# Lecture Notes: September 21, 2005

## 1. Savings.

Investment does not equal savings because of the possibility of international trade. However, investment rates and savings rates are highly correlated for the obvious reasons.

*Understand the savings related poverty traps done in class. Differences in Savings Rates Arising from Differences in Incomes*

If countries differed only in terms of savings rates, then GDP would differ by a factor of:

$$\frac{y_{i,ss}}{y_{j,ss}} = \left(\frac{\gamma_i}{\gamma_j}\right)^{\frac{\alpha}{1-\alpha}}$$

We simply take the ratio of the steady states and assume all else to be equal. The above expression shows that steady state levels of output will differ by this amount, if the only difference is in savings rates.

Brief Graph

## 2. Population Growth

We will now relax our assumption that population remains constant in determining steady-state income per capita in the Solow Model. That is, we add the effects of population growth to the evolution of capital.

Previously, the level of capital was determined by two things: 1) investment and 2) depreciation. With the assumption that population is changing, the level of capital per worker must be affected. We can find out exactly how this works through a bit of math.

**Using Calculus:**

First, simply write the equation for capital per worker and take the derivative with respect to time. Keeping in mind that we have two variables (K and L), we get the following:

$$\frac{\partial \frac{K}{L}}{\partial t} = \dot{k} = \frac{\left(\frac{\partial K}{\partial t}\right)L - K\left(\frac{\partial L}{\partial t}\right)}{L^2},$$

$$\dot{k} = \left(\frac{\left(\frac{\partial K}{\partial t}\right)}{L}\right) - \left(\frac{K}{L}\right)\left(\frac{\left(\frac{\partial L}{\partial t}\right)}{L}\right),$$

$$\dot{k} = \gamma(y) - \delta(k) - (k)\hat{L}$$

$$\dot{k} = \gamma(y) - \delta(k) - n(k)$$

$$\dot{k} = \gamma(y) - (\delta + n)k$$

So our new equation for the change in capital over time adds the effect of capital dilution due to increased population.

**Without Using Calculus:**

For those of you that are a bit uncomfortable with calculus, we can derive similar results for the effect of population growth on capital per worker. The following derivation is not completely accurate, as it is done in discrete time.

$$k_t - k_{t+1} = \frac{K_t}{L_t} - \frac{K_t}{L_{t+1}} = \frac{K_t}{L_t} - \frac{K_t}{(L_t)(1+n)} = \frac{K_t(1+n) - K_t}{(L_t)(1+n)} = \left(\frac{n}{1+n}\right)\frac{K_t}{L_t} = \left(\frac{n}{1+n}\right)k_t \cong nk_t.$$

The amount that capital per worker is diluted by is approximately *n* (the growth rate of the population) multiplied by the existing level of capital per worker. We now subtract this value from investment because it works similarly to the effects of depreciation.

$$\dot{k} = \gamma y - \delta k - nk,$$

$$\dot{k} = \gamma y - (\delta + n)k$$

Again, we get the same equation.

**Steady States**

We use the same methodology as before. The investment curve is graphed identically as before. However, the former depreciation curve is transformed into a depreciation and population growth curve that effectively changes the magnitude of the slope of the curve.

(look at graph)

For a production function given as: $y = Ak^{\alpha}; \dot{k} = \gamma Ak^{\alpha} - (\delta + n)k$, the steady state level of capital per worker now becomes:

$$k_{ss} = \left(\frac{A\gamma}{\delta + n}\right)^{\frac{1}{1-\alpha}}; y_{ss} = A^{\frac{1}{1-\alpha}}\left(\frac{\gamma}{\delta + n}\right)^{\frac{\alpha}{1-\alpha}}$$

**Variations on Population Growth Rates**

*Understand the population related poverty traps done in class. Differences in fertility rates, whether exogenous to income or endogenous to income.*

If countries differed only in terms of population growth rates, then GDP would differ by a factor of:

$$\frac{y_{i,ss}}{y_{j,ss}} = \left(\frac{\delta + n_j}{\delta + n_i}\right)^{\frac{\alpha}{1-\alpha}}$$

We simply take the ratio of the steady states and assume all else to be equal, except for population growth rates. The above expression shows that steady state levels of output will differ by this amount, if the only difference is in population growth rates.

**3. Understanding Steady States.**

By the version of the Solow Model we have discussed, one should realize that output:

1) Will converge to a steady state
2) Growth occurs only when output is converging to a steady state

To see this, let's take a look at how output will change over time.

$$\hat{y} = \hat{A} + \alpha\hat{k}$$

Output will grow only if *A* or *k* continues to grow. For the latter term,

$\hat{k} = \frac{\dot{k}}{k} = \frac{0}{k_{ss}} = 0$ in the steady state.

Therefore, the parameter *A* is the only thing that can sustain growth over an extended period of time.