The Measurement of Intellectual Influence*

Ignacio Palacios-Huerta† and Oscar Volij‡

April 18, 2002

PRELIMINARY DRAFT

Abstract

We examine the problem of measuring influence based on the information contained in the data on communication between scholarly publications, judicial decisions, patents, web pages, and other entities. The measurement of influence is useful to address several empirical questions such as reputation, prestige, aspects of the diffusion of knowledge, the markets for scientists and scientific publications, the dynamics of innovation, ranking algorithms of search engines in the World Wide Web, and others. In this paper we ask why any given methodology is reasonable and informative applying the axiomatic method. We find that a unique ranking method can be characterized by means of four axioms: anonymity, invariance to citation intensity, homogeneity for two-journal problems, and consistency. This method is easily implementable and turns out to be different from those regularly used in social and natural sciences, arts and humanities, and computer science.

*We thank Yiu T. Poon and participants in Iowa State's NFL seminar for very helpful comments. We also thank Debora Lewi and Salwa Hammami for their research assistance.

†Department of Economics, Brown University, Providence, RI 02912, USA Email: ipalacios@brown.edu. http:// www.econ.brown.edu/~ iph.

‡Department of Economics, Iowa State University, Ames, IA 50011, USA. Email: oscar@volij.co.il. http://volij.co.il.

1 Introduction

The quality of scientists' research, publications, and achievements plays an important role in the reward and productivity structures within academia. These structures have been the subject of keen interest to economists, sociologists, historians of science, and other scientists. Of particular interest to economists have been the determinants of scholarly productivity, the role of incentives as promoters of the growth of knowledge, and the implications of competition among scientists within the structure of rewards.

Academic journals have played an increasingly important role in both the dissemination of new knowledge and the certification of scientific merit throughout the past century, in economics as in other disciplines. In an attempt to judge the various qualities of scientific publications, several efforts have been made to measure influence. Citations, as a broad form of influence, are often used in these efforts.

Not only do citations figure prominently in scholarly journals and books, but they also appear in many other forms of documentation, such as patents, newspapers, legal opinions, and magazine articles, and in the link structure of the World Wide Web. The literature on citation analysis is by now vast and growing.[1] Citation analysis is growing mainly because it enables, at a relatively low cost, to make a first attempt at rigorously quantifying elusive but important socioeconomic phenomena such as reputation, influence, prestige, celebrity, the diffusion of knowledge, the quality of scholarly output, the quality of journals, the rise and decline of journals and schools of thought, changes in the publishing process and in the incentives for publication, the returns to publication, the basing of judicial decisions on previous decisions, and the productivity of scholars, judges, and academic departments.[2]

[1] See Posner (2000) and many references therein.

[2] For example, citations have been used to examine the distribution of influence across journals and changes in this distribution; the extent to which the status of a journal leads to a lengthening of the review process; changes in quality standards and in the distribution of quality in response to increased competition, and other aspects of the publication process (Ellison (2000b)). Citations are also relevant for examining tradeoffs associated with different aspects of paper quality, including the role of social norms for weighing different aspects of quality (Ellison (2000a)); the extent of favoritism on the part of editors (Laband and Piette (1994b)); the incentive for authors to publish, including the monetary value to article publication in terms of direct salary increments, promotion-related salary increments, faculty mobility, and life-cycle productivity profiles (Tuckman and Leahey (1975), Hammermesh, Johnson, and Weisbrod (1982)); the benefits of intellectual collaboration (Laband and Tollison (2000)), and the extent to which academic scientists receive differential returns to publishing articles of varying quality (Sauer (1988)). Citations are also useful to examine the relationship between scholarly significance, academic status, and

Citation analysis also plays an important role in the World Wide Web in the analysis of "links," where search engines combine sophisticated text matching techniques with a vast link structure to create web search algorithms that find and rank pages according to their importance. Last, but not least, citation analysis also is used widely as a management tool for making decisions on hiring, promotion, salary, and other personnel decisions.

The use of citation analysis is so extensive that *"counting* citations is already a well-established method of empirical research in law, economics, sociology, and academic administration" (Posner (2000), p. 382, italics added). We emphasize the word "counting" because it captures the basis of this methodology. Within our discipline, for instance, several studies have approximated productivity, quality, and influence by simple citation counts. These counts are then often used to examine various questions of interest such as those mentioned in footnote 2. However, the use of citation counts must be approached with caution. A principal criticism is that the number of citations is a poor proxy of what is really of interest, whether it is reputation, influence, impact, or the quality and magnitude of a person's achievement.

Some studies attempt to account for this criticism by weighting citations in certain intuitive but ad hoc ways. For instance, the general idea in the evaluation of journals and other scholarly publications is that citations by low impact journals should be given less credit than citations by high impact journals. As a result, there are several arbitrary methods to produce a ranking of journals. For instance, many studies use the "impact factor" constructed by the Social Science Citation Index and published by the Institute for Scientific Information, while others follow Liebowitz and Palmer (1984) who proposed various "impact-adjusted" methods for ranking journals. Over the last few years, several variations of these methods have been applied extensively in many studies in different disciplines.

Given the proliferation of rankings, and the important role that they often seem to play

---

public fame (Posner (2001)). In the legal profession, citation analysis is extensive. These include studies of judicial citation practices, judicial influence and legal precedent (Landes, Lessig, and Solimine (1998), Landes and Posner (1976)), the role of citations both to and in judicial opinions in adjudication and legal decision making, the productivity and influence of judges, the durability of precedents, and the rankings of scholars, books, journals, and schools. See, for instance, the legal citation studies in the conference volume "Interpreting Legal Citations" (*Journal of Legal Studies*, supplement, (2000)). On the usefulness of patents and citations data on the economics of innovation, see Jaffe and Trajtenberg (2002).

in personnel decisions and in the study of many socioeconomic phenomena of interest, it is somewhat surprising that neither the authors that propose these ranking methods nor those who use them, have tried to justify them.[3] In other words, given that there is, in principle, a plethora of different arbitrary ranking methods, there is no obvious reason why one should prefer one method over another. Adopting a method because it looks reasonable or because it yields introspectively intuitive results is, to say the least, not the best scientific practice. Without investigating the properties of these methods it is simply not possible to establish a reliably meaningful measure of impact or intellectual influence.

Posner (2000, p. 383) summarizes these criticisms by indicating that "citation analysis is not an inherently economic methodology; most of it has been conducted without any use of the theories or characteristic techniques of economists." The result is that, in terms of quality, the information content of the network of journals and publications that play a paramount role in the exchange, dissemination, and certification of scientific knowledge is little understood. This, in turn, has implications for all the empirical applications mentioned above.

