices are not called and trade takes place in pairwise meetings of buyers and sellers. Although the study of such markets has grown rapidly in recent years, their normative properties have rarely been analyzed.[1] Whenever each trader derives utility from the consumption of a single good and information is symmetric, efficiency is not an issue since the equilibrium agreement is reached without delay.[2] In a model with incomplete information on private values, Samuelson (1992) finds inefficiency caused by delay. However, he does not explore whether the inefficiency persists when time discounting is removed (there should be more delay, but it is less costly). We ask precisely this question in the common values asymmetric information framework suggested by Wolinsky (1990), finding that information revelation and efficiency are neatly related in these economies.

Suppose hereafter that information regarding the state of the world, say the quality of the good being traded, is not symmetric between traders. If the market were centralized, that is, if there were an auctioneer who calls prices, uninformed traders could learn the private information of informed traders from the market clearing price. In a decentralized market, where prices are not called, such transmission of information cannot occur.

Wolinsky constructs a decentralized model with uninformed traders on both sides of the market, in which trade takes place in pairwise meetings. Uninformed traders learn by meeting with potentially informed trading partners. The cost of learning is captured by the discount factor δ. He shows that even when the market becomes approximately frictionless (when discounting is removed) a non-negligible fraction of those who are uninformed end up transacting at a price which is not ex-post individually rational.[3] Thus, the equilibrium prices in Wolinsky's economy do not converge, as $\delta \to 1$, to prices which would have arisen in a fully revealing rational expectations equilibrium (FRREE) of the corresponding centralized economy.

Serrano and Yosha (1993) show that when information is one sided, that is, when there are uninformed traders only on one side of the market, typically all equilibrium sequences (indexed by δ) are asymptotically ex-post individually rational.[4] Thus, the equilibrium prices converge to FRREE prices.

The issue of information revelation is conceptually distinct from that of efficiency. Suppose information is one sided. Although uninformed traders do eventually learn, they do so after

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[1] Osborne and Rubinstein (1990) provide a survey of these models.

[2] Peters (1992) introduces an outside option for the traders (which can be interpreted as the consumption of a substitute good) and shows that when matching is purely random inefficiencies arise.

[3] We refer to this as absence of asymptotic ex-post individual rationality.

[4] The word typically is used because Serrano and Yosha could not rule out, for a particular region of the parameters, trade at a price which is not ex-post individually rational.

having sampled a large number of trading partners. As staying in the market is costly, the welfare gain from trade at an ex-post individually rational price could be offset by the cost of delay. Similarly, when information is two sided, the question is whether the absence of learning by the uninformed is compensated by a transfer to other traders. We establish that an equilibrium sequence is asymptotically ex-post individually rational if and only if it is asymptotically efficient (ex-ante, interim and ex-post).[5] Thus, when information is one sided, the pairwise meetings economy typically has desirable learning and efficiency properties, while the two sided information economy does not.

2 The Model

Time runs discretely from $-\infty$ to ∞. All periods are identical. In the beginning of each period M sellers and M buyers enter the market. Sellers want to sell one unit of an indivisible good, and buyers want to buy one unit of it. Each period sellers and buyers are randomly matched. Each meeting results in an agreement or a disagreement. Those who agree transact and exit the market, and those who disagree stay in the market to be matched anew. There are two states of the world. In state H buyers have a high valuation u_H for the good, and the cost of the good to the sellers c_H is also high. In state L the valuation u_L and the cost c_L are low.

When matched, traders make simultaneous announcements. Each trader can send one of two possible messages: h or l. If both traders say h they trade at price p^{hh}. If both say l they trade at price p^{ll}. If the buyer says h and the seller l they trade at price p^{hl}. Finally, if the seller says h and the buyer l there is disagreement. It is convenient to refer to messages as "tough" (h for the seller and l for the buyer) and "soft" (l for the seller and h for the buyer). It is assumed that

$$c_L < p^{ll} < u_L < p^{hl} < c_H < p^{hh} < u_H. \tag{1}$$

In state H the only ex-post individually rational price for buyers and sellers is p^{hh}. Similarly for p^{ll} in state L.

