**EC 151 Homework VIII**
**Solutions**
**Credit Markets**
**Due December 4th in class**

## 1) Risk Premium

In various models in class, we have seen that imperfections in informal credit market arise because of there is a risk of default. One such imperfection is the fact that lenders usually charge an interest rate that is above the opportunity cost (e.g. a lower rate they could get in the formal sector). The difference between the minimum rate at which the lender is willing to lend and the opportunity cost is called "risk premium."

a. Why do lenders charge a risk premium?

The difference between the minimum rate at which the lender is willing to lend and the opportunity cost is called a "risk premium". In informal credit markets, lenders usually charge an interest rate that is above the opportunity cost. This is partly because repayment of informal loans is uncertain, and so the interest paid by performing loans must compensate the losses from nonperforming loans. In the following calculations, p is the risk premium.

b. Question 1, Chapter 14 a-d (You can skip part about "explain concept of risk premium in words")

The premium is what is charged over the formal sector interest rate (.1).
(a) $0.5 \cdot 1000(1.1 + p) - 1100 = 0 \Rightarrow p = 110\%$.
(b) $0.5 \cdot 1000(1.1 + p) + 0.5 \cdot 1000 - 1100 = 0 \Rightarrow p = 10\%$.
(c) $(1/3)*1000*(1.1 + p) + (1/3)*1000 - 1100 = 0$ p $\Rightarrow$ 120%.
(d) $0.5 \cdot 1000(1.1 + p) + 0.5 \cdot (0.5*500) - 1100 = 0 \Rightarrow p = 85\%$.

## 2) Risk of default and size of loan

Let's see how the risk of default factors into the lender and borrower's willingness to agree on a credit contract.

Say we have a borrower who will use a loan of size L at interest rate i to invest in a project that yields return R with probability p (and he repays) and 0 with probability 1-p (he defaults). But if he defaults, he will suffer reputation damage worth "F". Note however, that the banks still gets nothing if he defaults.

a. Set up an equation that describes shows given L and p, what is the maximum price this borrower is willing to pay for this loan? Describe how this price varies with p, L, R, F

See attached

b. Using your results from part a, set up an equation that show the highest amount of risk that the bank is willing to tolerate? That is, what is the threshold level of p above which the bank will not be willing to make the loan? And how does this vary with L, R, F (i should not be in your equation)?

The way to think about threshold probability p* is as follows:
Say your probability of high return is p=½. In this problem you will calculate the minimum probability p* the bank wants you to have in order to make the loan at interest rate i*. This p* will depend on some parameters F, L, R that is, p*= f(F,L,R). Now say that F,L,R happen to be such that f(F,L,R)=3/4. In this situation p<p*, so the bank will not make you the loan because you are too risky. That is, your probability of high return (and thus of them getting repaid) is

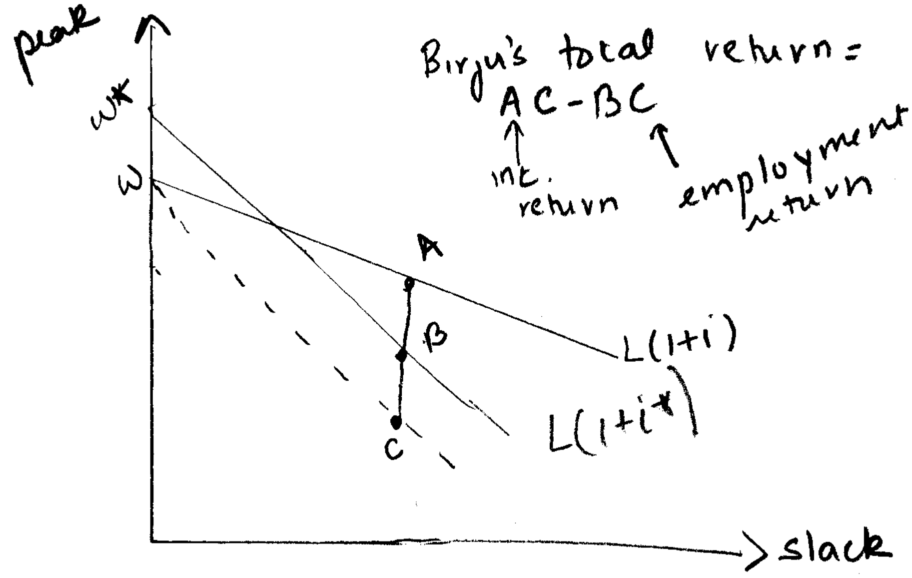
only ½ whereas they want it to be **at least** ¾ in order to give you a loan at interest rate i* . See the attached sheet for how to actually compute p*.