In this paper we bring economic methodology to bear on the ranking problem. Specifically, we apply the axiomatic method often used in social choice, game theory, and other areas, and present an axiomatic model for measuring intellectual influence.[4] Thus, the approach we take is different from that in the literature. Rather than *assuming* arbitrarily a ranking method on intuitive grounds or introspection, we *derive* a ranking method by requiring a few simple properties. The main result of the analysis is that there is a unique ranking method that satisfies the proposed properties simultaneously. As it turns out, this method is different from the ones typically used in economics and other sciences to rank journals and departments, and to measure productivity, influence, and prestige. Moreover,

[3] For instance, Bush, Hamelman, and Staaf (1974), Borokhovich, Bricker, Brunarski, and Simkins (1995), Dusansky and Vernon (1998), Laband and Piette (1994a), Kalaitzidakis, Mamuneas, and Stengos (2001), and many others only describe the method they use, or simply adopt previous methods, and report the resulting rankings. Liebowitz and Palmer (1984), claim that their ranking "probably comes closest to an ideal measure of the impact . . . of manuscripts published in various journals," and that their ranking is probably "the closest to 'journal quality'." However, no justification whatsoever is provided for any of these statements.

[4] Arrow's impossibility theorem, Gibbard-Satterthwaite theorem, Nash's derivation of the Nash bargaining solution, Harsanyi's characterization of the utilitarian social welfare function, Peleg's characterization of the core, and Segal's (2000) characterization of the relative utilitarian social preference are just a few examples of the successful application of the axiomatic method.

it is easily implementable.

We examine the following properties in the context of the ranking of journals and other scholarly publications. The first is the *anonymity* of the method. Roughly, a ranking method is anonymous if it does not depend on the names of the journals under consideration. The second property, which we call *invariance to citation intensity*, requires that the absolute number of references a journal cites should not affect the ranking of the journals, as long as the distribution of these references does not change. A citing article awards value to the articles it cites. This value, which depends exclusively on the content of a paper, is distributed among the cited articles. Thus, the longer the list of the references, the smaller the value awarded to each cited reference. In other words, given the content of the paper, the total value awarded to the articles it cites cannot be increased or decreased by changing the amount of references.

The third property concerns the ratio of mutual citations. This property, which we call *homogeneity for two-journal problems,* is based on Stigler, Stigler, and Friedland (1995), who stress that an important measure of the impact of one journal on another is the ratio of citations of one journal by the other to the citations of the latter to the former.[5] This property requires that in two-journal problems, the ratio of the journals' valuations be in a fixed proportion to the ratio of mutual citations.

The last property we consider is *consistency*, which allows us to extend a ranking method from problems with few journals to problems with more journals. The idea behind this property is that if we know how to rank a small problem, we should be able to extend the ranking method to a big problem in a consistent way. We follow the notion of consistency that has been applied for axiomatizing rules and solution concepts in diverse problems.[6]

We show that these four properties characterize a *unique* ranking method of journals

[5]Stigler, Stigler, and Friedland (1995) also show that an advantage of these ratios is that it is possible to fit a statistical model to the data in terms of simple univariate scores. See also Stigler (1994) for an application of the model to journals in statistics and their relationship with econometrics and economics.

[6]For example, Peleg and Tijs (1996) characterize the Nash equilibrium correspondence, Lensberg (1988) axiomatizes the Nash bargaining solution, Peleg (1985), (1986) characterize the core of NTU and TU games respectively, Hart and Mas-Colell (1989) axiomatize the Shapley value, and Dagan (1994) characterizes the Walrasian correspondence in the context of private ownership economies. In these papers and many others consistency plays a crucial role. Thomson (1990), (1995) offers comprehensive surveys on consistency and its applications.

based on their overall impact. We call this method the Invariant method, and it turns out that it is different from the methods regularly used to evaluate journals and scholarly publications, judges, academic departments, web pages, and to measure reputation, influence, and prestige.

We then turn to the problem of evaluating the influence of journals according to their per-manuscript impact. That is, while a ranking method according to the overall impact of the journals measures impact of the whole journal irrespective of its size, a per-manuscript ranking method measures the impact of the average paper published in each journal. Adding an axiom that requires invariance to splitting of journals to axioms analogous to the ones used in the characterization of the overall impact of journals, we are able to uniquely characterize the extension of the Invariant method to the per-manuscript ranking problem. The new axiom requires that the splitting of a journal into several identical but smaller journals does not affect the ranking.

Our purpose in this paper is not to claim that any given ranking method based on citations is the correct way of measuring impact, much less quality. Citation analyses, however sophisticated it may be, cannot be a substitute for critical reading and expert judging. However, to the extent that the data on the communication between scholarly publications, judicial decisions, patents, web pages, etc., contain valuable information that can be used to address several empirical questions of interest in different disciplines and problems, we should ask why a method is reasonable and informative. We thus evaluate this question by characterizing and comparing different methods according to the properties they satisfy and those they fail to satisfy.

The rest of the paper is organized as follows. Section 2 presents the basic problem of overall journal ranking and characterizes a ranking method by means of four independent properties. Section 3 deals with the problem of ranking journals according to the average impact of each of its articles. We extend the ranking method characterized in the previous section to this more general case. In Section 4 we illustrate the use of these methods with a simple example. Using the citations by articles published in the year 2000 we measure the impact of the articles published during the period 1993-1999 in a set of economics journals. The example shows that differences in the measurement of impact according to different methods can be substantial. Section 5 concludes.

2 The Intellectual Influence of a Scholarly Journal

In this section we analyze the problem of ranking journals according to their impact, as measured by the citations they generate. We shall be dealing with ranking methods which are functions that take some data as input and return a vector of valuations as output. These valuations should be interpreted as the relative values of the different journals as a whole, and not as the value of the representative article published in these journals. An alternative interpretation is that all journals under consideration have the same number of published papers. The problem of measuring the value of the representative article in a journal will be analyzed in Section 3.

