MARKET POWER AND INFORMATION REVELATION IN DYNAMIC TRADING

Piero Gottardi
Università di Venezia

Roberto Serrano
Brown University

Abstract
We study a strategic model of dynamic trading where agents are asymmetrically informed over common value sources of uncertainty. There is a continuum of buyers and a finite number n of sellers. All buyers are uninformed, while at least one seller is privately informed about the true state of the world. When $n = 1$, full information revelation never occurs in equilibrium and the only information transmission happens in the first period. With $n > 1$ the outcome depends both on the structure of the sellers' information and, even more importantly, on the intensity of competition allowed by the existing trading rules. When there is intense competition (absence of clienteles), information is fully and immediately revealed to the buyers in every equilibrium for n large enough, regardless of the number of informed sellers. On the other hand, for trading arrangements characterized by less intense forms of competition (presence of clienteles), for any n we always have equilibria where information is never fully revealed. Moreover, in that case, when only one seller is informed, for many parameter configurations there are no equilibria with full information revelation, even for large n. (JEL: C72, C78, D82, D83)

1. Introduction

This paper studies a strategic model of dynamic trading where (1) nonnegligible agents interact with negligible ones—a finite number of sellers serve a continuum of buyers—and (2) there is asymmetric information about the quality of the good being traded, an instance of common values uncertainty. In particular, all buyers are uninformed, while at least some sellers are informed.[1] In this situation, the "size" or market power of a seller is determined along two dimensions: his market

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E-mail addresses: Gottardi: gottardi@unive.it; Serrano: roberto_serrano@brown.edu.

1. Having no information on the buyers' side is a simplifying assumption, but it is in line with the oligopolistic literature that treats all consumers symmetrically and as having no market power. Our main focus is the study of competition among strategic traders (sellers, in our case) under asymmetric information.

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share, as in standard oligopoly, and his informational status (i.e., whether or not he is informed). Furthermore, trade extends potentially over infinitely many periods; hence, current actions may have important effects on a trader's size in the future.

When no trader has any appreciable market power, the most often used equilibrium notion in economies with common values asymmetric information is given by the rational expectations equilibrium (REE).[2] Like Walrasian equilibria, REE do not rely on a specific trading mechanism (in this sense we can say they are institution free) but simply on a price function specifying a price in each state, taken as given by all traders. The economy is "competitive", although not exactly in a standard sense. The observation of a realized price from the equilibrium price function not only defines the agents' trading possibilities in that state but also aggregates and disseminates information, allowing uninformed traders to learn about the true state. In fact, in almost every economy REE prices fully reveal all the agents' private information, without any delay. Not specifying the trading rules is no doubt an advantage in terms of robustness, but the "black box" feature of the price function leaves open an important question: How does the private information held by some traders get incorporated into the REE price function?

To address this question, we must investigate the strategic foundations of REE, which requires the specification of trading procedures where the transmission of information among the agents in the market is explicitly modeled. This issue has already received some attention in the literature, primarily within the set-up of static models and of other trading mechanisms, such as auctions or market games, for which it has been shown that the nonexclusivity of traders' information plays an important role in obtaining convergence to REE when the number of traders increases.[3]

When the analysis is extended to a dynamic trading set-up, a new trade-off arises: by revealing his private information a trader may reap some short-run gains, but this also entails a long-run cost because it means dissipating future informational rents. In evaluating this trade-off and hence determining the net benefits of revelation, the properties of the trading mechanism also matter, in addition to the information structure. In this paper we identify in particular two such properties that play an important role: (i) the *transparency* of the market (i.e., the extent to which the prices at which transactions take place in the market are observable by other traders); and (ii) the *degree of competition* allowed by the trading rules (i.e., the ease with which buyers are free, at any moment in time, to choose from whom they buy).

The importance of these factors can be seen by considering the negative results on convergence and information revelation obtained by Wolinsky (1990) and Blouin and Serrano (2001), among others, which are independent of the

2. See Radner (1982) for a survey.

3. See Section 8 for further details on the related literature.

information structure. These authors consider economies with infinitely many trading dates and where, in each period, agents chosen from a continuum are randomly matched in pairs and within each pair they bargain over the terms of trade (rejection of all offers and hence refusal to trade is always possible). In this setup it is found that even with a continuum of traders, many of them informed (and hence with no exclusivity of information), in the limit as discounting goes to zero[4] a sizeable portion of uninformed traders transact at prices such that their ex post utility is lower than the no-trade payoff.

In the trading mechanism considered in these pairwise meetings models, both the degree of transparency of the market and the degree of competition are fairly limited.

(A) No transparency: Agents have no access to public market signals/prices, nor do they have any information over the terms at which trade occurred in the past in the market (each trader observes and remembers only his own history of trades).

(B) Local monopoly: Each pairwise meeting (which can be thought of as a buyer visiting a particular store) represents a "local monopoly" in its own right, in the sense that during that period the buyer can buy the good only from that store. Of course, he can walk out and visit a different store, but this comes at a cost (which is captured by discounting). Likewise, because of the face-to-face trading interaction, a seller cannot obtain instantaneous, discrete increases in his market share.

We can view both these factors as frictions that characterize a situation in which, at the same time, revealing information across traders in the market is difficult and the short-run gains to information revelation are limited. Therefore, one can argue that the pairwise meetings technology goes too far in limiting the possibilities of information transmission among traders. It is in fact plausible to model a situation in which traders receive some market signal. In this paper we shall assume that, in each period, buyers, before they agree to buy the good, can observe all prices posted by sellers: the market is highly transparent.[5] Still, as our results will show, even if one removes friction (A) by having full observability of prices, restrictions to competition similar to (B) may suffice to prevent information revelation.

Our approach to better understand the mechanisms behind information revelation will be to develop models that "lie between" the pairwise meetings market and REE. Accordingly, we shall analyze variants of a model that differ in terms

4. Discounting is the cost of acquiring information through price sampling.

5. See Peters (1991) for a related model, where agents' matching is nonrandom because it is affected by the posted prices.

of the degree of competition allowed by the trading rules and the degree of exclusivity of the agents' information, thus identifying the conditions under which information revelation occurs; across the versions of the model, the short-run gain to information revelation varies while the long-run costs stay constant. In each case, we provide a complete characterization of equilibria in terms of their revelation and efficiency properties. The specifics of our model follow.

We study a market for an indivisible good of uncertain quality with infinitely many trading dates. All the units of the good in each period are of the same quality, either high or low. Thus, there are two states of the world: H and L. There is a continuum of buyers, all of them uninformed about the state. Among the finite number n of sellers, at least one is informed. Every period, each seller simultaneously chooses whether to post a high or a low price for the good.[6] Upon observing the prices posted in a period, every buyer can either: refuse to buy, and be present again in the market the next period; or accept to buy from one of the sellers at the price he posted, and leave the market.[7]

As a building block in our analysis, we consider first the case of an informed monopolist ($n = 1$). We find that, at equilibrium, information revelation is minimal. The monopolist reveals some information only when buyers' prior beliefs are too pessimistic about the state being H; in this case he reveals only part of his information and only in the first period. As the only seller in the market, he gains nothing by revealing any further information. Thus, continued randomization is not optimal; once the buyers are convinced that consuming the good brings them a nonnegative expected utility, the single informed trader has no further incentive to reveal his information. The presence of competitors among the sellers will change this result a great deal.

The paper proceeds to analyze the case of oligopoly under two opposite information structures.[8]

i) All sellers are informed, so that information is nonexclusive. In this case, as n grows, sellers are "informationally small" (see Gul and Postlewaite 1992) and their market power—as determined by their market share—becomes negligible.

6. The restriction of having only two possible prices strips the model of unnecessary complications (in particular, by limiting the role of the beliefs of the uninformed) and allows us to provide a characterization of all the equilibrium outcomes. As argued in Appendix B, the substance of our results remains valid even if this restriction is dropped.

7. We assume each buyer wishes to consume only one unit of the indivisible good, so that the only decision he has to make at any time is whether or not to trade (this rules out experimentation). Also, consumption takes place only after all trading rounds have been completed (or buyers leave the market after completing their transactions), so there is no communication among buyers. Thus, information is transmitted only through the price announcements of sellers.

8. Our main qualitative results extend to the case of less extreme information structures, because the relevant incentive constraints are similar.

ii) Only one seller is informed, so his information is exclusive. Here, as n grows, the informed seller remains "informationally large" while his market share decreases (with that of all sellers), so that information is the only possible source of market power.

It turns out, however, that more important qualitative differences (than those based on the information structure) can be traced to two alternative trading rules characterized by different intensities of price competition. In the first regime—the model without clienteles—each buyer is free to buy the good from the seller that is offering it at the lowest price (as in the classic Bertrand model). In the alternative model *with* clienteles, within each period any seller can reach only a fraction of all buyers (resembling less extreme forms of competition, such as Cournot or Bertrand with capacity constraints).

We show that, in the model without clienteles, information is fully and immediately revealed in every equilibrium for large enough n, no matter what is the structure of the information. The intensity of price competition generated by the lack of a clientele attached to any seller means that, by posting a low price (in State L), a seller can steal a significant part of his competitors' market share. When the number of sellers is large enough, the short-run benefits of doing this will be larger than the long-run gains of continuing to hide the private information about the good's true value. This proves to be the case not only with many informed sellers—for whom collusion to hide their information is fragile and survives with small n only—but also when there is just one informed seller who prefers to give up his exclusive information in order to increase his sales.

The situation is rather different in the model with clienteles. When all sellers are informed there is again an equilibrium with full revelation because their information is nonexclusive, but there also exists a collusive equilibrium where some information is not revealed at any date (as in the monopoly case) whatever the number n of sellers. Moreover, when only one seller is informed, for many parameter values there are no revealing equilibria even for arbitrarily large n. Price cuts in this case bring a limited increase in market share, even when the number of sellers is large: once the information is revealed by the announcement of a low price, all competitors will respond by cutting their own price next period, which will allow them (in this setup) to retain most of their market share.

The lack of information revelation obtained in the model with clienteles resembles the one created by the "local monopoly" of the pairwise meetings technology.[9] Our results show that, even allowing full transparency (removing friction (A)), a model that retains an element of friction (B)—such as the presence

9. Note that the face-to-face restriction of the pairwise meetings can be approximated as the limit of the restriction to market shares of the model with clienteles as $n \to \infty$. Of course, the cardinality of the continuum of buyers is technically larger than the limit of a sequence of a finite number of sellers, but the economic forces at work are well captured by this heuristic argument.

of clienteles—may yield that full revelation of information is not guaranteed. The results also suggest that, although both the nonexclusivity of the information and intense competition among sellers encourage information revelation, the effect of the latter proves stronger than that of the former: with clienteles we always have equilibria with no information revelation, whether or not information is exclusive. These findings shed light on some of the reasons behind strategic information revelation and contribute also to our understanding of full revelation results as in REE. The reader may find the following summary diagram instructive of how our models “fit between” the two paradigms with which our discussion began.

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Pairwise meetings	Market with clienteles	Market without clienteles	REE
no transparency	high transparency	high transparency	high transparency
“local monopoly” friction	“local monopoly” friction	no “local monopoly” friction	no “local monopoly” friction
strategic model	strategic model	strategic model	nonstrategic model
⇓	⇓	⇓	⇓
no revelation	both revelation and no revelation	full revelation	full revelation

The organization of the rest of the paper is as follows. Section 2 describes the general setup. The case of an informed monopolist is studied first, in Section 3; its findings will also be useful in the characterization of collusive equilibria in the subsequent sections. Sections 4–6 contain the results for the oligopolistic models. Assuming all sellers are informed, Section 4 studies an oligopoly without clienteles and Section 5 with clienteles. In both cases, we analyze the properties of the equilibria as the number of sellers becomes large. As already anticipated, the results differ greatly across these two sections. In Section 6 we explore the consequences of maintaining exclusive information (as under monopoly) by considering the case where there are several sellers but only one of them is informed. The welfare properties of the equilibria obtained in the various cases are discussed and compared in Section 7. Section 8 is devoted to the related literature. Since the main objective of our paper lies in the comparison of results across models, for ease of discourse we have relegated all formal proofs to Appendix A. Finally, the robustness of our results is discussed in Appendix B.