c. Think a bit more about the intuition behind the relationship between threshold p from above and F. What kind of policy could you implement to change this relationship (that is, make it more negative or more positive) to make a contract more likely to happen?

From part b, we see that F does not influence the constraint[1]. This means that changes in social sanctions will not influence the threshold probability at which the bank makes a loan at interest rate i*. This makes sense since after all F is not something the bank internalizes. If somehow the bank could actually use F in some way, then social sanctions would have a stronger relationship with p. Say, social sanctions provided information to banks. That is, once someone defaulted, their face was on posters all around town so other lenders would know they are risky. Now a higher social sanction provides information, so the threshold probability for banks to make loans can be lower. Social sanctions matter more. This is complicated to incorporate in a simple model but the intuition is simple enough.

## 3) Interlinked Transaction

The two panels in Figure 14.4 show two types of contracts of the for (w*, i*) Anka and Birju can agree on. That is, one where w*<w & i*>i and second, where w*<w & i*<i. Draw the case where w*>w & i*>i. Also show on the diagram what Birju's total return is.




[1] More precisely speaking, ceteris paribus, at smaller levels of F, p will increase as F increases since L<R. However, as F get larger, p is approximately unchanging. You can graph this function to see how it behaves.

2) RISK OF DEFAULT & SIZE OF LOAN

loan size : $L$

interest rate : $i$

$$\text{Return} \begin{cases} R & \text{w/ prob. } p \\ 0 & 1-p \end{cases}$$

$F$ : reputation damage.

a. Maximum interest rate (price) that borrower willing to pay is one that allows him to just break even i.e his expected return is zero.

$$p\left[R - L(1+i^*)\right] + (1-p)(-F) = 0$$

$$\Rightarrow pR - pL(1+i^*) - (1-p)F = 0$$

$$\Rightarrow pL(1+i^*) = pR - (1-p)F$$

$$\Rightarrow i^* = \frac{pR - (1-p)F}{pL} - 1$$

$$i^* = \frac{R}{L} - \frac{(1-p)F}{pL} - 1$$

Ceteris Paribus,

as $p\uparrow$, $i\uparrow$

as $L\uparrow$, $i\uparrow$

as $F\uparrow$, $i\downarrow$

as $R\uparrow$, $i\uparrow$

b. The bank knows $i^*$ is the max. int. rate they can charge borrower. Charging this, under what conditions will they make loan

The bank makes loans as long as their profit is positive. i.e expected $\Pi \geq 0$

exp

$$\Pi = p\left[L(1+i) - L\right] - (1-p)L \geq 0$$

We can substitute $i^*$ in for $i$ above and then solve for the $p$, the prob. below which bank will not lend

$$p\left[L\left(\cancel{1+}\left(\frac{R}{L} - \frac{(1-p)F}{pL} \cancel{-1}\right)\right) - L\right] - (1-p)L = 0$$

$$pL\left(\frac{R}{L} - \frac{(1-p)F}{pL}\right) - pL - (1-p)L = 0$$

$$p\left(R - \frac{(1-p)F}{p}\right) \cancel{-pL} - L \cancel{+pL} = 0$$

$$pR - (1-p)F - L = 0$$

$$\Rightarrow pR - F + pF - L = 0$$

$$\Rightarrow p(R+F) - (F+L) = 0$$

$$\Rightarrow p = \frac{F+L}{R+F}$$

if $p$ below $\frac{F+L}{R+F+L}$, then borrower too risky and bank will not lend at $i^*$ ceteris paribus

when $R\uparrow$, $p\downarrow$ (Remember – lower $p$ means higher chance of default)

$F\uparrow \to$ does not change very much.

$L\uparrow$ $p\uparrow$