2.1 Characterization

Let $\mathcal{J}$ be a countable set of journals. This set is to be interpreted as the universe of all journals. Let $J \subseteq \mathcal{J}$ be a subset of journals. A *citation matrix* for J is a $|J| \times |J|$ non-negative matrix (c_{ij}) where for each $i, j \in J$, c_{ij} is the number of citations to journal i by journal j. For $j \in J$, we denote by $\bar{c}_j$ the vector $(c_{ij})_{i \in J}$ of citations by journal j, and the sum of all citations by journal j is denoted by c_j, namely $c_j = \sum_{i \in J} c_{ij}$. All vectors are *column* vectors. For a vector v, $\|v\|$ denotes the 1-norm of v, namely $\|v\| = \sum_{i \in J}^{n} |v_i|$. The diagonal matrix with $d_1, \ldots, d_n$ as its main diagonal entries is denoted by $\operatorname{diag}(d_1, \ldots, d_n)$. Given a matrix of citations $C = (c_{ij})$, we define $D = \operatorname{diag}(c_j)_{j \in J}$ to be the diagonal matrix with the sums of the citations by the journals as its main diagonal. Further, the matrix CD^{-1} will be called the normalized matrix of C and it is readily seen to be a stochastic matrix (the entries of each of its columns add up to one).

Given a matrix of citations C for J, we say that journal i *is cited* by journal j if $c_{ij} > 0$. We say that journal i *impacts* journal j if there is a finite sequence $i_0, \ldots i_n$, with $i_0 = i$ and $i_n = j$, such that for all $t = 1, \ldots, n$, journal i_{t-1}, is cited by journal i_t. Journals i and j *communicate* if either $i = j$ or if they impact each other. It is easy to see that the communication relation is an equivalence relation and, therefore, it partitions the set J of journals into equivalence classes, which we call communication classes. A *discipline* is a communication class $J' \subseteq J$ such that no journal in $J \setminus J'$ impacts any journal inside J'. If a matrix of citations C has two disciplines, this means that there are two communication classes in J that are disconnected. Namely, there is no chain of citations that go from a journal in one discipline to a journal in another and vice versa. Note that as long as a discipline J contains more than one journal, each $\bar{c}_j$ is a non-zero vector of citations for each $j \in J$. Since we are interested in rankings within a single discipline, we will restrict attention to citation matrices with only one discipline.[7] This leads to the following definition:

Definition 1 A *ranking problem* is a pair $\langle J, C \rangle$ where $J \subseteq \mathcal{J}$ is a set of journals and $C = (c_{ij})_{(i,j) \in J \times J}$ is a citation matrix with only one discipline.

The primitives of a ranking problem consist of the relevant set of journals and the corresponding matrix of citations. As indicated earlier, the number of articles published in each journal is not taken into account at this point because we first want to measure the impact of the journal as a whole, rather than the impact of the representative article published in that journal. Thus, the assumption is that two journals with the same profile of citations have the same impact as a whole, independently of their respective number of published papers.

We are interested in building a cardinal ranking of the journals in J, namely a non-zero vector of non-negative valuations $(v_j)_{j \in J}$. Each v_j is to be interpreted as the overall value of journal i. Since only relative values matter, we can normalize the vector of valuations so that they add up to 1. Denote the set of all possible vectors of valuations of J by Δ_J. That is, $\Delta_J = \{(v_j)_{j \in J} : v_j \geq 1, \sum_{j \in J} v_j = 1\}$. Further, $\Delta = \cup_{J \in \mathcal{J}} \Delta_J$.

Clearly, the choice of the relevant set of journals will generally affect the results of the implementation of a ranking method. Thus, it is important to make a good choice of journals. In our analysis, however, we take as given the set of journals, and just deal with the problem of measuring influence within this set.

Definition 2 Let $\mathcal{R}$ be the set of all ranking problems. A *ranking method* is a function $\phi : \mathcal{R} \to \Delta$, that assigns to each ranking problem $\langle J, C \rangle$ a vector of valuations $v \in \Delta_J$.

Examples:

[7] Citation matrices with only one discipline are known in the language of matrix theory as non-negative, irreducible matrices.

1. The Egalitarian method is the function that assigns the same value to every journal. Formally, $\phi_E : \mathcal{R} \to \Delta$ is defined by $\phi_E(J, C) = (1/|J|, \ldots, 1/|J|)^T$.

2. The Counting method awards each journal the proportion of its citations out of the total citations. Formally: $\phi_C : \mathcal{R} \to \Delta$ is defined by $\phi_C(J, C) = (\frac{\sum_{j \in J} c_{ij}}{\sum_{k \in J} \sum_{j \in J} c_{kj}})_{i \in J}$.

3. The Modified Counting method awards each journal the proportion of its non-self-citations out of the total number of non-self-citations. Formally: $\phi_{MC} : \mathcal{R} \to \Delta$ is defined by $\phi_{MC}(J, C) = (\frac{\sum_{j \in J \setminus \{i\}} c_{ij}}{\sum_{k \in J} \sum_{j \in J \setminus \{k\}} c_{kj}})_{i \in J}$.

4. The Liebowitz-Palmer method $\phi_{LP} : \mathcal{R} \to \Delta$ assigns to each ranking problem $R = \langle J, C \rangle$ the only fixed point of the operator $T : \Delta_J \to \Delta_J$ defined by $T(v) = \frac{Cv}{\|Cv\|}$.

5. The Invariant method ϕ_I assigns to each ranking problem $R = \langle J, C \rangle$, the unique invariant distribution of the normalized matrix CD^{-1}. That is, $\phi_I(R) = v \in \Delta_J$ where $CD^{-1}v = v$.

The Counting method was first used in Bush, Hamelman, and Staaf (1974).[8] This method was later used by Liebowitz and Palmer (1984) (see their Table 1, column 2 (Rankings Based on Citations)). The Modified Counting method was used in Bush, Hamelman, and Staaf (1974) (see their Table 1, column "Diagonal Excluded"). As to the Liebowitz-Palmer method, it was proposed by Liebowitz and Palmer (1984) and used in the construction of the journal ranking that appears in their Table 1, column 3 (Rankings Based on Impact Adjusted Citations). To the best of our knowledge, the Egalitarian method and the Invariant method have never been used for ranking journals or other publications in any discipline.

The reader may wonder whether the Liebowitz-Palmer and the Invariant methods are well-defined. After all, the operators that are used to define them may have more than one fixed point. That the Invariant method is well-defined follows from the fact that the normalized matrix CD^{-1} is an irreducible stochastic matrix and that every irreducible stochastic matrix has a unique invariant distribution. The fact that the Liebowitz-Palmer

[8]Although these authors do not assign numerical values to each of the journals they consider, their ranking in Table 1, column "Diagonal Included" is built according to the Counting method.

method is well defined is a corollary of the Perron-Frobenius theorem for irreducible matrices. Specifically, note that $\phi_{LP}(J,P)$ is a characteristic vector of C that belongs to Δ_J.[9] One of the results of the Perron-Frobenius theory is that every irreducible, non-negative, $|J| \times |J|$ matrix C has exactly one eigenvector in Δ_J (see Minc (1988), Theorem 4.4). Therefore, the Liebowitz-Palmer method can be defined alternatively as assigning to each ranking problem $\langle J,C \rangle$ the only eigenvector of C in Δ_J. Incidentally, Liebowitz and Palmer (1984), Laband and Piette (1994a), and other papers that use the Liebowitz-Palmer method actually calculate the eigenvector by means of an iterative procedure known as the Power Method. For a discussion of this method see Wilkinson (1965), chapter 9.