A fraction x_S of sellers and a fraction x_B of buyers who enter the market each period are informed, i.e. know the state of the world. The rest have a common prior belief $\alpha_H \in (0,1)$ which is the probability that the state of the world is H. If $x_S < 1$ and $x_B < 1$ the information structure is two sided. If $x_S = 1$, $x_B \in (0,1)$ or $x_B = 1$, $x_S \in (0,1)$ it is one sided.[6] All traders

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[5] Asymptotic efficiency means that inefficiencies can be made arbitrarily small as discounting is removed. In section 3 we provide definitions of the efficiency concepts we use, and relate them to the notions of efficiency proposed by Holmström and Myerson (1983) and used in Laffont (1985).

[6] For the case $x_S = 1$, $x_B = 0$ or $x_B = 1$, $x_S = 0$ see the remark at the end of section 3.

discount future payoffs by a constant factor $\delta \in (0,1)$. The payoff of perpetual disagreement is zero.

Since disagreement occurs only if both buyer and seller play "tough," a trader's strategy can be completely described by the number of periods during which he will play "tough." Let n_{SH} be this number for a seller who knows that the state is H. Similarly for n_{SL}, n_{BH}, and n_{BL}. Let n_B and n_S be these numbers for uninformed traders. Uninformed traders extract information from the announcement of their (potentially informed) trading partners. By playing "tough" an uninformed trader ensures that if trade occurs, it will be at a price which is advantageous for him. If trade does not occur he will learn, updating α_H. Hence, a strategy n_B (or n_S) can be interpreted as a decision to keep sampling trading partners for n_B (or n_S) periods, provided that exit has not yet occurred. The cost of sampling an additional trading partner is captured by δ.

Let S_H^h be the proportion of the total number of sellers in the market who in state H say h. Similarly for S_L^h, B_H^l, and B_L^l. Traders know these distributions of announcements amongst their trading partners in each state of the world. Let K_H and K_L be the total number of sellers (and therefore of buyers) in the market in state H and in state L. The market is said to be in steady state when these six numbers are constant through time.

Let $V_B(n; \alpha_H, S_H^h, S_L^h)$ be the expected payoff of strategy n to an uninformed buyer. Similarly for $V_S(n; \alpha_H, B_H^l, B_L^l)$. Let $V_{SH}(n; B_H^l)$ be the expected payoff of strategy n to an informed seller in state H. Similarly for $V_{SL}(n; B_L^l)$, $V_{BH}(n; S_H^h)$, and $V_{BL}(n; S_L^h)$.

The market is said to be in equilibrium if each trader maximizes his expected payoff and the market is in steady state. When information is two sided, an equilibrium is fully described by twelve numbers: n_S, n_B, n_{SH}, n_{SL}, n_{BH}, n_{BL}, S_H^h, S_L^h, B_H^l, B_L^l, K_H and K_L. The equilibria of the model are determined by six best response conditions and the following six steady state equations:

$$M = K_H(1 - S_H^h B_H^l), \tag{2}$$

$$M = K_L(1 - S_L^h B_L^l), \tag{3}$$

$$K_H(1 - S_H^h) = M[x_S (B_H^l)^{n_{SH}} + (1 - x_S)(B_H^l)^{n_S}], \tag{4}$$

$$K_H(1 - B_H^l) = M[x_B (S_H^h)^{n_{BH}} + (1 - x_B)(S_H^h)^{n_B}], \tag{5}$$

$$K_L(1 - S_L^h) = M[x_S (B_L^l)^{n_{SL}} + (1 - x_S)(B_L^l)^{n_S}], \tag{6}$$

$$K_L(1 - B_L^l) = M[x_B (S_L^h)^{n_{BL}} + (1 - x_B)(S_L^h)^{n_B}], \tag{7}$$

When information is one sided, say when all sellers are informed, the terms involving n_S in equations (4) and (6), and the corresponding best response condition are dropped.[7] Also, equation (5) turns out to be redundant (it is identical to equation (2); Serrano and Yosha 1993, pp.484-5.) In order to complete the system, equation (5) is replaced by

