Econ 121
Problem Set 5
Solutions

Problem 6.2.

To show that the unemployment rate evolves over time to the steady-state rate, let's begin by defining how the number of people unemployed changes over time. The change in the number of unemployed equals the number of people losing jobs (sE) minus the number finding jobs (fU). In equation form, we can express this as:

$$U_{t+1} - U_t = \Delta U_{t+1} = sE_t - fU_t.$$

Recall from the text that $L = E_t + U_t$, or $E_t = L - U_t$, where L is the total labor force (we will assume that L is constant). Substituting for E_t in the above equation, we find:

$$\Delta U_{t+1} = s(L - U_t) - fU_t.$$

Dividing by L, we get an expression for the change in the unemployment rate from t to $t + 1$:

$$\Delta U_{t+1}/L = (U_{t+1}/L) - (U_t/L) = \Delta[U/L]_{t+1} = s(1 - U_t/L) - fU_t/L.$$

Rearranging terms on the right-hand side, we find:

$$\begin{aligned}\Delta[U/L]_{t+1} &= s - (s + f)U_t/L \\ &= (s + f)[s/(s + f) - U_t/L].\end{aligned}$$

The first point to note about this equation is that in steady state, when the unemployment rate equals its natural rate, the left-hand side of this expression equals zero. This tells us that, as we found in the text, the natural rate of unemployment $(U/L)^n$ equals $s/(s + f)$. We can now rewrite the above expression, substituting $(U/L)^n$ for $s/(s + f)$, to get an equation that is easier to interpret:

$$\Delta[U/L]_{t+1} = (s + f)[(U/L)^n - U_t/L].$$

This expression shows the following:

- If $U_t/L > (U/L)^n$ (that is, the unemployment rate is above its natural rate), then $\Delta[U/L]_{t+1}$ is negative: the unemployment rate falls.
- If $U_t/L < (U/L)^n$ (that is, the unemployment rate is below its natural rate), then $\Delta[U/L]_{t+1}$ is positive: the unemployment rate rises.

This process continues until the unemployment rate U/L reaches the steady-state rate $(U/L)^n$.

Problem 6.5.

a. The demand for labor is determined by the amount of labor that a profit-maximizing firm wants to hire at a given real wage. The profit-maximizing condition is that the firm hire labor until the marginal product of labor equals the real wage,

$$MPL = \frac{W}{P}.$$

The marginal product of labor is found by differentiating the production function with respect to labor (see the appendix to Chapter 3 for more discussion),

$$MPL = \frac{dY}{dL}.$$
$$= \frac{d(K^{1/3}L^{2/3})}{dL}$$
$$= \frac{2}{3}K^{1/3}L^{-1/3}.$$

In order to solve for labor demand, we set the *MPL* equal to the real wage and solve for L:

$$\frac{2}{3}K^{1/3}L^{-1/3} = \frac{W}{P}$$
$$L = \frac{8}{27}K\left(\frac{W}{P}\right)^{-3}.$$

Notice that this expression has the intuitively desirable feature that increases in the real wage reduce the demand for labor.

b. We asume that the 1,000 units of capital and the 1,000 units of labor are supplied inelastically (i.e., they will work at any price). In this case we know that all 1,000 units of each will be used in equilibrium, so we can substitute them into the above labor demand function and solve for $\frac{W}{P}$.

$$1{,}000 = \frac{8}{27}1{,}000\left(\frac{W}{P}\right)^{-3}$$
$$\frac{W}{P} = \frac{2}{3}.$$

In equilibrium, employment will be 1,000, and multiplying this by 2/3 we find that the workers earn 667 units of output. The total output is given by the production function:

$$Y = K^{1/3}L^{2/3}$$
$$= 1{,}000^{1/3}1{,}000^{2/3}$$
$$= 1{,}000.$$

Notice that workers get two-thirds of output, which is consistent with what we know about the Cobb–Douglas production function from the appendix to Chapter 3.

c. The congressionally mandated wage of 1 unit of output is above the equilibrium wage of 2/3 units of output.