Note that both the Liebowitz-Palmer and the Invariant methods assign to journal i a value that is a weighted average of some function of the citations it gets: $v_i = \sum_{j \in J} \alpha_{ij} v_j$. For the Invariant method, $\alpha_{ij} = \frac{c_{ij}}{c_j}$, while for the Liebowitz-Palmer method, $\alpha_{ij} = \frac{c_{ij}}{\|Cv\|}$. According to these measures, not all citations have the same value. Citations by important journals are more valuable than citations by less important journals. But the importance of a journal is determined endogenously and simultaneously with the importance of all other journals.

One can think of an infinite number of ranking methods, the above examples being just a few. In applications, it would be desirable to choose an appropriate one. What is appropriate, however, depends on the different properties that the ranking method may satisfy. Therefore, in order to analyze and distinguish among different methods, it is appropriate to evaluate the properties that each method satisfies. Next, we follow this approach and characterize a ranking method by means of some basic, desirable properties.

The first property we consider is *anonymity.* It says that the ranking of a set of journals should not depend on the names of the journals. Before we formalize this property, recall that a permutation matrix is a (0,1)-matrix that has exactly one 1 in each row and each column.

Definition 3 A ranking function ϕ satisfies *anonymity* if for all ranking problems $R = \langle J,C \rangle$ and for all $|J| \times |J|$ permutation matrices P, $\phi(J, PCP^T) = P\phi(J,C)$.

[9] That is, $\phi_{LP}(J,P)$ is a non-zero vector that solves $Cv = \lambda v$ for some λ.

The following diagram exemplifies the property of anonymity for two-journal problems:

$$\text{If}\quad \begin{array}{c c} & \begin{array}{cc} i & j \end{array} \\ \begin{array}{c} i \\ j \end{array} & \begin{pmatrix} a & b \\ c & d \end{pmatrix} \end{array} \xrightarrow{\phi} \begin{pmatrix} v_i \\ v_j \end{pmatrix} \quad \text{then} \quad \begin{array}{c c} & \begin{array}{cc} i & j \end{array} \\ \begin{array}{c} i \\ j \end{array} & \begin{pmatrix} d & c \\ b & a \end{pmatrix} \end{array} \xrightarrow{\phi} \begin{pmatrix} v_j \\ v_i \end{pmatrix}.$$

That is, a permutation of the matrix of citations results in the same permutation of the valuations.

In order to motivate the next property, consider a ranking problem $\langle J, C\rangle$ where for each journal $j \in J$, journal j's list of references is given by the vector $\bar{c}_j = (c_{ij})_{i\in J}$. The vector $\bar{c}_j$ represents journal j's opinions about the journals in J. These opinions do not change if journal j were to modify the number of references by multiplying it by a constant $\lambda_j > 0$, thus turning the vector $\bar{c}_j$ into the vector $\lambda_j \bar{c}_j$. The second property requires from the ranking method that it not be affected by such changes. In other words, the length of the reference section should not matter.

Definition 4 A ranking function ϕ satisfies *invariance with respect to citation intensity* if for every ranking problem $\langle J, C\rangle$ and for every non-negative diagonal matrix $\Lambda = \mathrm{diag}(\lambda_j)_{j\in J}$ with some positive diagonal entries, $\phi(J, C\Lambda) = \phi(J, C)$.

The following diagram exemplifies this property:

$$\text{if}\quad \begin{array}{c c} & \begin{array}{cc} i & j \end{array} \\ \begin{array}{c} i \\ j \end{array} & \begin{pmatrix} a & b \\ c & d \end{pmatrix} \end{array} \xrightarrow{\phi} \begin{pmatrix} v_i \\ v_j \end{pmatrix} \quad \text{then} \quad \begin{array}{c c} & \begin{array}{cc} i & j \end{array} \\ \begin{array}{c} i \\ j \end{array} & \begin{pmatrix} \lambda_i a & \lambda_j b \\ \lambda_i c & \lambda_j d \end{pmatrix} \end{array} \xrightarrow{\phi} \begin{pmatrix} v_i \\ v_j \end{pmatrix}.$$

The idea behind this property is that, given the content of a journal, each journal should have one vote. If journal j cites many articles in different journals, then j's single vote is divided among the cited journals. Making an analogy, given the content of a paper, the value of a paper is distributed among its coauthors. All else equal the greater the number of coauthors, the smaller the credit each one gets. That is, the value of a paper cannot be modified by leaving intact its content and changing the number of authors.

The next property says that in two-journal problems where both journals have the same number of cited references, the relative valuation of a journal should be proportional to the ratio of their mutual citations. Formally,

Definition 5 Let $R = \langle \{i, j\}, C \rangle$ be a two-journal problems such that $c_{ii} + c_{ji} = c_{ij} + c_{jj}$. The ranking function ϕ satisfies *homogeneity for two-journal problems* if there is $\alpha > 0$ (that may depend on $\{i, j\}$ but not on C), such that for all such problems, $\phi_i(R)/\phi_j(R) = \alpha c_{ij}/c_{ji}$.