$$B_H^l = \frac{(1-x_B)n_B}{(1-x_B)(n_B+1)+x_B}. \tag{8}$$

The analysis is performed in terms of the fraction of transacting uninformed buyers who in state L end up trading at a price which is not ex-post individually rational,

$$f_B = \frac{K_L(1-B_L^l)}{M(1-x_B)}, \tag{9}$$

and the fraction of transacting uninformed sellers who in state H end up trading at such a price, $f_S = \frac{K_H(1-S_H^h)}{M(1-x_S)}$.

Definition. *If along a sequence of equilibria such that* $\delta \to 1$, $\lim_{\delta\to 1} f_B = \lim_{\delta\to 1} f_S = 0$, *(or* $\lim_{\delta\to 1} f_B = 0$ *when information is one sided) we say that the sequence is asymptotically ex-post individually rational.*

Wolinsky (1990) has shown that when information is two sided, along any sequence of equilibria which has a limit, either $\lim_{\delta\to 1} f_B > 0$ or $\lim_{\delta\to 1} f_S > 0$. Serrano and Yosha (1993) have shown that when information is one sided, there always exist equilibrium sequences such that $\lim_{\delta\to 1} f_B = 0$. Furthermore, for an open region of the parameters of the model, all equilibrium sequences satisfy $\lim_{\delta\to 1} f_B = 0$.

3 Welfare Analysis

As the equilibria of the model involve costly delay, they are inefficient in the sense that a planner can induce a Pareto improving allocation which involves no delay. We propose such an allocation for the ex-ante stage (before informed traders learn the state of the world), the interim stage (after informed traders learn the state of the world), and the ex-post stage (after uninformed traders learn the state of the world). Furthermore, we show that this allocation is itself ex-ante, interim, and ex-post efficient. Most of the section is devoted to the question whether the inefficiency of equilibrium sequences vanishes as $\delta \to 1$. This is done by checking whether the limit payoffs along the sequences converge to the payoffs obtained in the above mentioned efficient allocation. We turn to the formal analysis.

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[7] From now on, whenever we refer to one sided information we have in mind this case. The case where all buyers are informed is symmetric.

Consider the following price function: p^{ll} if the state of the world is L, p^{hh} if the state of the world is H. Once the state is revealed through this function, all agents are willing to trade in the respective states of the world at the above prices, so the market clears. If the planner knows the state of the world, he can make this price function known to traders as they arrive to market, and then call the market clearing price. The market will then be in a FRREE.[8]