d. Firms will use their labor demand function to decide how many workers to hire at the given real wage of 1 and capital stock of 1,000:

$$L = \frac{8}{27}1{,}000(1)^{-3}$$
$$= 296,$$

so 296 workers will be hired for a total compensation of 296 units of output.

e. The policy redistributes output from the 704 workers who become involuntarily unemployed to the 296 workers who get paid more than before. The lucky workers benefit less than the losers lose as the total compensation to the working class falls from 667 to 296 units of output.

f. This problem does focus the analysis of minimum-wage laws on the two effects of these laws: they raise the wage for some workers while downward-sloping labor demand reduces the total number of jobs. Note, however, that if labor demand is less elastic than in this example, then the loss of employment may be smaller, and the change in worker income might be positive.

Additional Exercises:

(1) Consider a country in which there are two sectors, called Sector 1 and Sector 2. The production functions in two sectors are:

$$Y_1 = L_1^{\frac{1}{2}}$$

$$Y_2 = L_2^{\frac{1}{2}}$$

Where L_1 is the number of worker in employed in Sector 1 and L_2 is the number of workers employed in Sector 2. The total number of workers in the economy is L.

i) Calculate the number of workers in each sector if workers are paid their marginal products in each sector, assuming no unemployment. (*hint: answer is given in terms of L*)

Workers will be allocated between the two sectors such that the marginal product of both sectors will be equal. Therefore, one must first figure out the expression for the marginal products of each sector. If we denote the letter *i* in order to generalize between sector 1 and sector 2, and we differentiate with respect to labor, we get the following expression:

$$MPL_i = \frac{1}{2}(L_i)^{\frac{-1}{2}} = \frac{1}{2\sqrt{L_i}}$$

The latter expression is written for convenience. Then, equating the marginal products in each sector, we can solve for the amount of labor that would result in equilibrium.

$$MPL_1 = MPL_2$$

$$\frac{1}{2\sqrt{L_1}} = \frac{1}{2\sqrt{L_2}}$$

$$\therefore L_1 = L_2$$

Since the total number of workers in the economy is L, and the only condition for the MPLs to equate is that equal numbers be allocated in each sector, it follows that the number of workers in each sector is given by $\frac{L}{2}$.

ii) Calculate the prevailing wage rate in each sector of the economy.

Because workers are paid their marginal products in each sector and marginal product between sectors are equal, one can calculate the prevailing wage by substituting in for

L_i the amount of workers. Since (L/2) number of workers are in each sector, the prevailing wage rate will be:

$$MPL_i = MPL_{economy} = wage = \frac{1}{2\sqrt{L_i}} = \frac{1}{2\sqrt{\frac{L}{2}}} = \frac{1}{\sqrt{2L}} = (2L)^{\frac{-1}{2}}$$

iii) Suppose that a minimum wage is set by the government at $(L/2)^{\frac{-1}{2}}$. What are the number of workers employed in each sector, and what is the total unemployment rate in the economy?

Because the minimum wage is set above the unrestricted equilibrium wage of the economy, there will be some level of unemployment in the economy. This arises because firms are willing to hire workers only up to the point where the marginal product of labor equals the cost (ie. the wage). In fact, the number of workers employed in each sector will be:

$$wage = MPL_i$$

$$\frac{1}{\sqrt{\frac{L}{2}}} = \frac{1}{2\sqrt{L_i}}$$

$$2\sqrt{L_i} = \sqrt{\frac{L}{2}}$$

$$4L_i = \frac{L}{2}$$

$$L_i = \frac{L}{8}$$

Therefore, employment in each sector will be (L/8); economy-wide employment will be (L/4); total unemployment will be (3L/4); and so the unemployment rate will be 75%.

iv) Suppose the same as in (iii) but that the minimum wage is set at $(L)^{\frac{-1}{2}}$. What are the number of workers employed in each sector and what is the total unemployment rate in the economy?