The following diagram exemplifies this property for two-journal problems:

$$\begin{array}{cc} & \begin{array}{cc} \quad i \quad & \quad\quad j \quad \end{array} \\ \begin{array}{c} i \\ j \end{array} & \begin{pmatrix} K - c_{ji} & c_{ij} \\ c_{ji} & K - c_{ij} \end{pmatrix} \end{array} \xrightarrow{\phi} \begin{pmatrix} v_i \\ v_j \end{pmatrix} = \frac{1}{c_{ji} + \alpha c_{ij}} \begin{pmatrix} \alpha c_{ij} \\ c_{ji} \end{pmatrix}.$$

The value c_{ij} is a measure of i's direct influence on j. Thus, the ratio c_{ij}/c_{ji} represents the direct influence of journal i on journal j relative to the direct influence of journal j on journal i. The importance of these ratios was stressed in Stigler, Stigler, and Friedland (1995), where these sender-receiver ratios, as they call them, were calculated for a group of nine core journals. Clearly, however, in building a desirable ranking method, one would like to take into account not only the direct influence of each journal on each of the others, but also the indirect influence, as well. This is why these ratios, though conveying important information, are not, per se, a perfect index of the journals' total impact. In a two-journal problem, however, the ratio c_{ij} is a measure of the total impact of journal i on journal j. Stigler, Stigler, and Friedland (1995) admit that "[t]hese sender-receiver ratios are influenced by the varying number of citations in articles published by each journal." They are not clear, however, as to whether that variation calls for correction. In any case, our homogeneity property requires that the ratio of valuations be proportional to the ratio of mutual citations *only* in two-journal problems where the total number of citations by each journal is the same. In this way, the varying number of citations and the effects of indirect influences across journals are not an issue.

Finally, the last property will allow us to relate large problems to small problems. Thanks to this property, if we know how to solve a ranking problem with a few journals, we will also know how to solve problems with a greater number of journals. The idea is to extend a ranking method of few journals to a ranking method of more journals in a consistent way. In order to formalize what we mean by *consistency,* we first need some definitions.

Let $C = (c_{ij})_{(i,j)\in J\times J}$ be a citation matrix for J, and let k be a journal in J. The reduced matrix with respect to journal k is $C^k = (c^k_{ij})_{(i,j)\in J\setminus\{k\}\times J\setminus\{k\}}$, where:

$$c^k_{ij} = c_{ij} + c_{kj}\frac{c_{ik}}{\sum_{t\in J\setminus\{k\}} c_{tk}} \qquad \text{for all } i,j \in J\setminus\{k\}.$$

The reduced problem with respect to k is denoted by $R^k = \langle J\setminus\{k\}, C^k\rangle$. Note that since C is irreducible, $\sum_{t\in J\setminus\{k\}} c_{tk} > 0$ and, hence, R^k is well-defined. Further, C^k is itself irreducible.

The reduced matrix represents the following situation. Suppose we want to rank the journals in J and our computer cannot deal with $|J|\times|J|$ matrices, but only with $(|J|-1)\times(|J|-1)$ matrices. Therefore we need to resize our problem and abstract from one journal in our data set, say journal k. Still, we are interested in the relative values of all the remaining journals. If we eliminated journal k from the matrix, namely if we eliminated the corresponding row and column, we would lose some valuable information. Therefore, we need to "retouch" the matrix so that the information of the missing journal is not lost. In the old matrix, c_{kj} was the number of citations to journal k by journal j. If we do not want to lose this information, we need to redistribute these citations among the other journals; the appropriate way to do so is in proportion to the citations (or opinions) by the missing journal k. In other words, journal j gave credit to journal k in the form of c_{kj} citations while journal k gave credit to the journals other than k according to the vector $\bar{c}_k$ of citations by k. Therefore, if we do not want to lose the information about the indirect impact of each of the journals in $J\setminus\{k\}$ on $J\setminus\{k\}$, we need to redistribute each of the c_{kj} citations, $j \in J\setminus\{k\}$, according to the relative impact of the journals on k; that is in proportion to the values in $\bar{c}_k$. If this was the correct method to recover the information lost by the need to use a smaller matrix, we should expect that our ranking method give the same relative valuations to the journals in $J\setminus\{k\}$, if applied either to the original problem or to the modified, reduced one. This is the requirement of the next property.

Definition 6 The ranking function ϕ satisfies *consistency* if for all $R = \langle J, C \rangle$, with $|J| > 2$ and for all $k \in J$,

$$\frac{\phi_i(R)}{\phi_j(R)} = \frac{\phi_i(R^k)}{\phi_j(R^k)} \qquad \text{for all } i, j \in J \setminus \{k\}.$$

The property of consistency requires from a ranking method that the relative valuations of the journals not be affected if we apply the method to the reduced problem with respect to k. One could argue that consistency is not a reasonable requirement for a method that does not satisfy invariance to citation intensity. After all, the reduced matrix takes into account the relative citations of journal k and not its absolute citations. For a method that is sensitive to the absolute number of citations, the reduced matrix, which takes into account the relative number of citations, may not be the relevant matrix. For this reason, one may consider the following weak version of consistency, which is applied only to problems for which the absolute number of citations is not an issue.

Definition 7 The ranking function ϕ satisfies *weak consistency* if for all $R = \langle J, C \rangle$, with $|J| > 2$ and for all $k \in J$, if $\sum_{i \in J} c_{ij} = \sum_{i \in J} c_{ik}$ for all $j \in J$, then

$$\frac{\phi_i(R)}{\phi_j(R)} = \frac{\phi_i(R^k)}{\phi_j(R^k)} \qquad \text{for all } i, j \in J \setminus \{k\}.$$

The following diagram exemplifies the weak consistency requirement:

$$\text{If} \quad \begin{array}{c c} & \begin{array}{ccc} i & j & k \end{array} \\ \begin{array}{c} i \\ j \\ k \end{array} & \begin{pmatrix} 30 & 16 & 10 \\ 15 & 35 & 20 \\ 15 & 9 & 30 \end{pmatrix} \end{array} \xrightarrow{\phi} \begin{pmatrix} v_i \\ v_j \\ v_k \end{pmatrix} \quad \text{then} \quad \begin{array}{c c} & \begin{array}{cc} i & j \end{array} \\ \begin{array}{c} i \\ j \end{array} & \begin{pmatrix} 30+5 & 16+3 \\ 15+10 & 35+6 \end{pmatrix} \end{array} \xrightarrow{\phi} \begin{pmatrix} \frac{v_i}{v_i+v_j} \\ \frac{v_j}{v_i+v_j} \end{pmatrix}.$$

Note that the 15 citations from journal i to journal k are distributed between i and j in the proportion $10/20 = c_{ik}/c_{jk}$ of the citations received from k. Similarly, the 9 citations from j to k are distributed among i and j in the same proportions.

We are now ready to characterize the only ranking method that satisfies all the properties described so far.

Theorem 1 There is a unique ranking function that satisfies anonymity, invariance to citation intensity, homogeneity for two-journal problems, and weak consistency. This is the Invariant method ϕ_I.