The FRREE allocation is ex-ante, interim, and ex-post efficient. The notion of efficiency we use is that of Holmström and Myerson (1983), who argue that the informational constraints of the planner should be taken into consideration in the definition of efficiency.[9] Hence, attention should be restricted to outcomes which arise as equilibria of incentive compatible mechanisms. Since the FRREE is an equilibrium, it is incentive compatible. We are therefore establishing ex-ante, interim, and ex-post *incentive* efficiency of the FRREE. We begin with interim efficiency. Suppose information is two sided. In the FRREE traders obtain the following interim payoffs: $V_{SH} = p^{hh} - c_H$, $V_{SL} = p^{ll} - c_L$, $V_{BH} = u_H - p^{hh}$, $V_{BL} = u_L - p^{ll}$, $V_S = \alpha_H(p^{hh} - c_H) + (1 - \alpha_H)(p^{ll} - c_L)$, $V_B = \alpha_H(u_H - p^{hh}) + (1 - \alpha_H)(u_L - p^{ll})$. We shall refer to the various kinds of traders as SH, SL, S, and so forth. Consider an SL trader. We show that this trader cannot be made strictly better off without another trader becoming strictly worse off. Assume that an SL trader is made better off by allowing him to sell at a price $p_1 > p^{ll}$. If he sells to a BL trader, this buyer is made strictly worse off. Thus he must be selling to a B trader. For the B trader not to be made strictly worse off when buying at p_1 in state L, he must purchase the good in state H for a price $p_2 < p^{hh}$. If he purchases from an SH trader, then this seller is made strictly worse off. Thus, the B trader must be purchasing from an S trader. For this S trader not to be made strictly worse off, he must sell the good in state L for a price $p_3 > p^{ll}$, and the buyer must be a B trader, and so on and so forth. As the number of traders in the market is finite, some trader, B or S, will eventually have to transact with an informed trader (SH or BL respectively) thereby making him strictly worse off. A similar argument can be constructed for any other kind of trader, when information is two sided as well as one sided.[10] Since ex-ante payoffs are wieghted averages of interim payoffs, a similar argument can be made to establish ex-ante efficiency of the FRREE. Finally, ex-post efficiency is implied by interim efficiency.

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[8] This procedure follows Wolinsky (1990, p.16).

[9] See also Laffont (1985).

[10] The argument makes use of the assumption that M is finite. This assumption is by no means necessary. Assume that an infinite number of buyers and sellers enter the market each period, and suppose that we want to make a positive fraction of the SL traders, say $\lambda(1 - x_S)$, $\lambda > 0$, better off by allowing them to sell at a price $p_1 > p^{ll}$. By an analogous reasoning, these traders must be selling to $\lambda(1 - x_S)$ uninformed buyers, who in turn must purchase the good in state H for a price $p_2 < p^{hh}$ from $\lambda(1 - x_S)$ uninformed sellers, and so on and so forth. As $\lambda > 0$, some traders, B or S, will eventually have to transact with a positive fraction of the informed traders population, making them strictly worse off.

Definition. *A sequence of equilibria is asymptotically ex-ante/interim/ex-post efficient if its limit payoffs, as $\delta \to 1$, coincides with the payoffs of an ex-ante/interim/ex-post efficient allocation.*

The meaning of this definition is that the limits of sequences of asymptotically efficient equilibria exhibit trade by all agents, and negligible costs of delay. We compare the limit payoffs of equilibrium sequences to those of the FRREE. We find that along any sequence of equilibria, the payoffs to all traders are strictly less that those of the FRREE, confirming the intuition that costly delay entails inefficiencies (see the proof of the Proposition below). The Proposition focuses on the limiting behavior of equilibrium sequences, establishing an equivalence between asymptotically ex-post individually rational and asymptotically ex-ante, interim, and ex-post efficient equilibrium sequences. That is, only the payoffs along asymptotically ex-post individually rational equilibrium sequences converge to the FRREE payoffs.

Proposition. *(a) If a sequence of equilibria is asymptotically ex-post individually rational then it is ex-ante (and hence interim and ex-post) asymptotically efficient; (b) if a sequence of equilibria is not asymptotically ex-post individually rational then it is not ex-post (and hence not interim nor ex-ante) asymptotically efficient.*

Proof of the Proposition. We shall refer to Wolinsky (1990) as W, and to Serrano and Yosha (1993) as SY.

(a) Asymptotically ex-post individually rational equilibrium sequences occur only when information is one sided. The ex-ante payoffs of buyers and sellers are weighted averages of the interim payoffs of the various types: $x_B[\alpha_H V_{BH} + (1-\alpha_H)V_{BL}] + (1-x_B)V_B$ and $\alpha_H V_{SH} + (1-\alpha_H)V_{SL}$. We shall show that every interim type of trader achieves, in the limit, his FRREE payoff, thereby establishing asymptotic ex-ante efficiency.

