Solutions
Homework I
Economic Growth

1) The number of people worldwide living on less than one dollar per day can be calculated using either market exchange rates or purchasing power exchange rates. Which will be larger? Explain why.
Answer: The number of people living on less than a dollar a day will be larger if we calculate it using market exchange rates instead of purchasing power exchange rates because market exchange rates only take into account the relative value of traded goods, which are relatively more expensive in poorer countries. Individuals in these countries will have low purchasing power for traded goods. By using the market exchange rate, we are assuming that traded goods and non-traded goods are the same price, and therefore individuals in poor countries will have low purchasing power for non-traded goods as well, which will make they appear poorer than they actually are.

2) Between 1970 and 2005, China's GDP per capita grew at an average rate of 7.3% per year while GDP per capita in the United States grew at an average rate of 2.2%. In 2005, U.S. GDP per capita was \$36,806 and Chinese GDP per capita was \$5,955. Assuming that the two countries continue to grow at these rates, in what year will China overtake the United States in terms of GDP per capita?
Answer: In order to calculate the year in which income per capita in China will overtake the income per capita in the United States, we first need to find *t*, the number of years it will take for the income per capita in both countries will be equal. That is,

$$(Y_{US,\,2005}) * (1+.022)^t = (Y_{China,\,2005}) * (1+.075)^t \,.$$

Since $Y_{US,\,2005}$ =\$36,806, $Y_{China,\,2005}$ = \$5,955, we then substitute in these values and solve for *t*.

$$(1+0.075/1+.022)^t = (\$36{,}806 / \$5{,}955)$$

Taking the Natural Log of both sides, and noting that $ln(x^y) = y\ ln\ (x)$, we get

$$t\ \ ln\ (1.05) = ln\ (\$36{,}806 / \$5{,}955)$$

$$t = 37.33$$

That is, in 37.33 years, assuming they grow at the current growth rates, the income per capita of China will surpass that of the United States. This year will roughly be 2005+*t*, ie. year 2042.

3) Suppose that the country of Xanadu saves 20% of its income and has a capital-output ratio of 4.
   a) Using the Harrod-Domar model, calculate the rate of growth of total GNP in Xanadu

Answer: Neglecting depreciation in this exercise, The Harrod-Domar model leads us to the equation: $g = s/\theta$, where g is the aggregate growth rate, s is the rate of savings, and θ is the capital-output ratio. Here $s = 1/5$ and $\theta = 4$. So $g = 1/20$, or 5% per year.

   b) If the population growth were 3% per year and Xanadu wanted to achieve a growth rate per capita of 4% per year, what would its savings rate have to be to get to this growth rate?

Answer: We know that the per-capita growth rate is the *aggregate* growth rate minus the population growth rate. Therefore, if the required per-capita growth rate is 4% and the population growth rate is 3%, the required aggregate growth rate is 7% per year, or 7/100. Using the Harrod-Domar equation, we see, therefore, that the required rate of savings is $g \times \theta$, which in this case is (7/100) × 4, or 28% of income.

c) Now go back to the case where the savings rate is 20% and the capital-output ratio is 4. Imagine now, that the economy of Xanadu suffers violent labor strikes every year, so that whatever the capital stock is in any given year, a quarter of it goes unused because of these labor disputes. If population growth is 2% per year, calculate the rate of per capita income growth in Xanadu under this new scenario.

Answer: You have to calculate what is the *effective* capital-output ratio in Xanadu because of the labor problems. Basically, if θ is the amount of capital you need to produce a single unit of output, you will now effectively end up using more than that. How much more? Well, it must be $\theta \times (4/3)$. If you take away a quarter of this, you will get back exactly θ. So the effective capital-output ratio is now 4 × (4/3) = 16/3. Using this in the
Harrod-Domar equation with a rate of savings is 1/5, we see that $g = 3/80$, which is 3.75% per year. Subtract the population growth rate. The answer for per-capita growth is 1.75% per year.

d) If you were a planner in Xanadu and could costlessly choose the savings rate for that country, how would you go about making your decision? Think about the pros and cons of changing the savings rate and record your opinion here.