2. The Model

2.1. The Economy

Agents are of two types: buyers and sellers. There are two commodities, an indivisible consumption good, initially owned by the sellers, and a perfectly divisible commodity ("money"), initially owned by the buyers. There is a continuum of buyers, with measure normalized to 1. All buyers are identical: each of them is willing to buy at most one unit of the consumption good. There are n sellers, $1 \leq n < \infty$, and each of them can sell an arbitrarily large number of units of the consumption good. Except possibly for their information, all sellers are also identical.

Trade takes place over time and there is uncertainty over the quality of the indivisible good. Quality can be of two types, high (H) or low (L), and all units of the commodity in all periods are of the same quality. Uncertainty can thus be described by two aggregate states of the world, H and L. A buyer's valuation for one unit of the commodity in the two states is u_H and u_L, respectively. Similarly, a seller's unit reservation value is c_H in State H and c_L in State L. Therefore, the valuation of the commodity is perfectly correlated across agents; we are in a situation of common values uncertainty. We assume that

$$u_H > c_H > u_L > c_L \geq 0.$$

Note that in each state there are positive gains from trade.

Let us denote by $\alpha_0 \in (0, 1)$ the prior belief, common to all agents in the economy, that the state of the world is H. In what follows, we will always assume that buyers are uninformed over the realization of the state of the world; hence their belief, when trading begins, is given by α_0. On the other hand, we will allow for different cases with regard to the information held by sellers.

2.2. Trading Rules

There are infinitely many trading dates. At each t ($t = 1, 2, \dots$) each of the n sellers simultaneously posts a price at which he is willing to sell the consumption good (to any number of buyers). After observing the n quoted prices, each buyer chooses whether or not to trade in that period. If a buyer accepts an offer made in the period, he buys one unit at the proposed price and then exits the market (we shall specify from which seller he buys in the next paragraph). If the buyer rejects all offers, he remains in the market and can then trade at some future date. Sellers remain in the market until all buyers are served, which may never happen. There are no new entrants in the market after the first period. The quality of the good becomes known to buyers (and to uninformed sellers) only after they leave the market. The reader should then think of a case where consumption—and hence experience of the quality of the commodity being traded—takes place after the market closes (as, for instance, in the case of financial markets).

When there are several sellers ($n > 1$), we shall distinguish two possible forms of competition as follows.

1. *Model without clienteles.* Each period t, following the announcement of the prices posted by the n sellers, every buyer is free to trade with any of them. In this situation sellers compete in prices under no capacity constraints; competition among sellers is then quite intense (as in the classical model of Bertrand competition).
2. *Model with clienteles.* Every period t, a fraction of size $1/n$ of the buyers who are still in the market is randomly assigned to each seller. In that period, each buyer can only buy from the seller he is assigned to. Since buyers observe all n posted prices and always have the option of refusing to trade, the temporary segmentation of the market introduced here does not eliminate price competition among sellers. Rather, it only mitigates its intensity, since undercutting prices has a less dramatic effect on the demand for each seller (resembling models of Cournot competition or of Bertrand competition with capacity constraints). Also, information leakages from an informed seller to all uninformed traders are not prevented by the presence of clienteles.

Even though quite stylized, to fix ideas one can think of the model as portraying a specific wine crop sold in a fair by the producers. Although buyers do not know the quality of the crop, some sellers do. One can then think of clienteles as buyers being inside the kiosk of a specific wine seller. At any given moment they can buy only from that seller, although information flows across kiosks. Another example is the market of derivatives on aggregate credit risk. In both situations, all units sold are of the same quality, buyers and sellers are clearly identified, and there are reasons for trade other than differences in information; also, the actual realization of the state becomes known to uninformed buyers and sellers only after all trading rounds are completed. More generally, one may view informational asymmetries in financial markets as being typically over common values uncertainty (e.g., over the asset returns).

Both buyers and sellers evaluate payoffs from future trades according to the common discount factor $\delta \in (0, 1)$. In this paper we are concerned with the limiting results as $n \to \infty$ rather than as $\delta \to 1$. However, all our results hold for any $\delta < 1$ sufficiently large. We should think, in fact, of trading dates as taking place with a very high frequency; the reader is then invited to fix δ at some arbitrarily large value.

In most of the paper we focus our attention on the case where sellers can propose only one of two given prices, a high price p_H ($c_H < p_H < u_H$), and a low price p_L ($c_L < p_L < u_L$).[10] Note that all the fully revealing REE of the static version of this economy would have a high price in State H and a low price in State L, with prices lying in the corresponding intervals shown above. In addition, this parsimonious formulation allows us to provide a complete characterization of equilibrium outcomes and to better focus on information revelation issues. We discuss the robustness of our findings in Appendix B, where we show that most of our qualitative results remain valid also when sellers can choose from a continuum of prices.

Various cases with regard to the number of sellers in the economy and their information will be considered. This will allow us to disentangle—with regard to the revelation of information—the role of market power given by the size of the sellers' market share from that given by their private information. In each case we characterize the perfect Bayesian equilibria of the trading game just described. From now on we refer to a perfect Bayesian equilibrium simply as an equilibrium.

3. Monopoly

Consider the case where there is only one seller ($n = 1$) who is fully informed of the realization of the uncertainty.

The following observations will allow us to simplify the definition of an equilibrium. Note first that when the seller observes State H (i.e., the seller is of "type H") he will always propose p_H at any trading date. Also, a buyer who is proposed a price p_L will always accept, no matter what is his belief concerning the realization of the uncertainty.

On the other hand, the type L seller faces a nontrivial choice between offering p_H and p_L; similarly the buyers, when they are proposed p_H, must decide whether to accept or reject. In both cases we will allow for the possibility that the agents may randomize in their choice. Let $q_S(t)$ denote the probability that the seller in State L proposes p_H at date t (given histories according to which, in all previous periods, the seller always proposed p_H and at least some buyers always rejected). Similarly, let $q_B(t)$ be the probability that a typical buyer accepts, at date t, if the seller proposes p_H (given histories in which the seller always proposed p_H in the past).[11] The payoff of a buyer in a period where trade takes place, say at price p_H in State L, is $u_L - p_H$; for the seller the payoff is then $p_H - c_L$ times the fraction of buyers accepting.

10. We consider strict inequalities to rule out nonrobust indifferences.

11. Additional equilibria can be found in which all buyers play pure strategies, with a proportion $q_B(t)$ accepting the high price at date t and the rest rejecting it. These equilibria are outcome equivalent to the ones we study. This is the sense in which there is no loss of generality in restricting attention to symmetric equilibria, as we shall do.

Each period t the buyers—after observing the price proposed by the seller—will also update their beliefs over the state of the world. If the seller proposes p_L the buyers' inference is irrelevant since, as already argued, their optimal action is always to accept. Let α_t denote the buyers' belief at date t that the state of the world is H if the seller proposed p_H at t and in all past periods; such belief is updated every period, using Bayes' rule and taking into account the strategy of the type L seller:

$$\alpha_t = \frac{\alpha_{t-1}}{\alpha_{t-1} + (1-\alpha_{t-1})q_S(t)} \qquad \text{for all } t \geqslant 1. \tag{1}$$

Note that α_t is always weakly increasing with t and is strictly increasing as long as $q_S(t) < 1$.

An equilibrium of the trading game with an informed monopolist is then described by the sequences $\{q_B(t)\}_{t\geq 1}$, $\{q_S(t)\}_{t\geq 1}$, and $\{\alpha_t\}_{t\geq 1}$ such that:

(i) $\{\alpha_t\}_{t\geq 1}$ satisfies equation (1);
(ii) at each t, after every partial history in which not all buyers accepted the seller's offer in one of the previous periods, $\{q_B(\tau)\}_{\tau\geq t}$ maximizes the buyers' discounted (to that date) payoff given $\{q_S(\tau), \alpha_\tau\}_{\tau\geq t}$, and $\{q_S(\tau)\}_{\tau\geq t}$ maximizes the type L seller's discounted payoff given $\{q_B(\tau)\}_{\tau\geq t}$.

We will provide a complete characterization of the equilibria of this game. It will be shown that we never have complete revelation of the seller's information. In particular, if the prior belief α_0 that the true state of the world is H is sufficiently high, no information is ever revealed in the trading process: all equilibria exhibit perfect pooling (of the two seller types) and no delay (all trades take place at the initial date). On the other hand, if the buyers' prior belief over H is not high enough, some information is always revealed in the first trading date (because, with some positive probability, the type L seller will propose a revealing price p_L). In this case, the monopolist will reveal the minimum amount of information needed to induce buyers to trade at a high price. After that is accomplished, no further information is ever revealed; thus, we have "partial pooling". In addition there is delay: whereas p_L is always immediately accepted by buyers, when p_H is proposed the buyers will reject, in all periods, with some positive probability. Delay can even be infinite.

Define $\bar{\alpha}$ to be such that $\bar{\alpha}u_H + (1-\bar{\alpha})u_L = p_H$. That is, $\bar{\alpha}$ is the belief that makes buyers exactly indifferent between trading at p_H with probability 1 and not trading at all. Formally, we have the following:

PROPOSITION 1. *In the model with an informed monopolist, the following equilibria obtain.*
(i) No information revelation. *When $\alpha_0 \geq \bar{\alpha}$, in any equilibrium we have $q_S(t) = 1$ for all t. In particular, if $\alpha_0 > \bar{\alpha}$, the equilibrium is unique with $q_B(t) = 1$ for*

every t. On the other hand, if $\alpha_0 = \bar{\alpha}$, there are also equilibria where $q_B(t) \in (0, 1]$ for all $t \geq 1$.
(ii) Partial and immediate revelation. *When $\alpha_0 < \bar{\alpha}$, in all equilibria we have $q_S(1) \in (0, 1)$ so that $\alpha_1 = \bar{\alpha}$, $q_S(t) = 1$ for all $t > 1$, and $q_B(t) \in (0, 1]$ for all $t \geq 1$.*

It is easy to see that the configurations we propose in the statement of Proposition 1 are indeed equilibria of the model. Most of the argument in the proof is devoted to showing that no other behavior conforms with equilibrium. To gain some intuition on the result, notice that in our setup the profit a monopolist can derive from his private information and his market power lies in the possibility of manipulating buyers' beliefs and inducing them to agree to trade at a high price and with minimal delay. Our result shows that the seller will always succeed in generating such beliefs by revealing the minimal amount of information (possibly zero) that is necessary, and all in the first period. There is, however, a cost because there may be—possibly considerable—delay in trade; without such a cost the seller has no motive to reveal any of his information in equilibrium. Furthermore, the monopolist, facing no competition, has no incentive to reveal any information beyond what is needed to induce buyers to trade at a high price.

Proposition 1 is important in its own right. It yields predictions that differ from those of other static and dynamic monopoly models (see our discussion in Section 8). In addition, it is technically useful: Proposition 1 is a building block for establishing some of our next results, in which nonrevealing and collusive equilibria can be sustained (see part (ii) in each of Propositions 2–5 to follow).

4. Oligopoly without Clienteles

In this section we examine the case of $n > 1$ sellers who compete in prices among themselves, in the absence of clienteles. As described in Section 2, this means that, in each period, sellers simultaneously announce a price in the set $\{p_H, p_L\}$ and buyers observe the list of announced prices and the identity of the seller behind each price and can freely choose with whom to trade. Hence, if both prices are called on by sellers, those who announce p_L split the entire market equally among them while those who announce p_H sell no units. In this situation, competition is quite fierce because each seller—by undercutting—can immediately steal his competitors' market share.

We explore this model (as well as its counterpart with clienteles in the next section) for the case where all sellers are informed regarding the true state of the world. We then show in Section 6 that the substance of the results extend to the case of informational asymmetries also among sellers.

