

EC 151 Homework VI
Solutions
Land
Due November 18th in class

1) Land Size & Optimal Production Structure

Chapter 12, Question 1

- a. Each acre produces \$3,000 worth of output. If it is leased, the tenant will have to pay \$1,100 in wages to each of the two required laborers. Therefore, the rent the family can hope to obtain is $\$3000 - 2 \cdot \$1100 = \mathbf{\$800}$.
- b. Each acre requires two people to farm it. Since output per acre is \$3,000, this amounts to \$1,500 per person. Say two of the family members are available to either work the land, or earn wages in the labor market. Working the land will yield \$3,000, while leasing the land and having both members work elsewhere yields \$800 from the lease and $2 \cdot \$1,000 = \$2,000$ in wages, for a total of \$2,800. **Therefore, the family will always find it optimal to work its land, and a family of six will only lease land if it owns more than 3 acres.**
- c. Recall that a supervisor can oversee any amount of laborers. For each acre that is worked in this way, there will be \$2,000 in wages. Therefore, the profits from hiring a supervisor and laborers, for a given acreage s is $\$1,000s - \$2,000$.

First, note that once the family hires a supervisor, it need no longer work the land. Indeed, if the family owns a total of a acres, and leases l acres, its income will be

$\$9,000 + \$800l - \$2,000 + \$1,000(a - 3 - l) = \$4,000 + \$1,000a - \$200l$
if it works the land, and

$6 \cdot \$1,000 + \$800l - \$2,000 + \$1,000(a - l) = \$4,000 + \$1,000a - \$200l$
if it works outside instead.

Now, from the equation above, we see that **if it hires a supervisor, the family will not lease any land** (it reduces total profits), thus leasing and hiring labor will not co-exist.

So, when will the profits from farming (3 acres of) its own land + leasing be less than the profits from hiring a supervisor? Whenever

$$\begin{aligned} \$4,000 + \$1,000a &> \$9,000 + \$800(a - 3) \\ \$200a &> \$2,600 \\ a &> 13 \end{aligned}$$

So, whenever the family owns more than 13 acres, it will hire a supervisor and not lease any land.

be less than the wage, so at that point farmer 2 would reduce her supply. In any equilibrium, farmer 2 will not work at all.

What happens to total output and labor effort? As the share that goes to farmer 1 rises, both her labor supply and total output rise. In the extreme where she gets the whole production, she will choose the optimal amount.

d. Now, we assume that each farmer has an increasing marginal cost of supplying labor $c(l)$, but for each farmer it is the same. If we were trying to maximize total surplus, we would have to find l_1 and l_2 such that

$$f'(l_1 + l_2) = c(l_1)$$

$$f'(l_1 + l_2) = c(l_2)$$

Since the marginal cost function is the same, and it is increasing, we would choose $l_1 = l_2$.

e. Notice that now, we can have both farmers supplying effort even when they get different shares. This is because at very low levels of individual effort, marginal cost is low, and this will make it possible to equate the marginal return from labor (which is a share of the marginal product) with its marginal cost. From any aggregate amount of labor, the marginal return for farmer 1 is higher than for farmer 2, so that at an equilibrium farmer 1 will supply more labor.