Consider the informed sellers in state L. In all equilibria, either $n_{SL} = 0$ or $n_{SL} \in \{0, \ldots, \infty\}$ (W, p.9; SY, p.488). Hence, we can compute the payoff to these traders using $n_{SL} = 0$, obtaining $V_{SL} = B^l_L(p^{ll} - c_L) + (1 - B^l_L)(p^{hl} - c_L)$. Let $B(\delta)$ denote the unique positive root of $V_{SL}(1; B^l_L) - V_{SL}(0; B^l_L) = 0$, which converges to unity as $\delta \to 1$ (SY, p.486). As in any equilibrium $V_{SL}(1; B^l_L) - V_{SL}(0; B^l_L) \leq 0$ (SY, p.488), we have $B^l_L \geq B(\delta)$, implying $\lim_{\delta \to 1} B^l_L = 1$, and hence $\lim_{\delta \to 1} V_{SL} = p^{ll} - c_L$. Symmetrically for V_{BH}, $\lim_{\delta \to 1} S^h_H = 1$, yielding $\lim_{\delta \to 1} V_{BH} = u_H - p^{hh}$.

We turn to the informed buyers in state L. When information is one sided, $\lim_{\delta \to 1} f_B = 0$ implies $\lim_{\delta \to 1} S^h_L < 1$ (SY, p.492). Thus, $\lim_{\delta \to 1} V_{BL} = \lim_{\delta \to 1} \frac{1-S^h_L}{1-\delta S^h_L}(u_L - p^{ll}) = u_L - p^{ll}$.

As for the informed sellers in state H, we will need the auxiliary claim 1. First note that an alternative expression for f_B (SY, p.492) is

$$f_B = (S_L^h)^{n_B}. \tag{10}$$

Claim 1: If $\lim_{\delta\to 1} f_B = 0$ then $\lim_{\delta\to 1} \delta^{n_B} = 1$.

Proof of claim 1: As $\lim_{\delta\to 1} \delta^{n_B} = \lim_{\delta\to 1} \left\{ \left[1 - \frac{1}{1/(1-\delta)}\right]^{1/(1-\delta)} \right\}^{n_B(1-\delta)} = (1/e)^{\lim_{\delta\to 1}[n_B(1-\delta)]}$, $\lim_{\delta\to 1} \delta^{n_B} = 1$ is equivalent to $\lim_{\delta\to 1}[n_B(1-\delta)] = 0$. Using (10) we have $\lim_{\delta\to 1}[n_B(1-\delta)] = \lim_{\delta\to 1} \left[\frac{\log f_B}{\log S_L^h}(1-\delta)\right] = \frac{\lim_{\delta\to 1}[(\log f_B)(1-\delta)]}{\lim_{\delta\to 1} \log S_L^h}$. As $\lim_{\delta\to 1} S_L^h < 1$, it is sufficient to show that $\lim_{\delta\to 1}[(\log f_B)(1-\delta)] = 0$.

Using $n_{BL} = \infty$ (SY, p.484), (3), (7), (10), and $\lim_{\delta\to 1} S_L^h < 1$ once more, we have: $\lim_{\delta\to 1}[(\log f_B)(1-\delta)] = \lim_{\delta\to 1} \left\{ \left[\log \frac{1-B_L^l}{(1-x_B)(1-S_L^h B_L^l)}\right](1-\delta) \right\} = \lim_{\delta\to 1}\{[\log(1-B_L^l)](1-\delta)\} = \lim_{\delta\to 1} \frac{\log(1-B_L^l)}{1/(1-\delta)} = \lim_{\delta\to 1} \frac{-B'(\delta)/[1-B(\delta)]}{1/(1-\delta)^2} = -\lim_{\delta\to 1} B'(\delta) \lim_{\delta\to 1} \frac{(1-\delta)^2}{1-B(\delta)} = -\lim_{\delta\to 1} B'(\delta) \lim_{\delta\to 1} \frac{-2(1-\delta)}{-B'(\delta)} = -2\lim_{\delta\to 1}(1-\delta) = 0$, which completes the proof of claim 1.

