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A comment on the Nash program and the theory of implementation

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Abstract

The Nash program for cooperative games is made compatible with the framework of the theory of implementation. It is shown that the core is the only major cooperative solution that is Maskin monotonic. The mechanisms in the Nash program are adapted into our model, closer to that of the theory of implementation. © 1997 Elsevier Science S.A.

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1. Introduction

This note attempts to clarify the role of the mechanisms used in the Nash program for cooperative games. Specifically, it argues that these mechanisms (appropriately adapted) may be useful to an incompletely informed designer, thereby making them part of the theory of implementation of social choice correspondences.

The Nash program and the abstract theory of implementation are often regarded as unrelated research agendas. Indeed, their goals are quite different: while the former attempts to gain additional support for cooperative solutions based on the specification of certain non-cooperative games, the latter tries to help an incompletely informed designer implement certain desirable outcomes. However, it is misleading to think that their methodologies cannot be reconciled. A common criticism that is raised against the mechanisms in the Nash program is that they are not performing real 'implementations' since their rules depend on the data of the underlying problem (say, the characteristic function) that the designer is not supposed to know.[1]

We will provide a novel interpretation of the characteristic function, under which the property of monotonicity due to Maskin (1977) can be readily checked. It is interesting to note that the core is the

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[1] One interpretation is that these are problems of feasible implementation in the sense of Hurwicz et al. (1995). According to this approach, the mechanism must depend on the underlying environment and a stage of verification of announcements must be attached to the game form. For the most part, we will get around this interpretation (see also Serrano and Vohra, 1997).

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only major cooperative solution that satisfies Maskin monotonicity. Thus, the implementation of most cooperative solutions needs to rely on refinements of the Nash equilibrium concept (like most papers in the Nash program do). Our interpretation will also serve to transform the mechanisms in the Nash program into standard game forms of the theory of implementation.

2. The model

Our basic model will be that of a game in characteristic or coalitional form $(N;V)$, where $N=\{1,2,\ldots,n\}$ is a finite set of players and V, the characteristic function, assigns to each coalition S a set $V(S)\subset\mathbb{R}^s$, where $s=|S|$. The set $V(S)$ is usually interpreted as the set of feasible payoffs for coalition S.

We shall assume that, underlying the characteristic function, there is a production economy with k commodities (inputs and outputs). The set of physical allocations of the economy is $\mathbb{R}^k$. As for preferences, let $u_i(x_i y_i)$: $\mathbb{R}^k\to\mathbb{R}$ be a standard utility function, where $(x_i y_i)$ is an output–input vector assigned to agent i. In addition, we endow each agent $i\in N$ with an extended utility function $U_i[S,(x_i y_i)_{i\in N}]:C_i\times\mathbb{R}^{kn}\to\mathbb{R}$, where $C_i\subset 2^N$ is the set of coalitions that contain i. This utility function is such that:

$$U_i[S,(x_i,y_i)_{i\in N}]=u_i(x_i,y_i)\text{ if and only if }u_i(x_i,y_i)_{i\in S}\in V(S),\text{ and }U_i[S,(x_i,y_i)_{i\in N}]=-\infty\text{ otherwise}^2.$$

The productive activity in the economy takes place under a coalition structure, i.e., a partition of the set N. In the above interpretation every allocation is 'feasible' for every coalition. However, given the exogenous conditions of the economy (legal system, budget constraints with the outside world, and so on) certain allocations [those that are not produced with the technology of coalition S and are therefore outside of $V(S)$] are considered very bad outcomes for the members of S. That is, stealing or borrowing from outside of the coalition have prohibitively high costs for the agents in the coalition.