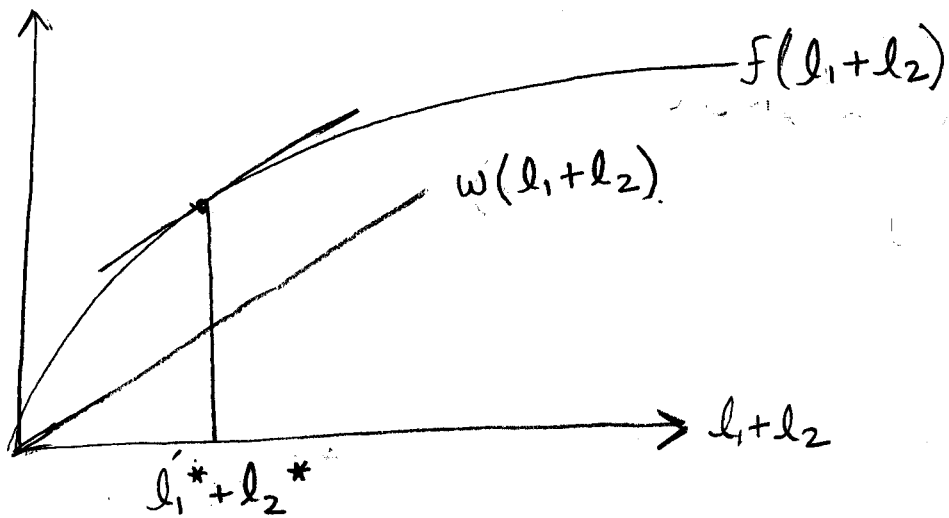
3) Risk

Say in an agricultural economy producing wheat there are two types of contracts: sharecropping and fixed rent. Under fixed rent contract, tenant gives R bushels of wheat to (risk-neutral) landlord. Under sharecropping, tenant gives "s" fraction of wheat harvest to landlord. There is also uncertainty/risk in the agricultural production due to the monsoon. A good monsoon will yield a high wheat harvest H and a bad monsoon season will yield a low wheat harvest L . The probability of a good monsoon is p .

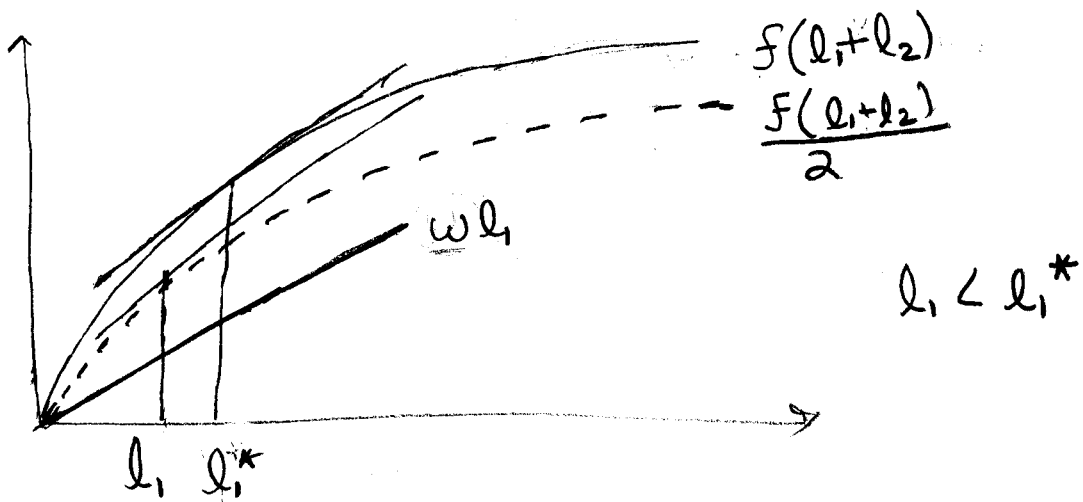
a. Show that if we consider sharecropping and fixed rent as two gambles with same expected outcome, sharecropping has a narrower spread

b. Using diagrams similar to Figure 12.6 show which contract would be preferred by tenant if he was risk-averse, risk-loving and risk-neutral.

5a)



b) For each farmer:



Similarly $l_2 < l_2^*$

So now, $l_1 + l_2 < l_1^* + l_2^*$

3. RISK

Landlord risk-neutral, tenant risk-averse

G: Good outcome
B: Bad outcome

R = rent from fixed rent
s = sharecropping share

$$p = \text{Prob}(G)$$

* Under a fixed rent contract (FR)
 Landlord gets R, tenant gets G-R if G
 " " R, " " B-R if B.

* Under sharecropping contract that provides exactly same expected return as FR

$$\text{Landlord gets } p s G + (1-p) s B = R$$

solving for s:

$$s = \frac{R}{pG + (1-p)B} \quad (*)$$

(expected return to tenant is also same)

The landlord is risk-neutral, so he just cares about fact that he gets R in both contracts.
 However, tenant is risk-averse, so which one does he prefer?

let's examine "Good" scenario

Under sharecrop. tenant gets $(1-s)G$
 " FR " " $G-R$

which is larger i.e. is $(1-s)G - (G-R) > 0$
 (sharecr. better) or $(1-s)G - (G-R) < 0$ (FR better)