Using (8) and claim 1, we have $\lim_{\delta\to 1} V_{SH} = \lim_{\delta\to 1} \frac{1-B_H^l}{1-\delta B_H^l}(p^{hh}-c_H) = \frac{1}{(1-x_B)\lim_{\delta\to 1}[n_B(1-\delta)]+1}(p^{hh}-c_H) = p^{hh} - c_H$.

We turn to the uninformed buyers, whose payoff is

$$\begin{aligned} V_B \;=\;& \alpha_H (S_H^h)^{n_B} \delta^{n_B} [S_H^h(u_H - p^{hh}) + (1-S_H^h)(u_H - p^{hl})] \\ &+ \alpha_H \tfrac{1-S_H^h}{1-\delta S_H^h}(u_H - p^{ll})[1 - \delta^{n_B}(S_H^h)^{n_B}] \\ &+ (1-\alpha_H)(S_L^h)^{n_B}\delta^{n_B}[S_L^h(u_L - p^{hh}) + (1-S_L^h)(u_L - p^{hl})] \\ &+ (1-\alpha_H)\tfrac{1-S_L^h}{1-\delta S_L^h}(u_L - p^{ll})[1 - \delta^{n_B}(S_L^h)^{n_B}]. \end{aligned} \tag{11}$$

When information is one sided $S_H^h = 1$ and $n_B < \infty$ (SY, pp.484-5), so the payoff in state H reduces to $\delta^{n_B}(u_H - p^{hh})$. Taking limits, using claim 1, (10), and $\lim_{\delta\to 1} S_L^h < 1$, we obtain $\lim_{\delta\to 1} V_B = \alpha_H(u_H - p^{hh}) + (1-\alpha_H)(u_L - p^{ll})$, which completes the proof of part (a).

(b) We show that in state L, along sequences such that $\lim_{\delta\to 1} f_B > 0$, uninformed buyers obtain limit payoffs strictly less than in the FRREE, while the limit payoffs to all other traders are bounded above by the FRREE payoffs. A symmetric proof can be constructed for state H, along sequences such that $\lim_{\delta\to 1} f_S > 0$.

For informed sellers in state L the argument is similar to that in the proof of part (a) since the inequality $B_L^l \geq B(\delta)$ continues to hold in equilibrium. Hence $\lim_{\delta\to 1} V_{SL} = p^{ll} - c_L$. Also, $\lim_{\delta\to 1} V_{BL} = \lim_{\delta\to 1} \frac{1-S_L^h}{1-\delta S_L^h}(u_L - p^{ll}) \leq u_L - p^{ll}$.

We turn to the uninformed sellers (who are present only when information is two sided).[11] Their

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[11] See footnote 4.

payoff is

$$\begin{aligned} V_S \quad = \quad & \alpha_H (B^l_H)^{n_S} \delta^{n_S} [B^l_H (p^{ll} - c_H) + (1 - B^l_H)(p^{hl} - c_H)] \\ & + \alpha_H \tfrac{1-B^l_H}{1-\delta B^l_H} (p^{hh} - c_H)[1 - \delta^{n_S} (B^l_H)^{n_S}] \\ & + (1 - \alpha_H)(B^l_L)^{n_S} \delta^{n_S} [B^l_L (p^{ll} - c_L) + (1 - B^l_L)(p^{hl} - c_L)] \\ & + (1 - \alpha_H) \tfrac{1-B^l_L}{1-\delta B^l_L} (p^{hh} - c_H)[1 - \delta^{n_S} (B^l_L)^{n_S}]. \end{aligned} \tag{12}$$