Next we formulate the implementation problem. We shall assume that both the set of agents N and the technology available to each coalition S are known to the designer, who, however, does not know the characteristic function V: the agents' true preference profile $U=(U_1U_2,\ldots,U_n)$ is the unknown to her. Suppose the designer wishes to implement a certain social choice correspondence (cooperative solution concept, in this context) $f(V)$ such that $U^{-1}[f(V)]=F(U)$. Notice that f lives in utility space, while F lives in the physical allocation space. To perform this task, given her incomplete information, the designer can impose on the set of agents N a mechanism or game form $G=[(M_i)_{i\in N},g]$, where M_i is agent i's set of allowable messages or strategies and $g:\prod_{i\in N}M_i\to\mathbb{R}^{kn}$ is an outcome function that assigns an allocation to each n-tuple of messages. Agents will then take the mechanism G as given and behave strategically within it. As usual, the designer is assumed to be able to enforce the allocations prescribed by the outcome function: in particular, if the outcome is not in $V(N)$, i.e., if some stealing is involved apart from normal production, she has the power to impose the 'death-all-

[2] Punishments to agents in this case can sometimes be made arbitrarily small, as in Serrano and Vohra (1997). We shall assume in this note that the technology is known to the designer. However, this formulation is more flexible and can be used even if the feasible set is unknown.

around' outcome. Alternatively, since she knows the technology available to each coalition, she can restrict the outcome function to yield feasible allocations.

We shall concentrate on the equilibria of G, under the assumption that rational strategic agents will end up playing an equilibrium $\boldsymbol{m}^* = (m_1^*, m_2^*, \ldots, m_n^*)$. We say that the mechanism G implements the social choice correspondence F over a certain domain of preferences if for all preference profiles $\boldsymbol{U}$ in the domain, we have that $g(\boldsymbol{m}^*(\boldsymbol{U})) = F(\boldsymbol{U})$, where $g(\boldsymbol{m}^*(\boldsymbol{U}))$ denotes the set of allocations obtained if the equilibria $\boldsymbol{m}^*$ of G are played when the true preference profile is $\boldsymbol{U}$.[3]

3. Maskin monotonicity in the model

Maskin (1977) shows that a necessary condition for implementability in Nash equilibrium of a social choice correspondence is the property of monotonicity. He also shows that in environments with at least one private good, monotonicity is also sufficient whenever there are at least three agents. We shall next adapt the concept of monotonicity to our model.

Denote by $L_i\{U_i[S,(x_i y_i)_{i\in N}]\}$ the lower contour set of U_i at a given point, i.e., the set of coalition-allocations that give agent i a utility no greater than a given one. In our context, we say that a correspondence $F(\boldsymbol{U})$ is monotonic if for all $(x_i y_i)_{i\in N} \in F(\boldsymbol{U})$ and for all allowable profiles $\boldsymbol{U}$ and $\boldsymbol{U}'$ such that for all $i \in N$

$$U_i[N,(x_i,y_i)_{i\in N}] = U_i'[N,(x_i,y_i)_{i\in N}] \tag{1}$$

and

$$L_i\{U_i[N,(x_i,y_i)_{i\in N}]\} \subset L_i\{U_i'[N,(x_i,y_i)_{i\in N}]\}, \tag{2}$$

we have that $(x_i y_i)_{i\in N} \in F(\boldsymbol{U}')$.[4]

That is, U_i and U_i' are two utility functions that represent a change in preferences for agent i. This change is such that the utility level at the given coalition-allocation is the same, but more coalition allocation pairs have been thrown into the lower contour set. The correspondence is monotonic if, after such a change in preferences, the given allocation remains in the correspondence. To check for monotonicity, the change in preferences from $\boldsymbol{U}$ to $\boldsymbol{U}'$ can be seen as a shrinking of the sets $V(S)$ such that the candidate payoff vector in the correspondence f remains in $V(N)$.

It is easy to see that the core satisfies monotonicity: if an allocation is in the core, it remains there when the sets $V(S)$ become smaller. However, all the other major cooperative solutions are not monotonic. The Shapley value, the nucleolus, the kernel, the Nash and the Kalai-Smorodinsky bargaining solutions are shown to be non-monotonic in Example 1. Example 2 shows that the different versions of the bargaining set are not monotonic either.