Answer: Economic well-being comes from a mix of both *current* consumption and *future* consumption. A higher savings rate benefits future consumption at the expense of current consumption. So our objective should not be to always raise savings rates, but find some intermediate
rate of savings that permits a desirable combination of current and future consumption.

4) Solow prediction versus actual relative incomes

Answer: Since we know productivity, A, and depreciation, δ , are the same, we know that they will cancel out in our steady state ratio analysis. Therefore, with α =1/3, our equation of interest boils down to

$$\frac{y_{1,ss}}{y_{2,ss}} = \left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{1}{2}},$$

for all three pairs of countries.

(a) Using a subscript T for Thailand and a subscript B for Bolivia, we rewrite the previous equation for Thailand and Bolivia as,

$$\frac{y_{T,ss}}{y_{B,ss}} = \left(\frac{\gamma_T}{\gamma_B}\right)^{\frac{1}{2}}.$$

Substituting in $\gamma_T = 0.303$ and $\gamma_B = 0.099$, we get the steady state ratio to be:

$$\frac{y_{T,ss}}{y_{B,ss}} = \left(\frac{0.303}{0.099}\right)^{\frac{1}{2}} \approx 1.75.$$

The actual ratio is,

$$\frac{y_T}{y_B} = \left(\frac{\$14,260}{\$6,912}\right) \approx 2.06.$$

Therefore, the Solow Model does a good job in predicting relative income for Thailand and Bolivia.

(b) Using a subscript *N* for Nigeria and a subscript *T* for Turkey, we rewrite the previous equation, with $\gamma_N = 0.075$ and $\gamma_T = 0.146$ to get,

$$\frac{y_{N,ss}}{y_{T,ss}} = \left(\frac{\gamma_N}{\gamma_T}\right)^{\frac{1}{2}} = \left(\frac{0.075}{0.146}\right)^{\frac{1}{2}} \approx 0.72.$$

The actual ratio is,

$$\frac{y_N}{y_T} = \left(\frac{\$3,648}{\$17,491}\right) \approx 0.21.$$

Therefore, the Solow Model does a poor job in predicting relative income for Nigeria and Turkey.

(c) Using a subscript *J* for Japan and a subscript *N* for New Zealand, we rewrite the previous equation, with $\gamma_J = 0.313$ and $\gamma_N = 0.207$ to get,

$$\frac{y_{J,ss}}{y_{N,ss}} = \left(\frac{\gamma_J}{\gamma_N}\right)^{\frac{1}{2}} = \left(\frac{0.313}{0.207}\right)^{\frac{1}{2}} \approx 1.23.$$

The actual ratio is,

$$\frac{y_J}{y_N} = \left(\frac{\$48,389}{\$43,360}\right) \approx 1.12.$$

Therefore, the Solow Model does a good job in predicting relative income for Japan and New Zealand.

5) True/False

Answer: (a) True. Here write down the Harrod-Domar equation. And then go on to mention that in the Solow model, long-run growth rate is determined simply by the exogenous rate of technical progress. The savings rate only dermines long-run capital stocks per-capita and the *level* of per-capita output, not its rate of growth.

(b) False. Simply write down the Harrod-Domar equation and argue that an increase in the capital-output ratio must *lower* the rate of growth.

(c) False. Studying countries that are *currently* rich introduces a bias towards convergence, as you are simply selecting *ex post* countries that were successful and so similar. You can mention Baumol's study as an example of this kind of mistake.
(d) True. Quah's study of mobility of countries shows that both very poor and very rich countries are unlikely to change world rankings all that much. In contrast, countries that were middle-income in 1960 have shown remarkable changes. A large fraction of them have become dramatically richer, while a large fraction have also become dramatically poorer.
(e) True. In the Solow model, population growth has only a level effect on long-run per-capita income. Here you may draw a quick diagram that describes the steady state in the Solow model and show what happens as population growth increases. Then point out that in the long-run, the rate of growth in the Solow model is just the rate of technical progress.
(f) False. Draw the production function relating output per head to capital per head. Output per head increases as capital per head increases (but it increases at a diminishing rate).