Because minimum is still restrictive, we follow a similar computational procedure as in part (iii). Doing so, we get the following

$$\text{minimum wage} = MPL_i$$

$$\frac{1}{\sqrt{L}} = \frac{1}{2\sqrt{L_i}}$$

$$2\sqrt{L_i} = \sqrt{L}$$

$$4L_i = L$$

$$L_i = \frac{L}{4}$$

Therefore, employment in each sector will be (L/4); economy-wide employment will be (L/2); total unemployment will be (L/2); and so the unemployment rate will be 50%.

Ignore Assumptions Given in (i)-(iv)

v) Calculate the number of workers (where the number of workers is given by L) in each sector if workers are paid their marginal products in Sector 1 but are paid their average product in Sector 2.

The marginal product of labor is given in part (i). However, the average product of labor must be computed. It is simply, output per worker, which means that all we must do is divide by L_i.

$$MPL_1 = \frac{1}{2\sqrt{L_1}}$$

$$APL_2 = \frac{Y_2}{L_2} = \frac{(L_2)^{1/2}}{L_2} = (L_2)^{-\frac{1}{2}} = \frac{1}{\sqrt{L_2}}$$

Equating the MPL with the APL, we derive the relative number of workers in each sector.

$$MPL_1 = APL_2$$

$$\frac{1}{2\sqrt{L_i}} = \frac{1}{\sqrt{L_2}}$$

$$L_2 = 4L_1$$

Therefore, the number of workers in sector 2 is FOUR times that of the number of workers in sector 1. Since the total amount of labor in the economy is given by L, the number of workers employed in sector 1 is (L/5); and the number of workers employed in sector 2 is (4L/5).

vi) Suppose that a minimum wage is set by the government. What must be the value of this minimum wage such that there is some unemployment in the economy? What must be the value of this minimum wage such that there is no unemployment in the economy?

The first step in solving this problem is to determine the equilibrium wage. Because the wages in sectors will be equal, we can substitute the number of workers in sector 1 in the MPL function for sector 1, or we can substitute the number of workers in sector 2 in the APL function for sector 2. Either way will yield the same answer.

$$APL_2 = \frac{1}{\sqrt{L_i}} = \frac{1}{\sqrt{\frac{4L}{5}}} = \sqrt{\frac{5}{4L}} = w*$$

Thus, if the minimum wage is set above *w**, then there will be unemployment in the economy, but if the minimum wage is set below *w**, then all workers will be employed (at least in this setup).

(2) Define your usual variables as provided in Mankiw for deriving the steady state level of unemployment. Now, suppose that some technological innovation leads to better monitoring of workers, such that the job separation rate, *s*, increases to a higher level *s**. Furthermore, this innovation increases the communication channels between employers and potential employees such that *f* rises to a higher level, *f**. Graphically depict the new possible steady state levels of unemployment.

When *f* rises, the slope of the curve depicting *fU* rises. Because the value of *f* is assumed to be positive, the curve becomes 'steeper'. What we see is a sweeping out of the curve around the origin.

When *s* rises, the slope of the curve depicting *sL-sU* also rises. However, the negative sign on the latter term of the above expression indicates that the curve will 'drop' faster, and instead of sweeping our around the origin, it will sweep around the y-axis intercept. In addition to the slope change, the y-intercept will change as well—getting higher. These two effects happen simultaneously for a given rise in *s*.

The intersection of these new curves will determine the new equilibrium steady state level of unemployment. If the resulting intersection is to the right (when view in terms of the x-axis) of the old level, the natural rate of unemployment will have risen. If the resulting intersection is to the left, then the natural rate of unemployment will have fallen. Either case, as well as the coincidental case in which unemployment levels do not change are feasible. For the case of rising unemployment, taking cross partials (not required for you to do) yields the stipulation that $s^* > f^*$. Otherwise, unemployment will likely fall. Using the guesstimate that *s* is usually less than *f* in empirical analysis…the most likely scenario is that both a rise in *s* and *f* will result in a lower natural rate of unemployment (albeit, the opposite is theoretically feasible).