[3] See Maskin (1985); Moore (1992) for two excellent surveys on the subject of implementation, and Abreu and Matsushima (1992) as an example of the recent successes of the theory when 'virtual' or approximate implementation is allowed.

[4] This definition has been made for cooperative solution concepts where the coalition structure is the grand coalition N. It can be readily modified for general coalition structures. Also, the requirement in Eq. (1) is not cardinal: since strategies in the mechanisms may induce lotteries, the theory uses preferences representable by a utility function and all its positive affine transformations.

Example 1: The Shapley value, the nucleolus, the kernel and the Nash and Kalai-Smorodinsky bargaining solutions are not monotonic. Let $N=\{1,2\}$. Suppose that we are in a production economy with one input y and one output x. The technology is described by the production function $x=y$ when $y \leq 1$, while $x=1$ otherwise. Suppose utility is linear and depends only on output.

Under the profile $\boldsymbol{U}$ the technology belongs to agent 1 and hence it can be used by the coalitions $\{1\}$ and $\{1,2\}$. In contrast, agent 2 by himself cannot produce anything. This economy can be represented by the following characteristic function:

$$V(1,2)=\{\boldsymbol{u}\in\mathbb{R}^2_+:u_1+u_2\leq 1\},\ V(1)=\{u_1\in\mathbb{R}_+:u_1\leq 1\},\ \text{and}\ V(2)=\{u_2\in\mathbb{R}_+:u_2\leq 0\}.$$

The Shapley value, the nucleolus, the kernel and the Nash and Kalai-Smorodinsky bargaining solutions prescribe a unique point in utility space: (1,0).

Suppose that the assignment of property rights changes so that the technology can be operated only by both agents. Equivalently, suppose the preferences of agent 1 change in such a way that he enjoys the output only in the company of agent 2. That is, we are in the presence of a new profile $\boldsymbol{U}'$ where every positive production of output by agent 1 alone has moved to his lower contour set of the solution point (1,0), i.e., to produce by himself has become 'illegal' and it gives him a payoff of $-\infty$. The new situation is described by the characteristic function V':

$$V'(1,2)=\{\boldsymbol{u}\in\mathbb{R}^2_+:u_1+u_2\leq 1\},\ V'(1)=\{u_1\in\mathbb{R}_+:u_1\leq 0\},\ \text{and}\ V'(2)=\{u_2\in\mathbb{R}_+:u_2\leq 0\}.$$

All the above solutions prescribe a unique point in utility space in this case: (1/2, 1/2), which corresponds to the allocation of equal split of the output. Thus, none of these solutions are monotonic.

Example 2: The bargaining set is not monotonic. Consider the economy $E=\{N,u_i,c\}$, where $N=\{1,2,3\}$ is the set of agents, $u_i(x_i,y_i)=x_i(1-y_i)$ is agent i's utility function with $x_i \geq 0$ being the output consumed by i, and $1 \geq y_i \geq 0$ being the input supplied by agent i. The function c represents the technological possibilities of the economy $y=c(x)=x^2$: an expression of the cost of production in terms of input units. This technology must be operated by at least two agents. Each agent holds initially 1 unit of input y_i and no units of output.

With these assumptions, each $V(i)$ in the associated NTU game is

$$V(i)=\{u_i\in\mathbb{R}_+:u_i\leq 0\},\ i\in N.$$

(The reader can easily calculate the sets $V(S)$ when $s \geq 2$).

Next consider the following allocation proposed to the grand coalition: $z^*=(x_i,y_i)_{i\in N}=(1/3,1/3)_{i\in N}$. This allocation gives a utility of 2/9 to each player. Notice that although z^* is Pareto efficient for N, it is not in the core. In fact, the three two-agent coalitions can object against it. For example, coalition $\{1,2\}$ can propose the allocation $(x_i,y_i)_{i\in\{1,2\}}=((2/3)^{1/2}/2,1/3)_{i\in\{1,2\}}$, which gives each of them a utility level of $(2/3)^{1/2}/3>2/9$. However, z^* belongs to the bargaining set (e.g., Aumann and Maschler (1964); Mas-Colell (1989)) as well as to the consistent bargaining set of Dutta et al. (1989).

