Intermediate Macroeconomics
Econ 121
Midterm I Solutions

Short Answer.

1. Explain the concept (in words **or** in math) of constant, decreasing, and increasing returns to scale. Be clear and concise.

IN WORDS…
A production function exhibits constant returns to scale if inputs are scaled by a certain amount and the subsequent outputs are scaled by the same amount.

A production function exhibits decreasing returns to scale if inputs are scaled by a certain amount and the subsequent outputs are scaled by less than that amount.

A production function exhibits increasing returns to scale if inputs are scaled by a certain amount and the subsequent outputs are scaled by more than that amount.

IN MATH…
Refer to example in the lecture notes. Note that if a student had just written some parameter bounds for the sum of α and β without specifying any type of production function or idea behind it, no credit was given.

2. If the production function $Y = F(K, L)$ exhibits constant returns to scale, what can we conclude about the marginal products of capital and of labor?

If a production function is constant returns to scale, then the marginal products of each input must diminish.

3. Give two endogenous variables in the simple version of the Solow Model.

Output, Capital

4. Give two exogenous variables in the simple version of the Solow Model.

Savings rates, Depreciation Rates, Population Growth Rates

5. Productivity can be decomposed into two distinct components. Name them.

Technology and Efficiency

6. The following equation states that Money Supply multiplied by the Velocity of Money is equal to the Price Level multiplied by Output: *MV=PY*. Write the expression that relates these four variables over time. That is, what is the mathematical relationship between the GROWTH RATE of the four variables? (*hint: ^)*

$$MV = PY$$

$$\ln(M) + \ln(V) = \ln(P) + \ln(Y)$$

$$\frac{\partial \ln(M)}{\partial t} + \frac{\partial \ln(V)}{\partial t} = \frac{\partial \ln(P)}{\partial t} + \frac{\partial \ln(Y)}{\partial t}$$

$$\frac{\dot{M}}{M} + \frac{\dot{V}}{V} = \frac{\dot{P}}{P} + \frac{\dot{Y}}{Y}$$

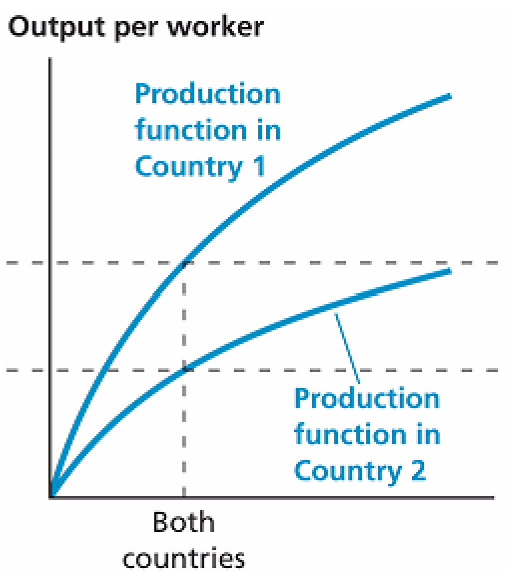
$$\hat{M} + \hat{V} = \hat{P} + \hat{Y}$$

7. The change in capital per worker is governed by what three rates of the economy?

Savings Rates, Depreciation Rates, and Population Growth Rates

8. Graph the production function for two economies that are identical in all ways but for the productivity parameter. Be certain to label each axis and each country with the appropriate productivity parameter.

The horizontal axis represents capital or factors of production. Country 1 has a higher level of productivity than does country two. $A_1 > A_2$



9. If the production function is given by the following, $Y = F(K, L) = K^{\alpha} L^{1-\alpha}$, then what is capital's share of output. That is, write an equation for alpha that is equal to a function of the Marginal Product of Capital.

$$\frac{MPK * K}{Y} = \alpha$$

10. If an economy is currently situated at its steady state level of output per worker (denoted by y_{ss}), what is the steady state level of consumption per worker, assuming your usual variables. (*hint: how is output split in a closed economy without government spending?*)

Since output can be divided up only between consumption and investment…all that is not invested is consumed. Simply, consumption per worker in the steady state is $(1-\gamma)f(k)$ *where gamma is the savings rate and f(k) is output.*

Longer Questions. Show all work and intermediate calculations.

I. (24 points; equally weighted) Suppose an economy is described by the following Cobb-Douglas Production Function:

$$Y = F(K,h,L) = AK^{\alpha}(hL)^{1-\alpha},$$

where Y is GDP, A is productivity, K is the physical capital stock in the economy, L is the number of workers, and α is a strictly positive parameter. h is the level of human capital per worker, a variable that augments the ability of workers to produce output. In addition, assume that physical capital evolves over time in the usual fashion. Physical capital depreciates at a rate δ each period, there is ***no*** population growth, and a fraction γ of output is invested into new physical capital each year.

i. Rewrite the production function in per worker terms where lower case variables represent the analogous per worker term (ie. $k = K/L$).

$$\frac{Y}{L} = \frac{AK^{\alpha}(hL)^{1-\alpha}}{L} = Ak^{\alpha}h^{1-\alpha}$$

ii. Solve for the steady state value of physical capital per worker, assuming that h is a constant.

$$\dot{k} = 0 = \gamma f(k) - \delta k$$

$$\gamma Ak^{\alpha}h^{1-\alpha} = \delta k$$

$$k^{1-\alpha} = \frac{\gamma Ah^{1-\alpha}}{\delta}$$

$$k_{ss} = \left[\frac{\gamma Ah^{1-\alpha}}{\delta}\right]^{\frac{1}{1-\alpha}} = \left[\frac{\gamma A}{\delta}\right]^{\frac{1}{1-\alpha}} h$$

iii. Solve for the steady state value of output per worker, again assuming that h is a constant.