As in the case of monopoly, in State H sellers will always charge p_H and, as soon as the price p_L is in the list of announced prices, all buyers remaining in the market will always buy at this price. If, on the other hand, all sellers announced p_H in the first t periods, the buyers' choice at t depends on their belief α_t that the state of the world is H. Let $q_B(t)$ denote then the probability that a buyer accepts, at t, if all sellers propose p_H given histories where all sellers offered p_H in the past. The formal definition of the strategies of buyers and sellers—and hence of an equilibrium—is otherwise the same as in the previous section (we denote by $q_S(t)$ the probability that each seller in State L charges p_H in Period t); similarly, α_t is buyers' belief at t that the state is H if all sellers proposed p_H at t and in each prior period.[12]

The nonexclusivity of the sellers' information, as well as the limits on their market power given by the presence of competitors, impose severe constraints on their ability to hide their information and manipulate buyers' beliefs as in the case of monopoly. To hide the information would in fact require repeated announcements of a high price even when the state is L. However, doing so now would give other sellers a strong incentive to undercut. Even though by undercutting the seller would reveal his information, the benefits from expanding his market share would be higher the larger is the number n of sellers; hence, for n sufficiently large, these gains will outweigh the costs of revealing the information.

We proceed to characterize the equilibria of the model in this case. Due to the nonexclusivity of the information that each seller holds, there is always an equilibrium where all the information is immediately revealed: each seller in Period 1 announces p_H in State H and p_L in State L with probability 1, while buyers always accept both p_L and p_H (if all sellers offered p_H). Evidently, if all competitors adopt a revealing strategy, a seller's best reply is also revealing.

When the number of sellers n is small enough, we can also have "collusive" equilibria, where sellers behave as the monopolist in the equilibria obtained in Section 3, or where they randomize in State L for the first T periods and then always propose p_L. However, as n grows, such collusive strategies cease to be part of any equilibrium, and there is no other equilibrium than full and immediate information revelation. We state this formally as follows.

PROPOSITION 2. *In the model with n informed sellers without clienteles, the following equilibria obtain.*

(i) Full and immediate information revelation. *For any $n \geq 2$, there always exists an equilibrium with $q_S(1) = 0$ and $q_B(1) = 1$.*

(ii) *For n small enough, the following equilibria also exist (for some parameter configurations).*

12. Again we can restrict our attention, without loss of generality, to symmetric equilibria. The purification argument outlined in footnote 11 continues to apply to the buyers. Moreover, by the simple structure of the model, one can show that there are no asymmetric equilibria among sellers—whence the notation $q_S(t)$ just presented.

(a) No revelation or partial immediate revelation. *All sellers (and hence all buyers) behave as in the monopoly equilibrium.*

(b) Full revelation but with delay. $q_S(t) \in (0, 1)$ *and* $q_B(t) \in (0, 1)$ *for* $1 \leq t < T$, *and* $q_S(t) = 0$ *and* $q_B(t) = 1$ *for* $t \geq T$.

(iii) Asymptotically, full and immediate revelation in all equilibria. *For n sufficiently large, all the equilibria in (ii) vanish and the unique equilibrium is the one described in (i).*

5. Oligopoly with Clienteles

We analyze here to what extent the results obtained in Section 4 (in particular, the fact that all information is immediately revealed to the buyers when there are sufficiently many sellers) remain valid when less extreme forms of price competition are considered. Toward this end, as anticipated in Section 2, we introduce the version of the model with clienteles. The main difference from the previous version is that now, each period, a buyer can buy only from his designated seller—hence the term clientele. This association lasts only one period because, at each time t, the buyers remaining in the market are randomly reassigned to sellers. By undercutting, each seller can now steal only a limited fraction of his competitors' market share. Again we proceed to study the model when all n sellers are fully informed.

If all other informed sellers choose p_L (i.e., to reveal their information) and if this is commonly observed by all buyers, then the best reply of a seller is clearly to do the same as long as there is some—even very weak—competition among sellers. Thus, full and immediate revelation of information remains an equilibrium in the model with clienteles because of the nonexclusivity of information.

On the other hand, we have already mentioned that the gains from undercutting are now much more limited. To see this more precisely, consider the situation where, at some date t in State L, all sellers announce p_H. We can now construct different equilibria supporting this outcome using different off-equilibrium beliefs when one seller deviates to p_L. If these beliefs are that the state is L with probability 1, buyers will reject all offers of p_H. Thus only the seller who announced p_L will sell to the $(1/n)$-th of the market constituting his clientele for the period. The remaining $((n-1)/n)$-th of the buyers in the market at t will then still be around at $t+1$ and will be equally split among the n sellers. At $t+1$ all sellers will offer p_L if the buyers' strategy is to keep rejecting all offers of p_H. Thus an informed seller who undercuts can only increase his market share from $(1/n)$-th to $([1+(n-1)/n]/n)$-th of the market; moreover, this increase in market share will take one period to materialize. As a consequence, collusive behavior among sellers—in particular hiding information to profitably manipulate buyers' beliefs—is now easier to sustain. If the collusive payoff in State L (i.e., $p_H - c_L$)

is tempting enough, the “local monopoly” power created by the presence of clienteles suffices for the existence of another (collusive) equilibrium regardless of the number of sellers: each seller behaves as in the monopoly equilibrium and so no information, or only the minimal amount necessary to induce trade at the high price, is revealed. Furthermore, with other off-equilibrium beliefs there exist collusive equilibria that do not necessitate the assumption of the collusive payoff being large enough; we give details in part (ii) of the next proposition.

These results stand in clear contrast to our findings for the model without clienteles. They reveal that, even in the absence of exclusivity of information, revelation might not occur at equilibrium—whatever the number of sellers—unless the intensity of competition among sellers is strong.

Formally, we have our next proposition.[13]

PROPOSITION 3. *In the model with n informed sellers with clienteles the following equilibria hold.*

(i) Full and immediate information revelation. *For all $n \geq 2$ and all α_0, there is always an equilibrium where $q_S(1) = 0$ and buyers accept p_H in Period 1 when all prices announced in that period are p_H.*

(ii) *For any $n \geq 2$, the following collusive equilibria (where sellers behave as in the case of monopoly) also exist.*

(a) No revelation. *If $\alpha_0 \geqslant \bar{\alpha}$, then $q_S(t) = 1$ for all t and buyers immediately accept p_H.*

(b) *Partial revelation. If $\alpha_0 < \bar{\alpha}$ and*

$$p_H - c_L \geqslant (p_L - c_L)\left[\delta + \frac{(1-\alpha_0)\bar{\alpha}}{\alpha_0(1-\bar{\alpha})}\right]$$

then $q_S(1) \in (0, 1)$, so that $\alpha_1 \geq \bar{\alpha}$ and buyers accept with probability 1 if all sellers announce p_H; for $t > 1$ we have $q_S(t) = 1$ and all buyers accept p_H.

6. The Role of Exclusive Information

We examine here how the properties of the equilibria obtained in the two previous sections change when we allow for information to be exclusive, as under monopoly. In particular, we consider the case where there is only one out of the n sellers who is informed and whose identity is commonly known.[14] As a consequence, the informed seller has the same possibility as the monopolist to

13. Here we use $q_S(t)$ to denote the probability that a seller proposes p_H in State L at Date t (again given histories where all sellers in the past always proposed p_H).

14. If the identity of the informed seller were not known the inferences of the uninformed traders from the observation of the proposals made would be more difficult. However, one can show that the validity of our results extends to that case due to the strong (weak) undercutting incentives in the absence (presence) of clienteles.

hide his information. However, in order to profit from this in the presence of (uninformed) competitors, the informed seller must successfully manipulate both the buyers' and the other sellers' beliefs so as to induce all of them to trade at a high price. As we shall see this may not always be optimal. In determining whether or not this is the case, the degree of competition allowed by the trading rules will once again play a key role.

In the absence of clienteles, if the number of sellers in the market is sufficiently large then the incentive of the informed seller to undercut and steal all his competitors' market share proves again too strong, even though doing so reveals all information. More specifically, when n is small we may have equilibria where no information is revealed (and there may even be no equilibrium with full revelation): the informed seller hides his information, partially or completely, and trade takes place with some positive probability at p_H in both states. However, when n is sufficiently large, there is a unique equilibrium where all information is immediately revealed to the buyers, as in the case where all sellers are equally informed. Therefore, the large number of sellers competing in the market under the absence of clienteles—rather than specific properties of the information structure—seems to be the key feature allowing information to be fully and immediately revealed to the consumers.

To characterize the equilibria formally, we must now describe separately both the strategy of the informed seller, in States H and L, and the uninformed sellers (and buyers). As before, the informed seller in State H will always charge p_H. We denote then by $q_I(t)$ and $q_U(t)$ the probability that the informed seller in State L and each uninformed seller (respectively) charge p_H at Date t following a history where no p_L has ever been charged. Let $q_B(t)$ be the probability that buyers accept p_H in Period t following a history where the only price announced by sellers has been p_H.

Some further notation will be needed to identify the relevant cutoff values in the beliefs of uninformed sellers.

> $\widetilde{\alpha}$ denotes the belief that makes uninformed sellers indifferent between trading at p_L and not trading at all: $\widetilde{\alpha}(p_L - c_H) + (1 - \widetilde{\alpha})(p_L - c_L) = 0$. Thus, for $\alpha < \widetilde{\alpha}$, an uninformed seller strictly prefers trade at p_L to no trade.
>
> $\hat{\alpha}$ denotes the belief that makes uninformed sellers indifferent between not trading at all and announcing p_L, when every other uninformed seller proposes p_H while the informed State L seller proposes p_L: $\hat{\alpha}(p_L - c_H) + (1 - \hat{\alpha})(p_L - c_L)/2 = 0$. For $\alpha > \hat{\alpha}$, no uninformed seller would offer p_L in this situation.

It can be easily verified that $\widetilde{\alpha} > \hat{\alpha}$.

PROPOSITION 4. *In the model with n sellers—of whom only one is informed—and without clienteles, the following equilibria obtain.*

(i) *For n large enough, there is always an equilibrium with full and immediate information revelation. Specifically, we have one of the following.*

(a) Full information revelation to all uninformed traders. *For large enough n and for all $\alpha_0 \geqslant \hat{\alpha}$, there exists an equilibrium where $q_U(1) = 1$, $q_I(1) = 0$, and $q_B(1) = 1$ (i.e., at $t = 1$ the uninformed sellers charge p_H, the informed seller charges p_H in State H and p_L in State L, and buyers accept both p_L and p_H provided all sellers offered p_H).*

(b) Full information revelation to buyers. *For large enough n and for all $\alpha_0 < \widetilde{\alpha}$, there exists an equilibrium where $q_U(1) = q_I(1) = q_B(1) = 0$ (i.e., at $t = 1$ the uninformed sellers charge p_L, the informed seller charges p_H in State H and p_L in State L, and buyers accept only p_L at $t = 1$).*

(ii) No full immediate revelation. *For n sufficiently small and some parameter configurations, there are equilibria where $q_I(t), q_U(t) \in (0, 1]$ and $q_B(t) \in (0, 1]$ for all $t \leq T$, for some finite $T \geq 1$.*

(iii) Asymptotically, full and immediate revelation in all equilibria. *As $n \to \infty$, the only equilibria are the ones in (i).*

The uninformed sellers' behavior also depends on their beliefs about the state. However, as in Sections 3 and 4, in this case there is still no need to specify off-equilibrium path beliefs after unilateral deviations (of the informed seller). The inference both of buyers and uninformed sellers is, in fact, irrelevant when the informed seller (or, for that matter, any seller) announces p_L, since the optimal response of buyers is always to accept in this case and hence the game ends immediately. On the other hand, there is no off-equilibrium deviation to p_H.[15]

Although information is monopolized by one seller, Proposition 4 makes it clear that, as n grows large, the intensity of price competition among sellers in the model with no clienteles gives too strong an incentive to undercut and hence all information is revealed to the buyers right away. The equilibrium in Proposition 4(i)(b), which also survives for all n, is characterized by the fact that all information is immediately revealed to the buyers yet there is no revelation to the uninformed sellers, who end up trading at p_L in State H. Their prior belief giving low probability to the state being high leads them to bear a payoff in that state that is not ex post individually rational.

The situation is again quite different in the presence of clienteles. In this case, as we saw in Section 5, hiding the information can be profitable when all sellers (whatever their number) are informed—in particular, when the unit profit from the sale at a high price is sufficiently higher than the profit from the sale at a low price. The same should be true a fortiori when there is only one informed seller. When information is nonexclusive, we have seen that a fully revealing

15. With regard to deviations by uninformed sellers, when $n > 2$ there will always be at least one uninformed offering p_L, thus leading to an immediate termination of the game.

equilibrium also exists for all n. In contrast, when information is exclusive the argument supporting such an equilibrium no longer holds.