Consider next the economy E', which differs from E only in that the technology can be operated just by coalitions $\{1,2\}$ and by the grand coalition N (or the preferences of agents 1 and 2 change so as to enjoy an input–output combination only if they are in each other's company). That is, the new characteristic function V' is like V except the following two changes:

$$V'(1,3)=\{(u_1,u_3)\in\mathbb{R}^2_+:(u_1,u_3)\leq(0,0)\}\text{ and }V'(2,3)=\{(u_2,u_3)\in\mathbb{R}^2_+:(u_2,u_3)\leq(0,0)\}.$$

Now z^* does not belong to any of the bargaining sets, since an objection raised by coalition $\{1,2\}$ cannot be countered.

Thus, except for the core, none of the major cooperative solutions can be implemented in Nash equilibrium. In light of this, it is not surprising that the mechanisms in the Nash program are typically given by extensive forms (a notable exception is the work of Nash (1953) and his pioneering virtual implementation of the Nash solution).

Of course, the canonical mechanism of Moore and Repullo (1988) can implement all the solutions in our framework. The mechanisms in the Nash program, though, have the advantage of being more realistic: they can be conceivably thought of as playable games.

4. Mechanisms in the Nash program

In this section we briefly discuss how some of the mechanisms in the Nash program can be adapted in our model, and, therefore, how they become a part of the theory of implementation properly understood. We are not trying to be exhaustive here, although we believe that most game forms in the literature of the Nash program can be incorporated into our model. An exception is the work by Bossert and Tan (1995): their game cannot be translated into the model since it depends on the utility representation.

Under the assumption that the designer knows the technology of every coalition, the mechanisms need a very small reformulation: the proposals that players make to each other should be made in the allocation space, instead of being points in the payoff space. Some mechanisms in the Nash program already use sets of messages in the physical allocation space. One of the classic examples in this respect is the virtual implementation of the Nash solution for bilateral bargaining problems, due to Binmore et al. (1986). The feasible allocations correspond to the different splits of the physical pie at different points in time. The designer knows the size of the pie, but is uninformed about the risk attitudes of the two bargainers. Then, within the class of strictly monotone and concave utility functions, Rubinstein (1982) alternatively offers game G_q, where time preference is replaced by the probability of breakdown q, virtually implements the Nash solution in subgame perfect equilibrium, i.e., as the probability of breakdown of negotiations q goes to 0, the unique subgame perfect equilibrium outcome of each game G_q converges to the agreement specified by the Nash bargaining solution. See Krishna and Serrano (1996) for a multilateral extension.

Notice that the designer does not need to know the size of the physical pie either. If we use our model, players can make proposals that add up to more than the size of the pie. In this case, and given that the physical allocations are verifiable, the designer will be able to check whether an accepted proposal is backed by a real allocation. If it is not, she will punish the players for their lack of seriousness.

In other cases (for example, Perry and Reny, 1994; Hart and Mas-Colell, 1996), the proposals are made in utility space. We should reinterpret each proposal in these mechanisms as a physical allocation that yields the utility levels proposed in the original papers. This is the only change needed if the designer knows the technology of each coalition. As before, if the designer does not, she will be able to enforce the contract that an accepted proposal represents by verifying that it is backed with the corresponding physical allocation. Then, the stationary subgame perfect equilibria of these extended games implement the core of totally balanced TU games and the consistent values of general NTU games, respectively.[5]

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[5] Serrano (1995a), (1995b) include implementations in which the designer has partial knowledge of the characteristic function: the mechanisms in these papers depend only on the half space $V(N)$. They could also be adapted into our general model.