Notice that, by the Implicit Function Theorem, the function $B(\delta)$ is differentiable. In SY (p.493) it is shown that $\lim_{\delta \to 1} B'(\delta) = \frac{p^{ll} - c_L}{p^{hh} - p^{ll}}$. This implies $\lim_{\delta \to 1} \frac{1-B^l_L}{1-\delta B^l_L} \leq \lim_{\delta \to 1} \frac{1-B(\delta)}{1-\delta B(\delta)} = \lim_{\delta \to 1} \frac{B'(\delta)}{1+B'(\delta)} = \frac{p^{ll}-c_L}{p^{hh}-c_L}$. Thus, using $\lim_{\delta \to 1} B^l_L = 1$, the limit of the last two terms of (12) is less or equal to $(1-\alpha_H)(B^l_L)^{n_S}\delta^{n_S}(p^{ll} - c_L) + (1-\alpha_H)\frac{p^{ll}-c_L}{p^{hh}-c_L}(p^{hh} - c_L)[1 - \delta^{n_B}(B^l_L)^{n_S}]$ which is equal to $(1-\alpha_H)(p^{ll} - c_L)$.

Finally, for the uninformed buyers, we need the following claim.

Claim 2: If $\lim_{\delta \to 1} f_B > 0$ then either $\lim_{\delta \to 1} \delta^{n_B} > 0$ or $\lim_{\delta \to 1} \frac{1-S^h_L}{1-\delta S^h_L} = 0$.

Proof of claim 2: $\lim_{\delta \to 1} f_B > 0$ implies $\lim_{\delta \to 1} S^h_L = 1$ (W, equation (21), p.14; SY, p.492). We then have $\lim_{\delta \to 1} \frac{1-S^h_L}{1-\delta S^h_L} = \frac{\lim_{\delta \to 1}(S^h_L)'(\delta)}{1+\lim_{\delta \to 1}(S^h_L)'(\delta)}$, where $(S^h_L)'(\delta)$ denotes the derivative with respect to δ of S^h_L along the sequence.

As $\lim_{\delta \to 1} \delta^{n_B} = (1/e)^{\lim_{\delta \to 1}[n_B(1-\delta)]}$ (see the proof of claim 1), it is sufficient to show that either $\lim_{\delta \to 1}[n_B(1-\delta)] < \infty$ or $\lim_{\delta \to 1}(S^h_L)'(\delta) = 0$.

Using (10) we have $\lim_{\delta \to 1}[n_B(1-\delta)] = \lim_{\delta \to 1} \left[\frac{\log f_B}{\log S^h_L}(1-\delta)\right] = \lim_{\delta \to 1} \log f_B \lim_{\delta \to 1} \frac{1-\delta}{\log S^h_L} = -\lim_{\delta \to 1} \log f_B \lim_{\delta \to 1} \frac{1}{(S^h_L)'(\delta)/S^h_L} = -\lim_{\delta \to 1} \log f_B \frac{1}{\lim_{\delta \to 1}(S^h_L)'(\delta)}$. Thus, either $\lim_{\delta \to 1}(S^h_L)'(\delta) > 0$, in which case $\lim_{\delta \to 1}[n_B(1-\delta)] < \infty$, or $\lim_{\delta \to 1}(S^h_L)'(\delta) = 0$. This completes the proof of claim 2.

By claim 2, and recalling (10), we have that either $\lim_{\delta \to 1}(S^h_L)^{n_B}\delta^{n_B} > 0$, in which case the limit of term 3 in (11) is strictly negative, or the limit of term 4 is zero. In either case the sum of terms 3 and 4 in V_B is strictly less than $(1-\alpha_H)(u_L - p^{ll})$, which completes the proof of the Proposition.

It is easily checked that for $\delta < 1$ payoffs in any equilibrium are strictly less than the FRREE payoffs.

Remark. When the information is extreme one sided ($x_S = 1$ and $x_B = 0$), in addition to the other equilibria found in the one sided model, there is an equilibrium which is not asymptotically ex-post individually rational (see Serrano and Yosha (1993) for details). However, as can be easily checked, it is asymptotically efficient. This equilibrium constitutes the only exception to the exact correspondence between the two concepts in this model.

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