We will show that, for many parameter configurations, separation is impossible at equilibrium and that an equilibrium exists where at least part of the information is never revealed. The only case in which hiding the information can prove too costly for the informed seller is when both the gain in the per-unit profit obtained by selling at p_H in State L is low enough and the prior belief of the uninformed sellers sufficiently optimistic that they may be willing to offer a low price.

The results are again in clear contrast to what we found in the model without clienteles. If information is exclusive, we should not expect it to be revealed in equilibrium when the competition intensity among sellers is not too strong.[16]

PROPOSITION 5. *In the model with n sellers—of whom only one is informed—and with clienteles the following properties of equilibria obtain.*
(i)(a) Full revelation is impossible for many parameter configurations. *For any $n \geq 2$, if $p_H - c_L > (p_L - c_L)(1 + \delta)$ (the collusive payoff is not too small) or if $\alpha_0 < (p_L - c_L)/(c_H - c_L)$ for all t (beliefs are sufficiently pessimistic), then at equilibrium we have $q_I(t) > 0$ for all t.*
(i)(b) Full revelation occurs only for some parameter values. *There exists n large enough such that, for $p_H - c_L < (p_L - c_L)(1 + \delta)$ and $\alpha_0 > (p_L - c_L)/(c_H - c_L)$, there is an equilibrium with $q_I(1) = 0$.*
(ii) *For any $n \geq 2$, the following additional equilibria exist.*

(a) No revelation. *If $\alpha_0 \geq \bar{\alpha}$, then $q_U(t) = q_I(t) = 1$ for all t (both the uninformed sellers and the informed seller in both states charge p_H) and buyers immediately accept.*

(b) Partial revelation. *If $\alpha_0 < \bar{\alpha}$ and if $(p_H - c_L) > (p_L - c_L)(1 + \delta)$, $q_I(1) \in (0, 1)$, and $q_U(1) = 1$ (the informed seller randomizes in State L in the initial period while the uninformed charge p_H), then for all $t > 1$ it follows that $q_I(t) = q_U(t) = 1$. Buyers randomize in every period between accepting and rejecting when all announced prices are p_H.*

Thus, for a large subset of the parameter region, full separation never occurs and only collusive equilibria exist—whatever the number n of sellers. For the complementary region, separation can be supported but collusive equilibria are also found.

Comparing these results to those of the previous sections shows that the intensity of competition among sellers, as captured by the absence of clienteles, appears to play a more important role than the exclusivity of information in determining whether or not information is fully revealed to buyers. The message

16. See footnote 13 regarding the notation employed here ($q_I(t)$, $q_U(t)$, etc.).

of the model with clienteles is somewhere between the information revelation findings—and hence convergence to fully revealing REE—of the model without clienteles, where competition is quite intense, and the nonrevelation results of the pairwise meetings literature, where the local monopoly power of each seller in every meeting is reinforced by the lack of observability of public signals that could help reveal the information.

7. Welfare Analysis

We discuss here the welfare properties of the equilibria we obtained. As known from the literature on REE, equilibria with full and immediate information revelation are ex post Pareto efficient. In our setup, where all traders are risk neutral, such equilibria are also interim efficient and are characterized by no delay in trading.[17]

An important issue is then whether information revelation implies simply a transfer of welfare from the uninformed to the informed agents or is rather an instance of inefficiencies manifesting themselves, for example, as delay in trading. In evaluating such efficiency properties, we should take into account the private nature of information and hence require improvements to satisfy incentive compatibility conditions (i.e., evaluate allocations according to the notion of interim incentive efficiency). As shown in Serrano and Yosha (1996) and Blouin and Serrano (2001), in the pairwise meetings models information revelation was always accompanied by efficiency, whereas no-revelation caused interim inefficiencies due to delay in learning. The same conclusion (i.e., revelation obtains if and only if welfare losses are negligible) does not emerge in this paper. For brevity, we will simply state the results and not provide the details of the arguments, which are available in an earlier version of our paper.[18]

7.1. Monopoly

(i) The equilibria with perfect pooling and immediate acceptance by the buyers,which are obtained when $\alpha_0 \geq \bar{\alpha}$, are clearly interim efficient: all gains from trade are exhausted and with no delay.
(ii) On the other hand, the equilibria with delay (obtained when $\alpha_0 \leq \bar{\alpha}$) are always interim incentive inefficient.

Thus, the equilibrium with no information revelation in (i) translates in a welfare transfer to the informed monopolist (with respect to the case of full and

17. Interim efficiency refers to the case where agents' welfare is evaluated conditionally on their information at the initial date, prior to the opening of markets.

18. Available at <http://www.sss.ias.edu/papers/econpaper27.pdf>.

immediate information revelation) without entailing any welfare loss. On the other hand, the partial revelation of information occurring at the equilibrium in (ii) creates delay and imposes a net loss in total welfare. We can show, in fact, that an (incentive compatible) improvement can be achieved in this case if buyers' behavior is kept the same as at this equilibrium while the seller charges p_L with probability 1 in State L: such a mechanism leaves the welfare of both types of the seller unchanged (with respect to the equilibrium outcome), but strictly improves buyers and is incentive compatible. Moreover, such inefficiency persists even with arbitrarily small discounting, as $\delta \to 1$, so that the cost of delaying trade by any finite number of periods become vanishingly small: this is because, as δ approaches 1, a significant fraction of trades takes place with longer and longer (possibly infinite) delay.

Although the comparison among the equilibrium payoffs of buyers and sellers in the equilibria of type (i) and (ii) (without and with delay) involves agents in different economies who are characterized by different values of the prior belief α_0, it is worth noting that both types of seller have a strictly lower payoff in the second equilibrium than in the first. Thus, information revelation comes at a cost to the seller.

7.2. Oligopoly without Clienteles

When all sellers are informed, we found that the only equilibrium surviving for any number of sellers n is the one with full and immediate information revelation—which, as we have argued, is not only ex post but also interim Pareto efficient. When compared to the pooling equilibrium obtained under monopoly, the sum of the payoffs of sellers in State L is now lower while buyers' payoffs are higher. This transfer to buyers is the cost to the sellers of information revelation, but—unlike the monopolist case—information revelation here does not destroy society's welfare.

When only one seller is informed, two types of equilibria exist for all n. Both are separating equilibria and the information, privately held by the only informed seller, is fully revealed to the buyers.

(i) In the equilibrium described in part (i)(a) of Proposition 4, the informed seller in State H shares the market with the uninformed sellers, whereas in State L he manages to appropriate the entire market. Each buyer continues to receive the same expected payoff as when all sellers are informed. This equilibrium is also interim efficient, and the total payoff of sellers and buyers is the same as with all sellers informed. The only difference worth stressing is the transfer of surplus from the uninformed sellers to the informed seller in State L. Again, information revelation is achieved without any welfare loss.

(ii) Although the equilibrium described in part (i)(b) of Proposition 4 is also interim efficient (because of the absence of delay in trading), its features are rather different. In this equilibrium, the more optimistic beliefs of the uninformed traders crowd out the informed seller in State H, who sells nothing. The informed seller in State L now shares the market with the uninformed sellers, who sell in both states. Buyers are of course the clear winners, paying a low price in both states and avoiding delays. Recall that even though this equilibrium is interim efficient, information is not revealed to the uninformed sellers: all trade occurs at p_L in State H so that uninformed sellers, driven by their optimistic beliefs, end up transacting at a price that is not ex post individually rational (in State H).

7.3. Oligopoly with Clienteles

With regard to the case of all informed sellers, one possible equilibrium outcome (Proposition 3(i)) is again full and immediate information revelation, as in Proposition 2. The other, collusive equilibria obtained in Proposition 3 are close to the ones found in Proposition 1 for the case of the monopolist. The reader is referred to the previous subsections for discussions of their welfare properties.

Consider next the equilibria with only one informed seller. When full revelation is sustained in equilibrium, all trade takes place at p_L in State L and at p_H in State H. The welfare properties of this equilibrium are similar to those of the separating equilibrium described in Proposition 4(i)(a), with two differences: (1) there is delay, though this should now be viewed as a trade friction generated by the presence of clienteles; and (2) the distribution between uninformed and informed sellers of their profits in State L is now more equal.

In contrast, at the equilibria where the informed seller hides his information, the total payoff of buyers and sellers is the same as at the equilibria obtained under monopoly. In some of these equilibria, delay occurs as a result of the buyers' randomization and this causes inefficiencies. Note also that the expected payoff for the informed and uninformed sellers is the same in this case (unlike in the equilibria of the model without clienteles). Thus, the existence of clienteles exerts a positive externality on the uninformed sellers and clearly a negative externality on buyers, who pay the high price in both states.

8. Related Literature

As mentioned in the Introduction, other papers examine the strategic foundations of REE while considering some specific trading mechanisms. The case in which there is a single informed trader acting strategically is examined by Grinblatt and Ross (1985) and Laffont and Maskin (1990) within a one-period model and by

Kyle (1985) in a dynamic setting. In both cases it was shown that the monopolist may choose not to completely reveal his private information and that equilibria differ from REE. To understand the differences with respect to our results, particularly in the case of Kyle's work, note that he considers a model with noise traders, where prices are determined each period by competitive market makers on the basis of the observation of aggregate trades. Thus, aggregate trades, a fortiori prices, cannot fully reveal the private information of the informed trader. Even if he were to act nonstrategically, full revelation could only be achieved in the limit—after infinitely many rounds of trade. Since he acts strategically, convergence of prices to their full information value is slower: the informed agent is able to "hide" his trades from the market. Our result for the monopolist case is different, because continued randomization (whereby information is progressively revealed over time) is never optimal, a result of discounting and the absence of noise traders.

Information revelation is also addressed in models of dynamic trading with more than a single informed trader by various papers extending the analysis of Kyle (1985). In particular, Vives (1993) considers a dynamic trading economy with a continuum of risk-neutral agents, of whom some fraction receives each period an imperfect and independent signal concerning the true (aggregate) state of nature. There are noise traders as well. Vives investigates the speed of convergence to full information and how this speed is affected by various factors (precision of private signals, proportion of informed traders, steepness of traders' adjustment costs, and variance of noise trades). In this setup—in contrast to our paper—there are no strategic effects, and private information is never a source of market power. The main issue is to what extent agents rely on their private signal (rather than each period's market price) when deciding the level of their trades, which in turn affects the effectiveness of prices in aggregating and transmitting the agents' private information. In our setup there is no issue of information aggregation, since signals are perfectly informative and there are no noise traders. Informed agents strategically choose how much information to reveal through their trading strategy. Thus, less information is revealed in some situations because the informed traders choose not to reveal it, not because they rely more on market than on private signals. Vives (1995) examines then a situation with a continuum of risk-averse, informed traders acting myopically as well as noise traders and risk-neutral market-makers, finding that convergence is rather fast due to the coordinating actions of the market makers (see however Medrano and Vives (2001), where the addition of a large strategic agent to the setup of Vives (1995) limits information revelation).

Several informed traders are introduced into the dynamic trading model of Kyle (1985) by Holden and Subrahmanyam (1992). They show, via numerical simulations, that strategic agents compete aggressively by sending very large trade orders, thereby neutralizing the effect of noise traders and leading to quick revelation (in line with our findings for the case in which all sellers are informed). However, such results no longer hold if the signals received by informed traders are only imperfectly correlated, as shown by Foster and Viswanathan (1996) (also via numerical simulations); again there are some analogies with our findings for the case of only one informed seller in the model without clienteles.

The case where trading takes place via market games à la Shapley and Shubik (1977), in the presence of a continuum of agents of finitely many types, is studied by Dubey, Geanakoplas, and Shubik (1987) and more recently by Forges and Minelli (1997). Each source of information is possessed by a continuum of agents. It is shown that, when the trading game is repeated, the Nash equilibria are such that the first stage is used to exchange information among traders (equilibrium prices act as public signals); in the subsequent stages, the outcome coincides (under appropriate conditions) with the fully revealing REE. A static market game where strategy sets are demand functions is analyzed by Kyle (1989) with finitely many informed traders: he shows the existence of equilibria where only part of the information is revealed and studies how the amount of information revelation varies with the share of informed traders relative to both uninformed and noise traders.