$$y_{ss} = Ak_{ss}^{\alpha} h^{1-\alpha} = A\left[\left[\frac{\gamma A}{\delta}\right]^{\frac{1}{1-\alpha}} h\right]^{\alpha} h^{1-\alpha} = A^{\frac{1}{1-\alpha}}\left[\frac{\gamma}{\delta}\right]^{\left(\frac{\alpha}{1-\alpha}\right)} h$$

iv. Now assume that the level of human capital per worker changes from year to year in the following manner:

$$\dot{h} = \mu h - \eta \ ,$$

where h is the level of human capital per worker, μ is some scale effect that increases human capital per worker and η is an amount lost from year to year. Determine the steady state level of human capital.

$$\dot{h} = 0 = \mu h - \eta$$

$$\mu h = \eta$$

$$h_{ss} = \frac{\eta}{\mu}$$

v. Determine the steady state level of physical capital and the steady state level of output, incorporating your results from question iv.

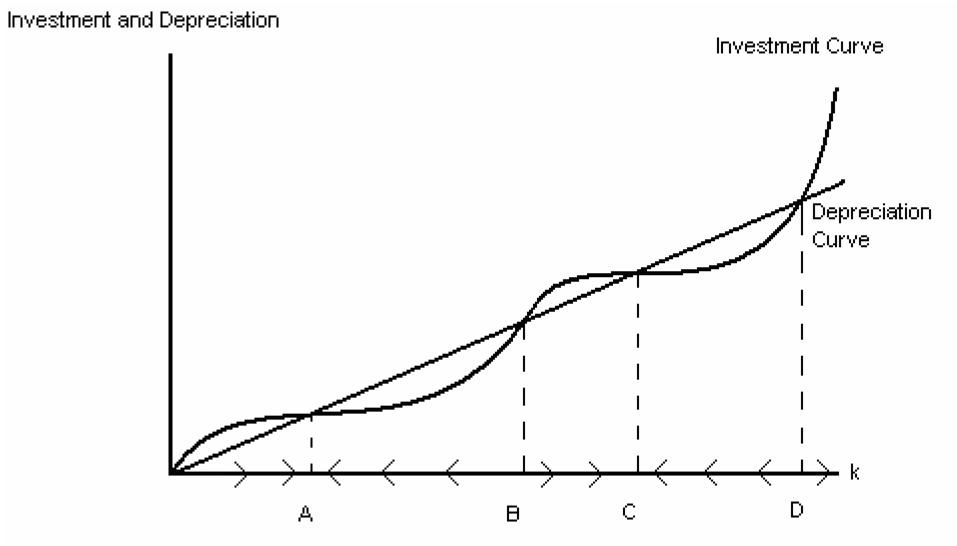
$$k_{ss} = \left[\frac{\gamma A}{\delta}\right]^{\frac{1}{1-\alpha}} h_{ss} = \left[\frac{\gamma A}{\delta}\right]^{\frac{1}{1-\alpha}} \frac{\eta}{\mu}$$

$$y_{ss} = A^{\frac{1}{1-\alpha}}\left[\frac{\gamma}{\delta}\right]^{\left(\frac{\alpha}{1-\alpha}\right)} h_{ss} = A^{\frac{1}{1-\alpha}}\left[\frac{\gamma}{\delta}\right]^{\left(\frac{\alpha}{1-\alpha}\right)} \frac{\eta}{\mu}$$

vi. Suppose that α is equal to one-third; the growth rate of productivity over the past year is 5%, the growth rate of h is 6%, and the growth rate of k is 3%. Then, what is the growth rate of output per worker?

$$\hat{y} = \hat{A} + \alpha\hat{k} + (1-\alpha)\hat{h} = 5\% + \left(\frac{1}{3}\right)3\% + \left(1 - \frac{1}{3}\right)6\% = 5\% + 1\% + 4\% = 10\%$$

II. (11 points). Suppose that stock of physical capital is dependent on only two things: the investment effort and the depreciation amount. The figure below depicts the level of investment and depreciation for a range of possible levels of capital.



i. Label all stable steady state values of physical capital per worker.

A and C are STABLE steady states. B and C are UNSTABLE steady states and capital does not converge to those states.

ii. Draw arrows on the horizontal axis showing how capital will move from year to year.

See figure above.

III. (15 points; equally weighted). Qualitatively state what will happen to the steady state level of output in each case. (fall, rise, uncertain, or no effect) In addition, provide a 'back of the envelope' diagram to illustrate how you obtained your result.

i. The desired number of children permanently increases for all people.

n rises and so y_{ss} must fall

ii. The banking system falters, leading to a permanent decrease in savings rates.

γ falls and so y_{ss} must fall

iii. A breakthrough in medicine enhances productivity but also, people live longer and are able to have more children.

A rises which should make y_{ss} rise, but n also rises which should make y_{ss} fall. The net effect is uncertain.

iv. A compound more durable than steel is discovered.

δ falls and so y_{ss} must rise

v. Half the population dies.

There is no effect on y_{ss}. We are simply off the equilibrium point as no rates have changed.

vi. Half the capital stock is lost.

Again, there is no effect on y_{ss}. We are simply off the equilibrium point as no rates have changed.