Proof : By its mere definition, the Invariant method satisfies invariance to citation intensity. To see that this method satisfies anonymity, let P be a permutation matrix and assume that v^* solves $CD^{-1}v = v$. We need to show that Pv^* solves $(PCD^{-1}P^T)v = v$. (Note that the normalized matrix of PCP^T is the permutation of the normalized matrix of C: $(PCP^T)(PD^{-1}P^T) = PCD^{-1}P^T$.) But if

$$v^* = CD^{-1}v^*,$$

then

$$\begin{aligned} Pv^* &= PCD^{-1}v^* \\ &= PCD^{-1}(P^TP)v^* \\ &= (PCD^{-1}P^T)Pv^*, \end{aligned}$$

which means that Pv^* is an invariant distribution of $PCD^{-1}P^T$. It is also straightforward to check that the Invariant method satisfies homogeneity for two-journal problems. Let us show that it also satisfies consistency. Let $R = \langle J, C\rangle$, let $(v_i)_{i\in J} = \phi_I(R)$, and let $k \in J$. Since ϕ_I satisfies invariance to citation intensity, we can assume that the entries of each column of C add up to one. It is enough to show that

$$\sum_{j\in J\setminus\{k\}} c^k_{ij} v_j = v_i.$$

By definition of c^k_{ij},

$$\sum_{j\in J\setminus\{k\}} c^k_{ij} v_j = \sum_{j\in J\setminus\{k\}} (c_{ij} + c_{kj}\frac{c_{ik}}{\sum_{t\in J\setminus\{k\}} c_{tk}})v_j$$

$$= \sum_{j\in J\setminus\{k\}} c_{ij}v_j + \sum_{j\in J\setminus\{k\}} (c_{kj}\frac{c_{ik}}{1-c_{kk}})v_j.$$

Since $v_i = \sum_{j\in J} c_{ij}v_j$, we have $\sum_{j\in J\setminus\{k\}} c_{ij}v_j = v_i - c_{ik}v_k$. Therefore,

$$\begin{aligned}
\sum_{j\in J\setminus\{k\}} c^k_{ij}v_j &= v_i - c_{ik}v_k + \frac{c_{ik}}{1-c_{kk}}\sum_{j\in J\setminus\{k\}} c_{kj}v_j \\
&= v_i - c_{ik}v_k + \frac{c_{ik}}{1-c_{kk}}(v_k - c_{kk}v_k) \\
&= v_i - c_{ik}v_k + \frac{c_{ik}}{1-c_{kk}}v_k(1-c_{kk}) \\
&= v_i.
\end{aligned}$$

We shall show next that a ranking method that satisfies the four axioms must be the Invariant method. For this, we show first that any ranking method that satisfies anonymity, invariance to citation intensity, and homogeneity for two-journal problems must coincide with the Invariant method in two-journal problems.

Lemma 1 Let $\phi : \mathcal{R} \to \Delta$ be a ranking method that satisfies anonymity, invariance to citation intensity, and homogeneity for two-journal problems. Then, for every two-journal problem $R = \langle\{i,j\}, C\rangle$, $\phi(R)$ is the vector of valuations awarded by the Invariant method.

Proof : Let $R = \langle\{i,j\}, C\rangle$ be the a ranking problem. We need to show that $\phi(R) = (c_{ij}/(c_{ij}+c_{ji}), c_{ji}/(c_{ij}+c_{ji}) = \phi_I(R)$. By invariance to citation intensity, we can assume that the columns of the citation matrix of R add up to one. By homogeneity for two-journal problems, there is a positive constant α such that:

$$\phi_i(R)/\phi_j(R) = \alpha\frac{c_{ij}}{c_{ji}}. \tag{1}$$

We will show that $\alpha = 1$. To see this, consider the auxiliary ranking problem $R' = \langle\{i,j\}, C'\rangle$ with

$$C' = \begin{pmatrix} c_{jj} & c_{ji} \\ c_{ji} & c_{jj} \end{pmatrix}.$$

By homogeneity for two-journal problems, $\phi_i(R')/\phi_j(R') = \alpha$. On the other hand, letting

$P = \binom{0\,1}{1\,0}$, by anonymity:

$$\begin{aligned} P\phi(R') &= \phi(\{i,j\}, PC'P^T) \\ &= \phi(\{i,j\}, C') \\ &= \phi(R'), \end{aligned}$$

where the second equality follows from the particular choice of the matrix C'. Therefore, we must have that $\phi(R') = (1/2, 1/2)$ and, consequently, $\alpha = 1$. As a result, it follows from (1) that

$$\phi_i(R)/\phi_j(R) = \frac{c_{ij}}{c_{ji}},$$

which together with $\phi_i(R) + \phi_j(R) = 1$ implies the desired result. $\square$

We shall now show that if ϕ satisfies consistency and coincides with the Invariant method for two-journal problems, then ϕ is the Invariant method. The proof is by induction on the number of journals. Assume that ϕ coincides with the invariant function for all n-journal problems and let $R = \langle J, C\rangle$ be an $n+1$ journal problem. For all $k \in J$ we have $\phi(R^k) = \phi_I(R^k)$. But then, by consistency of both ϕ and ϕ_I, for all $k \in J$

$$\frac{\phi_i(R)}{\phi_j(R)} = \frac{\phi_i(R^k)}{\phi_j(R^k)} = \frac{(\phi_I)_i(R^k)}{(\phi_I)_j(R^k)} = \frac{(\phi_I)_i(R)}{(\phi_I)_j(R)} \qquad \text{for all } i, j \in J.$$

This implies that $\phi(R) = \phi_I(C)$. $\square$

2.2 Independence of the Axioms

We show in this subsection that the four axioms used in the above characterization are logically independent.

In order to see that the homogeneity axiom is not implied by the other three, consider the Egalitarian method $\phi_E : \mathcal{R} \to \Delta$ defined by $\phi_E(J, C) = (1/|J|, \ldots, 1/|J|)$. It is easy to check that ϕ_E satisfies anonymity, invariance to citation intensity, and consistency, but it does not satisfy homogeneity for two-journal problems.

As for weak consistency, consider the function $\phi_{SC} : \mathcal{R} \to \Delta$ defined by $\phi_{SC}(J, C) = \phi_{MC}(J, CD^{-1})$, where again CD^{-1} is the normalized citation matrix. This method applies the Modified Counting method to the normalized matrix of citations. It satisfies invariance to citation intensity since it applies the Modified Counting method to the normalized matrix of citations. It can be readily checked that it satisfies homogeneity for two-journal problems. The fact that this method satisfies anonymity is shown in the Appendix. This method, however, does not satisfy weak consistency.