In a static auction context, Pesendorfer and Swinkels (1997) and Perry and Reny (2003) study conditions under which sequences of symmetric equilibria aggregate information and converge to REE. Closer to our work, Peters and Severinov (2002) study the issue in a sequential auction context, in which traders' observations of the actions of others allow them to make inferences about the true state.

The difficulties met in establishing the convergence of the equilibria of strategic models to competitive outcomes when there is asymmetric information over common values uncertainty should be contrasted with the case of complete information, or even of asymmetric information of the private values type. In these cases perfect competition arises as a fairly robust limit of game theoretic models when the number of traders increases (see e.g., Rustichini, Satterthwaite, and Williams 1994; Satterthwaite and Williams 2002; and Cripps and Swinkels 2004 in a double auction context). Note that this occurs even in models of random matching and pairwise meetings (see Gale 1987; Serrano 2002; Shneyerov and Satterthwaite 2003). We should also point out that, in a dynamic set-up, market power may vanish even in the case of monopoly: the Coase conjecture for a durable goods monopolist—uninformed about the demand curve he faces—shows in fact that monopoly pricing, under reasonable assumptions, converges to the competitive price as the frequency of trades increases (see e.g., Gul, Sonnenschein and Wilson 1986).

Finally, even though formally ours is not a repeated game, we should also mention the relationship between our work (particularly in the case of a single informed seller) and the literature on two-player repeated games with incomplete information, where partial revelation results in which information is revealed in the first stages are also obtained. See Forges (1992), Zamir (1992), Mertens, Sorin and Zamir (1994) and references therein.

Appendix A: Proofs

This appendix contains the proofs of our results.

Proof of Proposition 1. The proof proceeds by first establishing some properties of the equilibria as follows.

(A) Full and immediate information revelation never occurs (in finite time); at no date does the seller in State L propose p_L with probability 1. Formally, we have the following.

LEMMA 1. *At any equilibrium, $q_S(t) > 0$ for every t.*

Proof. Suppose not; then there exists a period t such that $q_S(t) = 0$. Recall that, if the state is H, the seller proposes p_H for certain, so it follows that there is full separation at t. Thus we have $\alpha_t = 1$. Upon observing p_H at t, the buyers can deduce the state is H for certain and so their optimal strategy must always be to accept p_H with probability 1 at date t ($q_B(t) = 1$). But then the optimal strategy of the seller in State L at t would be to propose p_H rather than p_L, a contradiction. □

(B) If at some t the buyers' strategy is to accept p_H for sure, then in that period the type L seller will propose p_H for certain. This in turn implies—when the buyers' belief α_t is sufficiently close to 1—that at each earlier date the buyers should also prefer to accept p_H for sure; hence the seller must prefer to propose p_H for certain. Hence we have the next Lemma.

LEMMA 2. *If, at an equilibrium, $q_B(t) = 1$ for some t, then we must also have $q_S(t) = 1$. If, in addition, $\alpha_t > \bar{\alpha}$ then we obtain that, for every $t' < t$, $q_B(t') = q_S(t') = 1$.*

Proof. This is easily established by backwards induction. If $q_B(t) = 1$, the seller's best response in period t is clearly $q_S(t) = 1$. Since $q_S(t) = 1$, it follows that $\alpha_{t-1} = \alpha_t$; thus, if $\alpha_t > \bar{\alpha}$ a buyer's payoff from accepting at $t - 1$ is positive and strictly higher than his payoff from accepting at t, for all $\delta < 1$. The buyers' best response in period $t - 1$ must then be $q_B(t - 1) = 1$. Iterating the argument, we find that the same must be true at all previous dates $t' < t$. □

(C) If in equilibrium buyers randomize for infinitely many periods, their belief α_t must jump to a sufficiently high level at the initial date and stay constant at that level forever after, as shown in the following result.

LEMMA 3. *If $q_B(t) \in (0, 1)$ at every date t, then $\alpha_t = \bar{\alpha}$ for all t.*

Proof. Note that the evolution of posterior beliefs α_t is determined from Bayes' rule using the seller's strategy and, accordingly, follows equation (1). As we already noticed, this sequence of posterior beliefs is nondecreasing in t. In addition, to sustain the buyers' randomization in every period, we need the following condition to hold at all t:[19]

$$\begin{aligned}\alpha_t u_H + (1-\alpha_t)u_L - p_H = \delta\big[&(1-\alpha_t)(1-q_S(t+1))(u_L - p_L)\\ &+ (1-(1-\alpha_t)(1-q_S(t+1)))\\ &\times (\alpha_{t+1}u_H + (1-\alpha_{t+1})u_L - p_H)\big], \quad \text{(A.1)}\end{aligned}$$

where the left-hand side is the payoff from accepting p_H and the right-hand side is the payoff from rejecting it at t and accepting it at $t+1$. Subtracting $\alpha_{t+1}u_H + (1-\alpha_{t+1})u_L - p_H$ from both sides yields:

$$\begin{aligned}\delta(1-\alpha_t)&(1-q_S(t+1))(u_L - p_L) - (\alpha_t - \alpha_{t+1})\,(u_H - u_L)\\ &= [1-\delta+\delta(1-\alpha_t)(1-q_S(t+1))]\\ &\quad\times (\alpha_{t+1}u_H + (1-\alpha_{t+1})u_L - p_H). \qquad \text{(A.2)}\end{aligned}$$

Because (by hypothesis) the randomization involves infinitely many periods, it follows that the infinite sequence of posteriors $\{\alpha_t\}$, which is monotone and bounded, has a limit. We will show that $\lim_{t\to\infty}\alpha_t = \bar{\alpha}$.

The convergence of the sequence $\{\alpha_t\}$ implies that both terms of the subtraction on the LHS of (A.2) tend to zero as $t \to \infty$ (this is evident for the second term; the same is clearly true for the first term if $\lim_{t\to\infty}\alpha_t = 1$, but even if $\lim_{t\to\infty}\alpha_t < 1$, since in that case by equation (1) we must have $\lim_{t\to\infty} q_S(t) = 1$, the term tends to zero). On the other hand, the first term of the product in the expression on the RHS of (A.2) is always strictly positive; this expression can then equal zero in the limit only if its second term tends to zero—that is, if $\lim_{t\to\infty}\alpha_t = \bar{\alpha}$.

Finally, we note that, in order to sustain the randomization of the buyers at any date t, we must have $\alpha_t \geq \bar{\alpha}$. This fact, together with the property $\lim_{t\to\infty}\alpha_t = \bar{\alpha}$ just established, implies that infinite randomization of the buyers requires $\alpha_t = \bar{\alpha}$ for all t. □

19. The same arguments apply if the randomization does not involve two consecutive periods. The right-hand side of (A.1) is then more involved, but the essence of the argument is identical.

We are now ready to establish the claim of Proposition 1. By Lemma 1, pure strategies where $q_S(t) = 0$ for some t can never be part of an equilibrium. Consider then the only other possible pure strategy of the type L seller: $q_S(t) = 1$ for all t.

If $\alpha_0 \geq \bar{\alpha}$, a best reply for the buyers to this strategy is $q_B(t) = 1$ for all t: their expected payoff is nonnegative, and any other strategy would only induce delay and still result in either no trade or trade at the same price, p_H, thereby yielding a lower (weakly if $\alpha_0 = \bar{\alpha}$) payoff. The strategy $q_S(t) = 1$ for all t is then also the seller's best reply to $q_B(t) = 1$ for all t, since in this case the payoff obtained by the type L seller is $p_H - c_L$, the highest possible. This establishes that $q_B(t) = q_S(t) = 1$ for all t is an equilibrium if $\alpha_0 \geq \bar{\alpha}$, as claimed in part (i) of the proposition's statement.

On the other hand, if $\alpha_0 < \bar{\alpha}$, the buyers' best reply to $q_S(t) = 1$ for all t is $q_B(t) = 0$ for all t. But then the seller's best reply is $q_S(t) = 0$ for all t, so we do not have a pure strategy equilibrium in this case.

Consider next the candidate equilibria where buyers randomize for infinitely many periods. From Lemma 3, such equilibria require the seller (a) to (possibly) randomize at the initial date, so as to induce the posterior belief $\alpha_1 = \bar{\alpha}$, and (b) to propose p_H with probability 1 at all later dates $t > 1$. This is clearly possible only if $\alpha_0 \leq \bar{\alpha}$ (hence, when $\alpha_0 > \bar{\alpha}$ this type of equilibrium does not exist).

Let us denote by $V_L(t)$ the present value, at t, of the discounted expected flow of payoffs of the type L seller, given that he always proposed p_H in the past (including the current period t), and was always rejected. Then $V_L(t)$ satisfies

$$V_L(t) = q_B(t)(p_H - c_L) + \delta(1 - q_B(t))V_L(t+1). \tag{A.3}$$

Any sequence of values $q_B(t) \in (0, 1)$ satisfying the conditions

$$\begin{aligned} p_L - c_L &= q_B(1)(p_H - c_L) + \delta(1 - q_B(1))V_L(2) \qquad \text{and} \\ p_L - c_L &\leq q_B(t)(p_H - c_L) + \delta(1 - q_B(t))V_L(t+1) \qquad \text{for all } t > 1 \end{aligned} \tag{A.4}$$

supports the strategy $q_S(1) \in (0, 1)$, $q_S(t) = 1$ for all $t > 1$ as the seller's best response to $\{q_B(t)\}_t$. It is immediate to verify that we can always find some (in fact many) sequences with this property. Furthermore, since $\alpha_t = \bar{\alpha}$ for all $t \geq 1$, any sequence of values $q_B(t) \in (0, 1)$ is a best reply for the buyers. Thus, as stated in Proposition 1(ii), such equilibria always exist if $\alpha_0 < \bar{\alpha}$. By a similar argument, any sequence of values $q_B(t) \in (0, 1)$ satisfying the inequality in the second line of (A.4) for all $t \geq 1$ and the seller's strategy $q_S(t) = 1$ for all $t \geq 1$ are a best response to each other (and hence constitute an equilibrium) when $\alpha_0 = \bar{\alpha}$.

To complete the proof of the proposition, it remains to consider the possibility of an equilibrium where buyers randomize for a positive but finite number of periods.[20] Suppose there is an equilibrium where $q_B(t) = 0$ for all t greater or equal than some date $T \geq 2$. Then the seller's best response, as we already argued, would be $q_S(t) = 0$ for all $t \geq T$, which by Lemma 1 cannot be part of an equilibrium.

On the other hand, if $q_B(t) = 1$ for all $t \geq T \geq 2$ is part of an equilibrium strategy, we must have $\alpha_T \geq \bar{\alpha}$. If $\alpha_T > \bar{\alpha}$ we again reach a contradiction, by Lemma 2. If $\alpha_T = \bar{\alpha}$ and $\alpha_t < \bar{\alpha}$ for $t < T$, we must have $q_S(T) \in (0, 1)$; for this choice of the seller to be optimal we need (from (A.4), taking into account that $q_B(T) = 1$), the equality $p_L - c_L = p_H - c_L$ to hold, which is impossible. We are then left with the case where $\alpha_T = \bar{\alpha}$ and where, for some $\bar{t} < T$, $\alpha_t = \bar{\alpha}$ for $\bar{t} < t < T$, so that $q_S(t) = 1$ for all $t > \bar{t}$ and $q_S(\bar{t}) \in (0, 1)$. Note first that this can be part of an equilibrium only if $\bar{t} = 1$ and hence $\alpha_t = \bar{\alpha}$ for all $t \geq 1$.[21] The conditions for the optimality of the seller's strategy are again given by (A.4), where $V_L(2)$ is defined recursively by (A.3) together with the equality $V_L(T - 1) = p_H - c_L$. It is easy to check that, for any $\delta < 1$, we can find T sufficiently high that these conditions are satisfied for some sequence $\{q_B(t)\}_{t\geq 1}$ exhibiting the property $q_B(t) = 1$ for all $t \geqslant T$. The closer is δ to 1, the larger is the minimal number of periods of randomization T required.

We conclude that, when $\alpha_0 \leq \bar{\alpha}$, there exist equilibria where buyers randomize both for an infinite and a finite number T of periods, with T larger the closer is δ to 1. On the other hand, if $\alpha_0 > \bar{\alpha}$ then there are no equilibria where buyers randomize. □

Proof of Proposition 2.

(i) The proof of the claim is immediate. If a seller anticipates that his $(n - 1)$ competitors offer price p_L in State L (i.e., choose to fully reveal their information), then his unique best response is to do the same (for any $n \geq 2$). In fact, the alternative is losing his market share entirely.

(ii)(a) Let $\alpha_0 \geq \bar{\alpha}$. Consider the seller's strategy in the corresponding monopoly equilibrium (Proposition 1(i)). When all other sellers follow this strategy and hence offer p_H with probability 1, the best possible deviation for a seller is to undercut and announce p_L at the initial date in State L. By doing so, he sells to the whole market at the price p_L, so that his profit (starting from that node) is $p_L - c_L$. On the other hand, the profit obtained by adhering to the collusive strategy is $(p_H - c_L)/n$ (i.e., the seller's share of the monopoly profit).[22] Therefore, adopting the monopoly strategy for all sellers remains optimal if and only if

20. It is immediate to see that we can only have an equilibrium where buyers never randomize if the seller also never randomizes, the case already considered at the beginning of the proof.

21. If $\bar{t} > 1$, so that $\alpha_t < \bar{\alpha}$ and hence $q_B(t) = 0$ for $t < \bar{t}$, then it follows from the inequality in the second line of (A.4) that $p_L - c_L \leq \delta V_L(t + 1)$ for $t < \bar{t}$. But this is impossible, since $q_S(\bar{t}) \in (0, 1)$ implies that $V_L(\bar{t}) = p_L - c_L$.

22. Strictly speaking, in the special case where $\alpha_0 = \bar{\alpha}$ the profit is less than or equal to this level.

$(p_H - c_L)/n \geq p_L - c_L$, which may only hold for small enough n. When this inequality is satisfied, each seller playing the proposed strategy and the buyers using the equilibrium strategy of Proposition 1(i) constitutes an equilibrium.

When $\alpha_0 < \bar{\alpha}$, we can construct an equilibrium similar to that of Proposition 1(ii): sellers randomize only in the first period in order to raise the buyers' posterior beliefs, when all sellers announced p_H, to $\alpha_1 = \bar{\alpha}$.

(ii)(b) The optimality of the prescribed strategy profile for sellers and buyers from period T onward follows from part (i). In addition, sellers are required to be indifferent in each of the first $(T-1)$ periods between offering p_H or p_L. In particular, for $t = T - 1$, we must have

$$(q_S(T-1))^{n-1}\left[q_B(T-1)\frac{p_H - c_L}{n} + (1 - q_B(T-1))\delta\frac{p_L - c_L}{n}\right]$$
$$= (q_S(T-1))^{n-1}(p_L - c_L) + R(T-1), \tag{A.5}$$

where $(q_S(T-1))^{n-1}$ is the probability that the other $(n-1)$ sellers charge p_H in period $(T-1)$. On the LHS of this expression we have the payoff from offering p_H (at $T-1$) and on the RHS the payoff from p_L (where $R(T-1)$ denotes the expected payoff to charging p_L in the event that some of the competitors also charge p_L). Since $R(T-1) > 0$, if n is sufficiently large then (A.5) cannot hold no matter what the values of $q_S(T-1), q_B(T-1) \in (0, 1)$ are. On the other hand, if n is small it is possible to verify that, for some parameter values, we can find $q_S(t), q_B(t) \in (0, 1),\ t < T$, satisfying (A.5) and the analogous equalities for $t = 1, \ldots, T-2$; in other words, an equilibrium with temporary collusion can sometimes be sustained.

(iii) We have already shown that, for n large, the equilibria in parts (a) and (b) of Proposition 2(ii) cannot exist. To complete the proof of the proposition, we must show that no other equilibrium exists. Parts (i) and (ii)(a) characterize the equilibria where sellers follow pure strategies or randomize for finitely many periods and then switch to p_H; part (ii)(b) describes the possible equilibria where sellers randomize for finitely many periods and then switch to p_L. It thus remains only to consider the case of sellers in State L randomizing for infinitely many periods. By a similar argument to the monopoly case, we can show the following.

LEMMA 4. *There is no equilibrium where the sellers in State L randomize for infinitely many periods.*

Proof. Observe that, in order for sellers to randomize at any period t $(0 < q_S(t) < 1)$, buyers have to accept p_H with positive probability: $q_B(t) > 0$. Moreover, for the game not to end with probability 1 in finite time, we must have $q_B(t) < 1$ for all t—that is, buyers must also randomize for infinitely many periods. Now recall Lemma 3: this result is still valid and implies that $\lim_{t\to\infty} \alpha_t = \bar{\alpha}$. But if sellers randomize during infinitely many periods, the sequence $\{\alpha_t\}_{t\geq 1}$ is strictly increasing and so, for any t, we have $\alpha_t < \bar{\alpha}$, which contradicts the fact that $q_B(t) > 0$. □

This completes the proof of Proposition 2. □

Proof of Proposition 3.

(i) We omit this proof because it is similar to that of Proposition 2(i).

(ii)(a) When the strategy of the buyers is to accept p_H if all sellers propose p_H and that of all the other sellers is to propose p_H in State L at Date 1, the profit for a given seller who adheres to offering p_H is $(p_H - c_L)/n$. On the other hand, if this seller were to undercut and charge p_L, he would sell immediately to the $1/n$ share of buyers constituting his clientele at $t = 1$. Assigning off-equilibrium path beliefs after this deviation that are equal to the beliefs on the equilibrium path and assuming that the strategy of each seller is still to offer p_H at any later date (i.e., the same equilibrium behavior as in Period 1), buyers will continue to accept all offers of p_H at $t = 1$. The payoff for undercutting is then only $(p_L - c_L)/n$, so that charging p_H is clearly a seller's best response.[23] Given the sellers' strategy, since $\alpha_0 \geq \bar{\alpha}$ it follows that all buyers prefer to accept immediately both p_L and p_H.

(ii)(b) Taking as given the strategies of the buyers and the other sellers as described in this part of the proposition, the payoff to a seller in Period 1 in the event that $r > 0$ other sellers announce p_L is

$$\delta(p_L - c_L)\frac{n-r}{n^2} \tag{A.6}$$

if he charges p_H and

$$\frac{p_L - c_L}{n} + \delta(p_L - c_L)\frac{n-1-r}{n^2} \tag{A.7}$$

if he charges p_L. The probability of this event is then $(q_S(1))^{n-1-r}(1-q_S(1))^r(n-1)!/(r!(n-1-r)!)$.

On the other hand, if $r = 0$ other sellers charge p_L, then his payoff is

$$\frac{p_H - c_L}{n}$$

if he charges p_H and

$$\frac{p_L - c_L}{n} + \delta(p_L - c_L)\frac{n-1}{n^2}$$

if he charges p_L, and the probability of this event is $(q_S(1))^{n-1}$.

23. When the off-equilibrium path beliefs following a deviation to p_L are such that the probability of L is 1 (as discussed in the third paragraph of Section 5), a similar collusive equilibrium can be constructed with continuation strategies such that trade takes place at p_L in State L in the second period. This holds under the condition $p_H - c_L \geq (p_L - c_L)(1 + \delta)$.

To sustain indifference, we need the expected payoff of p_H to equal the expected payoff of p_L. That is, the sum of the terms above describing the payoff associated to p_H (weighted by their respective probabilities) over all r running between 0 and $(n-1)$ must equal the sum of the corresponding terms describing the payoff associated to p_L. Noting that (for all $r > 0$) the difference between (A.6) and (A.7) equals $(\delta/n - 1)(p_L - c_L)/n$, we may simplify terms to obtain the following equality:

$$\left(1 - (q_S(1))^{n-1}\right)\frac{p_L - c_L}{n}\left[\frac{\delta}{n} - 1\right] + (q_S(1))^{n-1}\frac{1}{n}\left[(p_H - c_L) - (p_L - c_L)\left(1 + \delta\frac{n-1}{n}\right)\right] = 0,$$

which can be simplified to

$$\frac{p_L - c_L}{n}\left[\frac{\delta}{n} - 1 - (q_S(1))^{n-1}\delta\right] + \frac{p_H - c_L}{n}(q_S(1))^{n-1} = 0. \qquad \text{(A.8)}$$

Next, let $\bar{q}_S$ be the value of $q_S(1)$ that generates an updated belief of the buyers, after observing all sellers announcing p_H, of $\alpha_1 = \bar{\alpha}$:

$$\bar{q}_S^n = \frac{\alpha_0(1-\bar{\alpha})}{(1-\alpha_0)\bar{\alpha}}.$$

Observe that $q_S(1)$ can be set to take any value between $\bar{q}_S$ and 0, thus inducing a belief $\alpha_1 \geq \bar{\alpha}$ and hence supporting the buyers' choice to accept immediately both p_H and p_L. When $q_S(1) = 0$, the term on the LHS of (A.8) is clearly negative. On the other hand, when $q_S(1)$ is such that $q_S(1) = \bar{q}_S$, the sign of this term is equal to the sign of

$$(p_H - c_L)\left(\frac{\alpha_0(1-\bar{\alpha})}{(1-\alpha_0)\bar{\alpha}}\right)^{(n-1)/n} + (p_L - c_L)\left[\frac{\delta}{n} - 1 - \left(\frac{\alpha_0(1-\bar{\alpha})}{(1-\alpha_0)\bar{\alpha}}\right)^{(n-1)/n}\delta\right], \qquad \text{(A.9)}$$

which is positive for all n under the condition in the statement of part (ii)(b).[24] Therefore, it is always possible to find a value of $q_S(1) \in [0, \bar{q}_S]$ such that (A.8) is satisfied. □

24. The expression in (A.9) is positive if $(p_H - c_L) > (p_L - c_L)\{\delta + (1-\delta/n)[(1-\alpha_0)\bar{\alpha}/(\alpha_0(1-\bar{\alpha}))]^{(n-1)/n}\delta\}$. Noting that $(1-\alpha_0)\bar{\alpha}/(\alpha_0(1-\bar{\alpha}))$ is the reciprocal of a probability and hence is greater than 1, we have $(1-\alpha_0)\bar{\alpha}/(\alpha_0(1-\bar{\alpha})) > [(1-\alpha_0)\bar{\alpha}/(\alpha_0(1-\bar{\alpha}))]^{(n-1)/n}$. Combining this property with the condition in the statement of Proposition (ii)(b) establishes the validity of the inequality at the beginning of this note, and hence the positivity of (A.9).

Proof of Proposition 4.

(i)(a) Given the strategies for the other players described in this part of the proposition, if $\alpha_0 \geq \hat{\alpha}$ the expected profit for an uninformed seller from offering p_L is non-positive. Hence, in this situation an optimal choice for any uninformed seller is indeed to offer p_H, which yields a positive expected profit of $(p_H - c_H)\alpha_0/n$. The informed seller in State L then strictly prefers to charge price p_L if

$$p_L - c_L > \frac{1}{n}(p_H - c_L)$$

is always satisfied for n sufficiently high.

(i)(b) When the other uninformed sellers (as well as the informed one in State L) offer p_L, an uninformed seller also prefers to charge p_L if

$$\alpha_0(p_L - c_H)\frac{1}{n-1} + (1-\alpha_0)(p_L - c_L)\frac{1}{n} \geq 0;$$

this inequality is always satisfied (for n large enough) if $\alpha_0 < \bar{\alpha}$. It is then immediate to see that the informed seller's choice of offering p_L (in State L) is an optimal response to the uninformed sellers' strategy, since $(p_L - c_L)/n \geq 0$.

(ii) Let n be sufficiently small that $p_L - c_L \leq (p_H - c_L)/n$. Note that $p_L - c_L \geq (p_L - c_H)\alpha_0 + (1-\alpha_0)(p_L - c_L)$ for all $\alpha_0 \in [0, 1]$; the unit expected payoff of trading at p_L is always higher for the informed than for the uninformed seller because the former can choose to trade at this price only in State L.