To see that anonymity is not implied by the other axioms, let $\sigma : \mathcal{J} \to I\!N$ be a non-constant function. For each J let $H_J = \text{diag}(\sigma(j))_{j \in J}$ be the diagonal matrix whose diagonal entry H_{jj} is $\sigma(j)$. The function ϕ_σ assigns to each ranking problem $R = \langle J, C \rangle$, the unique solution $v \in \Delta_J$ to $(CD^{-1})H_J v = H_J v$, where CD^{-1} is the normalization of C. By its own definition, the method ϕ_σ satisfies invariance to citation intensity. It can be checked that it also satisfies homogeneity for two-journal problems: for all relevant problems $R = \langle J, C \rangle$,

$$(\phi_\sigma)_i(R)/(\phi_\sigma)_j(R) = \alpha c_{ij}/c_{ji},$$

where $\alpha = \sigma_i/\sigma_j$. The proof that this method satisfies consistency is analogous to the proof that the Invariant method satisfies consistency. The method ϕ_σ does not satisfy anonymity.

Lastly, the Liebowitz-Palmer method, satisfies anonymity, homogeneity and weak consistency, but does not satisfy invariance to citation intensity. To see that it satisfies anonymity, let P be a permutation matrix and assume that v^* solves $Cv = \lambda v$ where $\lambda = \|Cv^*\|$ is the maximum eigenvalue of C. We need to show that Pv^* solves $(P^T CP)v = \lambda v$. But if

$$\lambda v^* = Cv^*,$$

then

$$\begin{aligned} \lambda P v^* &= PCv^* \\ &= (PCP^T)Pv^*, \end{aligned}$$

which means that Pv^* is a maximal eigenvector of PCP^T.

The fact that this method satisfies homogeneity for two-journal problems follows from the fact that for any citation matrix of the form

$$C = \begin{pmatrix} \lambda - c_{21} & c_{12} \\ c_{21} & \lambda - c_{12} \end{pmatrix}$$

we have $C(c_{12}, c_{21})^T = \lambda(c_{12}, c_{21})^T$.

The proof that the Liebowitz-Palmer method satisfies weak consistency is similar to the proof that the Invariant method satisfies consistency and it is shown in the Appendix.

3 The Intellectual Influence of a Manuscript

The previous section dealt with the problem of measuring the *overall* impact of journals. It is also of interest to measure the impact *per manuscript* of each journal. Note that any ranking method that measures the overall impact of journals can also be thought of as a ranking method of the impact per manuscript in the case where all the journals under consideration have the same number of articles. Viewed in this light, the problem of deriving a general ranking method of the impact per manuscript of journals simply consists of the problem of extending a ranking method of journals with the same number of articles to a ranking method of journals with a potentially different number of articles.

The first thing to note is that we need to add to the primitives of the model the number of articles published by the various journals over the period of interest. Thus, each journal in $i \in \mathcal{J}$ now has a number $a_i > 0$ of articles published in the relevant period associated with it. Therefore a ranking problem is now given by $\langle J, (a_i)_{i \in J}, (c_{ij})_{(i,j) \in J \times J} \rangle$. We shall denote the set of all possible per manuscript ranking problems by $\mathcal{M}$. Hence, a per manuscript ranking method will be a function $f : \mathcal{M} \to \Delta$ where for each $R \in \mathcal{M}$, $f(R) \in \Delta_J$.

Given an overall journal ranking method ϕ, there is a straightforward way to extend it to a per manuscript ranking method. If $\phi(R) = (v_i)_{i \in J}$ is the vector of overall valuations corresponding to problem $R = \langle J, C \rangle$, the corresponding vector of per manuscript valuations should be proportional to $(v_i/a_i)_{i \in J}$. That is, the value of a manuscript in a journal that published a_i articles and whose overall valuation is v_i, should be v_i/a_i (properly normalized) so that the value of the journal equals the sum of the values of its published articles. More formally, given a vector $(a_i)_{i\in J}$ of number of articles, let $A = \text{diag}(a_i)_{i\in J}$ be the diagonal matrix whose generic diagonal entry is a_i. Given an overall journal ranking method $\phi : \mathcal{R} \to \Delta$, the *per manuscript ranking method associated with* ϕ is $f : \mathcal{M} \to \Delta$ defined by $f(J, a, C) = \frac{A^{-1}\phi(J,C)}{\|A^{-1}\phi(J,C)\|}$. In other words the per manuscript ranking method associated to the overall ranking method ϕ takes the per article vector of valuations according to ϕ and normalizes it so that its entries add up to one.

Examples:

1. The per manuscript ranking method associated with the Counting method ϕ_C is defined by $f_C(J, a, C) = \frac{A^{-1}\phi_C(J,C)}{\|A^{-1}\phi_C(J,C)\|}$. Equivalently, $f_C(J, a, C) = (\frac{\sum_{j\in J} c_{ij}/a_i}{\sum_{k\in J}\sum_{j\in J} c_{kj}/a_k})_{i\in J}$. Note that $f_C(J, a, C) = \phi_C(J, A^{-1}C)$.

2. The per manuscript ranking method associated with the Invariant method ϕ_I, or per manuscript Invariant method, assigns to each ranking problem $R = \langle J, a, C\rangle$, the vector $f_I(J, a, C) = \frac{A^{-1}\phi_I(J,C)}{\|A^{-1}\phi_I(J,C)\|}$. Equivalently, $f_I(R)$ assigns the unique vector $v \in \Delta_J$, that solves the equation $v = A^{-1}(CD^{-1})Av$.

3. The per manuscript ranking method associated with the Liebowitz-Palmer method ϕ_P assigns to each ranking problem $R = \langle J, a, C\rangle$, the vector $f_{LP}(J, a, C) = \frac{A^{-1}\phi_{LP}(J,C)}{\|A^{-1}\phi_{LP}(J,C)\|}$. Equivalently, $f_{LP} = \phi_{LP}(J, A^{-1}CA)$.[10]

To the best of our knowledge, none of the above are per manuscript methods used in the economics literature or in any other discipline. For instance, Liebowitz and Palmer (1984) propose a method that consists of applying the overall ranking method ϕ_{LP} to a corrected matrix of citations. Specifically, they propose to take the matrix of citations C and divide each row i by some measure Z_i, which could be the number of characters published by journal i or the number of articles published by journal i or any other number. For our purposes, it will be convenient to analyze the method that corrects the matrix of citations by the number of articles. Formally,

4. The *JEL* per manuscript ranking method $f_{JEL} : \mathcal{M} \to \Delta$ is defined by $f_{JEL}(J, a, C) = \phi_{LP}(J, A^{-1}C)$.[11]

[10]To see that the two definitions are equivalent, note that if $\phi_{LP}(J, C)$ is a positive eigenvector of C, then $A^{-1}\phi_{LP}(J, C)$ is a positive eigenvector of $A^{-1}CA$, and therefore $f_{LP}(J, a, C) = \phi_{LP}(J, A^{-1}CA)$.