First, we have equilibria that resemble the ones found in the monopoly section. That is, if $\alpha_0 \geq \bar{\alpha}$ offering p_H every period—both for the uninformed sellers and the informed seller in the H and L states—constitutes an equilibrium.[25]

In addition, there may be equilibria that involve randomization of the informed seller in State L for $T > 1$ periods and, in period $T+1$, a deterministic offer of p_L (this low price in the final trading date is needed to sustain the randomization of buyers, which in turn is required for the informed seller to be willing to randomize in earlier periods). In the first T periods, we can construct equilibria where the uninformed sellers charge p_H as well as others where they randomize between p_H and p_L; in Period $T+1$, all the uninformed sellers charge p_L (if $\alpha_{T+1} < \widetilde{\alpha}$), or they all charge p_H, or possibly they randomize.

(iii) We show finally that, as $n \to \infty$, the only equilibria are those with full and immediate separation as described in (i). Evidently, there is no equilibrium where the informed seller in State L charges p_H with probability 1 (as the first of the equilibria described in (ii)): with n large, undercutting is always preferred. By an argument similar to the one in the proof of Lemma 4, sellers cannot randomize

25. On the other hand, if $\alpha_0 < \bar{\alpha}$ there is an equilibrium where the informed seller in State L randomizes in the initial period to induce the belief $\alpha_1 = \bar{\alpha}$.

for infinite periods. Thus we need only examine the other equilibria described in (ii), where the type L seller randomizes for T periods before choosing p_L with probability 1.

Suppose first that the uninformed sellers choose p_H at $(T+1)$. Then, the indifference condition between p_L and p_H for the informed seller in period T is

$$(q_U(T))^{n-1}(p_L - c_L) + \sum_{r=1}^{n-1}(1 - q_U(T))^r (q_U(T))^{n-1-r} \frac{(n-1)!}{r!(n-1-r)!} \frac{p_L - c_L}{r+1}$$
$$= (q_U(T))^{n-1}\left[\frac{p_H - c_L}{n} q_B(T) + \delta(p_L - c_L)\right].$$

For n sufficiently large, the term on the RHS is approximately $(q_U(T))^{n-1}\delta(p_L - c_L)$. This is smaller than the first term on the LHS because $\delta < 1$, and in addition the rest of terms on the LHS are not negligible. Thus, the expression on the LHS exceeds the one on the RHS—a contradiction.

The same is true a fortiori if the uninformed choose p_L with positive probability at $(T+1)$. We conclude that these also cease to be equilibria for n large. □

Proof of Proposition 5.

(i)(a) Suppose full separation occurs in equilibrium in some Period t. Then, in State H the informed seller charges p_H in that period while in State L the informed seller charges p_L. By Bayes' rule, the buyers and the uninformed sellers must update their beliefs at t to $\alpha_t = 1$ upon observing the informed seller charging p_H and must update them to $\alpha_t = 0$ upon observing the informed seller charging p_L. If p_L is observed, the clientele of the informed seller must clearly accept. But so must the clientele of the informed seller if he charges p_H, because the only reason to reject would be to expect p_L in the future. However, the low price in the future could only come from the uninformed sellers, who believe now that the state is H with probability 1 and hence will never charge the low price. Therefore, the clientele of the informed seller must accept both p_H and p_L. It follows that, in State H, all units are sold in Period t because all buyers accept even p_H. When all the uninformed announce p_L (which may only occur when $\alpha_0 \leq \alpha_t < (p_L - c_L)/(c_H - c_L)$), the informed seller's profit from announcing p_L in State L is then $(p_L - c_L)/n$; this is always smaller than the profit from announcing p_H, which is given by $(p_H - c_L)/n$. On the other hand, when all the uninformed announce p_H, the informed's profit from announcing p_L in State L is $(p_L - c_L)/n + \delta(p_L - c_L)(n-1)/n^2$, again smaller than the profit from announcing p_H, $(p_H - c_L)/n$, under the conditions stated in (i)(a). Thus, under those conditions the informed seller in State L has an incentive to deviate and charge p_H.

(i)(b) The strategies supporting separation are as follows. The informed seller charges p_L in State L and p_H in State H in every period. The uninformed sellers also charge p_H in Period 1 but then switch to p_L if they observe the informed seller charging p_L. Buyers accept both prices if they observe the informed seller charging p_H; they accept only p_L if they observe the informed seller charging p_L.

The optimality of the buyers' strategy easily follows from the degeneracy of their beliefs (obtained by using Bayes' rule at all information sets where the informed charge p_H or p_L, thus assigning off-equilibrium beliefs equal to the ones on the equilibrium path when only uninformed sellers deviate). Consider now the informed seller in State L. Given that the collusive payoff is not too attractive (the first condition on the parameter values imposed in (i)(b)), there exists an n large enough such that p_L is a best response:

$$\frac{p_L - c_L}{n} + \delta(p_L - c_L)\frac{n-1}{n^2} > \frac{p_H - c_L}{n}.$$

Finally, the uninformed sellers prefer to charge p_H at $t = 1$ if

$$\alpha_0 \frac{p_H - c_H}{n} + \delta(1 - \alpha_0)\frac{(p_L - c_L)(n-1)}{n^2}$$
$$> \alpha_0 \frac{p_L - c_H}{n} + (1 - \alpha_0)(p_L - c_L)\left(\frac{1}{n} + \frac{\delta(n-2)}{n^2}\right),$$

which simplifies to

$$\alpha_0(p_H - p_L) > (1 - \alpha_0)(p_L - c_L)(1 - \delta/n).$$

The validity of this condition is ensured by the assumption made on α_0 in (i)(b) for large enough n.

(ii)(a) We construct here an equilibrium in which p_H is announced by all sellers in period 1. Therefore, the belief held by uninformed traders is equal to their prior α_0. Because $\alpha_0 \geq \bar{\alpha}$, a best response for buyers is accepting p_H. Furthermore, we consider the case where the off-equilibrium belief, following a deviation to p_L in period 1, remains α_0; hence p_H continues to be accepted by buyers. In this situation, charging p_H is clearly the best response for the informed seller in State L, since the effect of charging p_L would be to lower the profits from $(p_H - c_L)/n$ to $(p_L - c_L)/n$ (because of the clientele friction and given the assigned off-equilibrium beliefs, the only effect of undercutting would be to lower revenues over the same market share).[26]

26. As in Proposition 3, when the off-equilibrium path beliefs following a deviation to p_L are such that the probability of L is 1, a limited increase in the seller's market share is obtained if he undercuts. The deviation is again nonprofitable under the condition $p_H - c_L \geq (p_L - c_L)(1 + \delta)$.

By essentially the same argument, charging p_H is also a best response for any uninformed seller. If he were to deviate and charge p_L, the beliefs held by uninformed traders would remain unchanged and hence buyers would continue to accept p_H from all the other sellers. Thus his expected profit would be $\alpha_0(p_L - c_H)/n + (1 - \alpha_0)(p_L - c_L)/n$, lower than the one obtained when charging p_H, which is given by $\alpha_0(p_H - c_H)/n + (1 - \alpha_0)(p_H - c_L)/n$.

(ii)(b) Let q_B be the probability with which buyers accept p_H, in any period t, when all prices announced (at t and any earlier date) are p_H. By Bayes' rule, upon observing p_L charged by the informed seller and p_H by all uninformed sellers, the posterior belief is that the state is L with probability 1.

We begin by analyzing the incentives of the informed seller in State L. For him to be willing to randomize in Period 1, the following equality must hold:

$$\frac{p_L - c_L}{n}\left[1 + \delta\frac{n-1}{n}\right] = \frac{p_H - c_L}{n}\left[q_B + \frac{\delta(1 - q_B)q_B}{1 - \delta(1 - q_B)}\right]. \qquad \text{(A.10)}$$

Under the condition that the collusion payoff is attractive enough, it is easy to see that, for fixed δ and n, there exists a unique value of q_B that makes the equality hold. This will be the equilibrium value of q_B. At any later date $t > 1$ the payoffs from announcing p_H and p_L are again the same (when the off-equilibrium beliefs, following a deviation to p_L, are that the state is L with probability 1); hence charging p_H is (weakly) a best response.

Consider next the uninformed sellers. If one of them deviates to p_L we assign off-equilibrium beliefs equal to the ones on the equilibrium path. Clearly the payoff associated to this deviation is positive only if $p_L > \alpha_0 c_H + (1 - \alpha_0)c_L$. In that case, if the strategy of all traders following this deviation prescribes offering p_L, it is easily verified that the expected payoff for deviating to p_L is $[p_L - (\alpha_0 c_H + (1 - \alpha_0)c_L)][1 + \delta(n - 1)/n]/n$. This expression, taking into account (A.10), is strictly smaller than the expected payoff from p_H : $[p_H - (\alpha_0 c_H + (1 - \alpha_0)c_L)][q_B + \delta(1 - q_B)q_B/(1 - \delta(1 - q_B))]/n$. This shows that the deviation considered is unprofitable.

Note, finally, that the probability of the informed seller charging p_H at $t = 1$ is chosen so as to yield $\alpha_1 = \bar{\alpha}$. Thus, buyers are willing to randomize at $t = 1$ between accepting and rejecting upon the observation of all prices being p_H. Given that pooling on p_H takes place from Period 2 onward, the best response of buyers is to continue their randomization in any period $t > 1$. □

Appendix B: Robustness of Results

The results obtained on the characterization of the equilibria are robust to the extension of the model along several dimensions. We will formally establish these properties for the monopoly model and then discuss their extension to the oligopoly case.

Continuum of Prices

The characterization of the equilibria was obtained under the simplifying assumption that the seller can only propose one of two possible prices, p_H and p_L, which are exogenously given. We show here that the main qualitative properties of equilibria, in particular with regard to their information and efficiency properties, remain essentially the same when the seller is free to propose any price in a closed interval that is a subset of (c_L, u_H). Thus, for the issues we have studied, our simplifying assumption is without essential loss of generality.

(a) *We still have no equilibrium sequence with separation in finite time—that is, the seller charging different prices at some t in H and L with probability* 1.[27] This result follows by a straightforward extension of Lemma 1 in Appendix A. Suppose such an equilibrium existed. In State H the seller would offer a price $\bar{p} \geq c_H$ (sales at any price below c_H would result in losses). If buyers accept such price with probability 1, then also in State L the seller would prefer to charge $\bar{p}$ rather than the separating price, which is some price $\underline{p}$ less or equal than u_L—a contradiction. But neither are there equilibria in which the high price is always rejected with some positive probability: such randomization on the part of buyers would actually require—after Period t, when information is revealed—a decreasing sequence of prices in State H, which at some point would have to fall below c_H, another contradiction.[28]

(b) *If the prior belief α_0 is high enough that we can find prices (typically an interval of prices $I \subseteq [c_H, u_H)$) at which buyers and both types of seller are willing to trade, then pooling equilibria with immediate trading again exist—though now there is a continuum of them.* For any $p \in I$, there is in fact an equilibrium where the seller charges p both in State H and L and buyers accept. If the seller deviates to $p' \neq p$, off-equilibrium beliefs are such that $\alpha_1 = 0$ and so buyers accept p' if and only if $p' \leq u_L$, which ensures that no profitable deviation exists for any type of seller. This continuum of equilibria raises the complicated issue of equilibrium coordination, which we avoid with our simplifying assumption.

27. This stands in contrast with the result of Laffont and Maskin (1990) that a separating equilibrium always exists, obtained in the setup of a one-period trading model where the uninformed's demand is strictly decreasing in the price.

28. Note that if u_H were allowed as a possible price then an equilibrium of this type would exist: set $\bar{p} = u_H$, $\underline{p} = u_L$, and $q_B(t)$ constant and low enough to deter deviations by the State L seller.

(c) *If α_0 is small enough that there is no price at which buyers and the seller in both states are willing to trade, then the seller's strategy in State L must include some revelation of information as in the case considered before with only two prices.* Therefore, equilibria are always characterized by the fact that the State L seller randomizes in the initial period between two prices. The low price is always given by u_L. (To see this, observe that following such a low price announcement we have a complete information game between the State L seller and the buyers; in such a game, u_L is the only equilibrium price.) The high price is some price above c_H that is also charged by the seller in State H.[29] Buyers then randomize between accepting and rejecting.

(c1) *Equilibria always exist where, at all subsequent dates $t > 1$, the State L seller no longer randomizes and proposes the high price while buyers continue to randomize between accepting and rejecting.* Such equilibria have then the same features as the equilibrium with partial information revelation that we obtained in Proposition 1 for the case of low α_0. Again there is a continuum of such equilibria, associated to the different values that the high price can take.

(c2) *There may be additional equilibria, where the seller (in State L) randomizes for an arbitrary and possibly infinite number of periods.* At all equilibria where the State L seller randomizes for more than one period (so that $\alpha_{t+1} > \alpha_t \geq \bar{\alpha}$), buyers have to randomize and—by essentially the same argument as in the proof of Lemma 2—they must do so at all periods. This will in turn be optimal provided the following condition holds:

$$\begin{aligned}\alpha_t u_H + (1-\alpha_t)u_L - p_H(t) \\ = \delta(1-(1-\alpha_t)(1-q_S(t+1))) \\ \times (\alpha_{t+1}u_H + (1-\alpha_{t+1})u_L - p_H(t+1)),\end{aligned} \tag{B.1}$$

where $p_H(t)$ denotes the price charged by the seller in State H with probability 1 and in State L with probability $q_S(t)$, which may now vary with t. Since the sequence of posterior beliefs $\{\alpha_t\}_{t\geq 1}$ must converge and since the prices in State H, $\{p_H(t)\}_{t\geq 1}$, cannot fall below c_H as t tends to infinity, condition (B.1) can hold only if, for large t,

$$\alpha_t u_H + (1-\alpha_t)u_L - p_H(t) \cong \alpha_{t+1}u_H + (1-\alpha_{t+1})u_L - p_H(t+1) \cong 0. \tag{B.2}$$

29. It is possible that the seller randomizes in State H, also (and that the State L seller randomizes over the low price and more than one level of the high price). The argument in the text easily extends to this case.

Note that $\alpha_t u_H + (1-\alpha_t)u_L - p_H(t) \geq 0$ for all t, and (B.1) implies that the sequence $\{\alpha_t u_H + (1-\alpha_t)u_L - p_H(t)\}_t$ is increasing. Hence from (B.2) we obtain:

$$\alpha_t u_H + (1-\alpha_t)u_L - p_H(t) = 0 \qquad \text{for all } t; \tag{B.3}$$

that is, buyers are always indifferent between trading and not trading at $p_H(t)$, as in the equilibrium with one period randomization described in (c1). Moreover, from (B.3) it follows that the price $p_H(t)$ proposed in State H increases over time.

Thus, information can now be revealed over many periods. However, it is important to notice that, as shown below, the payoffs of buyers and the seller at all such equilibria are either equal or lower to the payoffs obtained in the equilibria where the seller in State L randomizes for only one period. The equilibria with more periods of information revelation (as in (c2)) are then interim Pareto dominated by the equilibrium with only one period of randomization (as in (c1)). More information revelation requires greater delays in trading and hence entails a welfare loss.

LEMMA 5. *At all equilibria in which the State L seller randomizes for more than one period (as in (c2)), the payoff of buyers and the State L seller is the same whereas the payoff of the State H seller is lower than in an equilibrium in which the State L seller randomizes for one period only (as in (c1)).*

Proof. From the above description it is immediate that the present value of the discounted expected flow of payoffs at an equilibrium as in (c2) is $u_L - c_L$ for the seller in State L and 0 for buyers (since, as we argued, $p_L = u_L$); thus for both it is the same as in the equilibrium in (c1). The payoff for the State H seller then has the same expression as in such equilibrium

$$p_L - c_L - \sum_{\tau=1}^{\infty}\left[\delta^{\tau-1} q_B(\tau)\left(\Pi_{l=1}^{\tau-1}(1-q_B(l))\right)\right](c_H - c_L),$$

though its value will typically be different (since buyers may randomize with different probabilities): in particular, we can show it will be lower. This follows because, as can be seen from (A.4), sustaining the State L seller's randomization with an increasing price sequence $\{p_H(t)\}_{t\geq 1}$ requires buyers to reject with higher probabilities ($q_B(\cdot)$ will be lower), and these are the only variables determining the State H seller's payoff (as we see from the previous expression). □

Finite Horizon

All the equilibria we obtained for the case of infinitely many trading dates remain (or are approximated by) equilibria for which there is only a finite number $\mathrm{T} \geq 1$ of trading dates. This is clear when the equilibrium in question involves a finite number of periods of trade; but even when there are infinite periods of trade—for example with infinite randomization on the part of buyers—for any T there is a payoff equivalent equilibrium where buyers randomize at each trading date.

To illustrate, consider the one-period ($\mathrm{T} = 1$) version of the model. In this case, rejection of a price leads to "no trade" and thus to a zero payoff. It is easy to see that, if $\alpha_0 > \bar{\alpha}$, the unique equilibrium continues to yield trade at p_H in both states. If $\alpha_0 < \bar{\alpha}$ then the unique equilibrium prescribes that the seller in State L randomize between p_H and p_L so as to induce the belief $\alpha_1 = \bar{\alpha}$, while buyers must accept the high price with probability $q_B(1)$ (where $q_B(1)$ is set at a level such that the seller in L is exactly indifferent between charging p_L and p_H).

When $\mathrm{T} = 2$, the equilibrium with $\alpha_0 > \bar{\alpha}$ continues to have pooling on p_H and immediate trade with no delay. When $\alpha_0 < \bar{\alpha}$, with $\mathrm{T} = 2$ we continue to have a unique equilibrium in which $q_S(1)$ is chosen to yield $\alpha_1 = \bar{\alpha}$, while $q_B(1)$ and $q_B(2)$ are chosen to make the seller in State L indifferent between charging p_L and p_H at $t = 1$ and to (weakly) prefer p_H at $t = 2$; the information is then partially revealed only in period 1.

We can also show that no other equilibria exist; in particular, there is no equilibrium where both buyers and the seller in State L randomize at each trading date. In fact in this case we must have $\alpha_t > \bar{\alpha}$ for all t; thus, buyers at the terminal date accept p_H for sure and so the seller always proposes p_H at T, but then buyers cannot be indifferent between accepting and rejecting p_H at $\mathrm{T} - 1$.

We thus conclude that the qualitative properties of the set of equilibria are essentially the same when $\mathrm{T} < \infty$ and $\mathrm{T} = \infty$, so our findings are not generated by a possible discontinuity at $\mathrm{T} = \infty$.

Robustness of Results in the Oligopoly Models

The robustness analysis can build, to a large extent, on the properties derived in this regard for the monopoly case. In particular, it is easy to see that the full and immediate revelation result of the model without clienteles for n sufficiently large is again found with a continuum of possible prices, though now there will be a continuum of such equilibria; also, for n large enough, no collusive equilibria exist. Analogously, the model with clienteles has equilibria with no or only partial information revelation, whatever is n. Similar findings are obtained in the corresponding finite-horizon versions of these models.

References

Blouin, Max, and Roberto Serrano (2001). "A Decentralized Market with Common Values Uncertainty: Non-Steady States." *Review of Economic Studies,* 68, 323–346.

Cripps, Martin, and Jeroen M. Swinkels (2004). "Efficiency in Large Double Auctions." Working Paper, Washington University in St. Louis.

Dubey, Pradeep, John Geanakoplos, and Martin Shubik (1987). "The Revelation of Information in Strategic Market Games: A Critique of Rational Expectations Equilibrium." *Journal of Mathematical Economics,* 6, 105–137.

Forges, Francoise (1992). "Repeated Games of Incomplete Information: Non-0 Sum." In *Handbook of Game Theory with Economic Applications,* vol. I, edited by Robert J. Aumann and Sergiu Hart. North-Holland.

Forges, Francoise, and Enrico Minelli (1997). "Self-Fulfilling Mechanisms and Rational Expectations." *Journal of Economic Theory,* 75, 388–406.

Foster, Douglas F., and S. Viswanathan (1996). "Strategic Trading When Agents Forecast the Forecast of Others." *Journal of Finance,* 51, 1437–1478.

Gale, Douglas (1987). "Limit Theorems for Markets with Sequential Bargaining." *Journal of Economic Theory,* 43, 20–54.

Grinblatt, Mark, and Stephen Ross (1985). "Market Power in a Securities Market with Endogenous Information." *Quarterly Journal of Economics,* 100, 1143–1167.

Gul, Faruk, and Andrew Postlewaite (1992). "Asymptotic Efficiency in Large Exchange Economies with Asymmetric Information." *Econometrica,* 60, 1273–1292.

Gul, Faruk, Hugo Sonnenschein, and Robert Wilson (1986). "Foundations of Dynamic Monopoly and the Coase Conjecture." *Journal of Economic Theory,* 39, 155–190.

Holden, Craig W., and Avanidhar Subrahmanyam (1992). "Long-Lived Private Information and Imperfect Competition." *Journal of Finance,* 47, 247–270.

Kyle, Albert (1985). "Continuous Auctions and Insider Trading." *Econometrica,* 53, 1315–1335.

Kyle, Albert (1989). "Informed Speculation with Imperfect Competition." *Review of Economic Studies,* 56, 317–356.

Laffont, Jean-Jacques, and Eric Maskin (1990). "The Efficient Market Hypothesis and Insider Trading on the Stock Market." *Journal of Political Economy,* 98, 70–93.

Medrano, Luis A., and Xavier Vives (2001). "Strategic Behavior and Price Discovery." *RAND Journal of Economics,* 32, 221–248.

Mertens, Jean-Francois, Sylvain Sorin, and Shmuel Zamir (1994). "Repeated Games." Working Papers 9420, 9421, and 9422, CORE, Universitè Catholique de Louvain.

Perry, Motty, and Philip Reny (2003). "Toward a Strategic Foundation for Rational Expectations Equilibrium." Working Paper, University of Chicago.

Pesendorfer, Wolfgang, and Jeroen M. Swinkels (1997). "The Loser's Curse and Information Aggregation in Common Values Auctions." *Econometrica,* 65, 1247–1281.

Peters, Michael (1991). "Ex Ante Price Offers in Matching Games: Non-Steady States." *Econometrica,* 59, 1425–1454.

Peters, Michael, and Sergei Severinov (2002). "Internet Trading Mechanisms and Rational Expectations." Working Paper, University of British Columbia.

Radner, Roy (1982). "Equilibrium under Uncertainty." In *Handbook of Mathematical Economics,* vol. II, edited by Kenneth J. Arrow and Michael D. Intriligator. North-Holland.

Rustichini, Aldo, Mark Satterthwaite, and Steven Williams (1994). "Convergence to Efficiency in a Simple Market with Incomplete Information." *Econometrica,* 62, 1041–1063.

Satterthwaite, Mark, and Steven R. Williams (2002). "The Optimality of a Simple Market Mechanism." *Econometrica,* 70, 1841–1863.

Serrano, Roberto (2002). "Decentralized Information and the Walrasian Outcome: A Pairwise Meetings Market with Private Values." *Journal of Mathematical Economics,* 38, 65–89.

Serrano, Roberto, and Oved Yosha (1996). "Welfare Analysis of a Market with Pairwise Meetings and Asymmetric Information." *Economic Theory,* 8, 167–175.

Shapley, Lloyd, and Martin Shubik (1977). "Trade Using One Commodity as Means of Payment." *Journal of Political Economy,* 85, 937–968.

Shneyerov, Artem, and Mark Satterthwaite (2003). "Convergence of a Dynamic Matching and Bargaining Market with Two-Sided Incomplete Information to Perfect Competition." Working Paper 03-12, University of British Columbia.

Vives, Xavier (1993). "How Fast Do Rational Agents Learn?" *Review of Economic Studies,* 60, 329–347.

Vives, Xavier (1995). "The Speed of Information Revelation in a Financial Market Mechanism." *Journal of Economic Theory,* 67, 178–204.

Wolinsky, Asher (1990). "Information Revelation in a Market with Pairwise Meetings." *Econometrica,* 58, 1–23.

Zamir, Shmuel (1992). "Repeated Games of Incomplete Information: 0-Sum." In *Handbook of Game Theory with Economic Applications,* vol. I, edited by Robert J. Aumann and Sergiu Hart. North-Holland.