[11]This method was proposed and first used in Liebowitz and Palmer (1984). We call this method JEL,

Note that the JEL per manuscript ranking method is not the per manuscript method associated with the Liebowitz-Palmer overall ranking method. In fact, these two methods may give very different results.[12]

It is useful to compare the per manuscript Invariant and the JEL methods. Both methods calculate a positive eigenvector of an appropriately corrected matrix of citations. The JEL method calculates the positive eigenvector of the matrix $A^{-1}C$ while the Invariant method calculates the positive eigenvector of the matrix $A^{-1}CD^{-1}A$. The entry c_{ij}/a_i of the matrix $A^{-1}C$ is the average number of citations that an article in journal i gets from journal j. This is the underlying measure of direct impact (of a typical article in i on a typical article in j) that the JEL method takes into account. As we will later show, this method does not satisfy invariance to citation intensity. The per-manuscript Invariant method controls for citation intensity by dividing the value c_{ij}/a_i by $\sum_k c_{kj}/a_j$, that is, by the average length of the list of references of the articles in j. Therefore, the measure of direct impact of journal i on journal j that underlies the Invariant method is the average number of citations of an article in i out of the average number of citations by a typical article of j.

so that it is not confused with the per manuscript method associated with the Liebowitz-Palmer overall ranking method and because it was adopted subsequently by Laband and Piette (1994a), an article that was also published in the *Journal of Economic Literature* to build a ranking of economic journals.

[12]This can be seen in the following example. Consider a ranking problem with only two relevant journals: $J = \{1, 2\}$. Journal 1 has 250 published articles during the relevant period while Journal 2 has only 100 published articles during the same period: $a^T = (250, 100)$. The matrix of citations if given by

$$C = \begin{pmatrix} 30 & 10 \\ 10 & 30 \end{pmatrix}.$$

If one applies the Liebowitz-Palmer overall ranking method to the ranking problem $R = \langle J, C\rangle$, we get that $\phi_{LP}(R) = (1/2, 1/2)^T$. In this case, any anonymous ranking method ϕ yields $\phi(R) = (1/2, 1/2)^T$. The interpretation is that both journals have the same overall impact. However, if we divide the overall impact of each of the journals by their respective number of articles, after the appropriate normalization, we get that:

$$f_{LP}(R) = \frac{A^{-1}\phi_{LP}(R)}{\|A^{-1}\phi_{LP}(R)\|} = (2/7, 5/7).$$

This means that each article published in Journal 2 during the relevant period has an impact of 2.5 times the impact of an article published in Journal 1. However, if we apply the JEL per manuscript ranking method to the ranking problem $R' = \langle J, a, C\rangle$, we get that $f_{\text{JEL}}(R') = (1/6, 5/6)^T$, meaning that each article published in Journal 2 during the relevant period has 5 times more impact than each article published in Journal 1. As one can see, the overall and per manuscript ranking methods proposed by Liebowitz and Palmer (1984) are two different methods, not clearly related to each other.

The properties of overall journal-ranking methods can be adapted to per manuscript ranking methods as follows:

Definition 8 A per manuscript ranking function f satisfies *anonymity* if for all ranking problems $R = \langle J, a, C \rangle$ and for all $|J| \times |J|$ permutation matrices P, we have $\phi(J, Pa, PCP^T) = P\phi(J, a, C)$.

Definition 9 A per manuscript ranking function f satisfies *invariance with respect to citation intensity* if for every ranking problem $\langle J, a, C \rangle$ and for every positive, diagonal, $|J| \times |J|$ matrix $\Lambda = \text{diag}(\lambda_i)_{i \in J}$, we have that $f(J, a, C\Lambda) = f(J, a, C)$.

Definition 10 Let $R = \langle \{i, j\}, (a_i, a_j), C \rangle$ be a two-journal problem such that $c_{ii} + c_{ji} = c_{ij} + c_{jj}$ and $a_i = a_j$. The per manuscript ranking function f satisfies *homogeneity for two-journal problems* if there is a $\alpha > 0$ (that may depend on $\{i, j\}$ but not on C), such that for all such problems, $f_i(R)/f_j(R) = \alpha c_{ij}/c_{ji}$.

Definition 11 The per manuscript ranking function f satisfies *weak consistency* if for all $R = \langle J, a, C \rangle$, with $|J| > 2$ and such that $a_i = a_j$ for all $i, j \in J$, and for all $k \in J$, if $\sum_{i \in J} c_{ij} = \sum_{i \in J} c_{ik}$ for all $j \in J$, then

$$\frac{f_i(R)}{f_j(R)} = \frac{f_i(R^k)}{f_j(R^k)} \qquad \text{for all } i, j \in J \setminus \{k\}.$$

In order to motivate the next property of per manuscript ranking methods, consider a ranking problem $\langle J, a, C \rangle$ and suppose that a journal $j \in J$ splits into two identical journals. Specifically, j splits into journal $(j, 1)$ and journal $(j, 2)$, each with $a_j/2$ articles. Further, for these two new journals to be equivalent, the citations are also split: the vectors of citations by journals $(j, 1)$ and $(j, 2)$ to the other journals are equal and given by $\bar{c}_j/2$. Also, the citations of journal j are equally split: each journal $i \neq j$ is cited $c_{ij}/2$ times by each of the newly-born journals. Lastly, the self-citations c_{jj} are equally split between the two journals, giving $c_{(j,\alpha),(j,\beta)} = c_j/4$ for $\alpha, \beta = 1, 2$. For a case of a two journal problem, this split is illustrated as follows:

$$\begin{array}{cc} & \begin{array}{cc} i & j \end{array} \\ \begin{array}{c} i \\ j \end{array} & \begin{pmatrix} c_{ii} & c_{ij} \\ c_{ji} & c_{jj} \end{pmatrix} \end{array} \longrightarrow \begin{array}{cc} & \begin{array}{ccc} i & (j,1) & (j,2) \end{array} \\ \begin{array}{c} i \\ (j,1) \\ (j,2) \end{array} & \begin{pmatrix} c_{ii} & c_{ij}/2 & c_{ij}/2 \\ c_{ji}/2 & c_{jj}/4 & c_{jj}/4 \\ c_{ji}/2 & c_{jj}/4 & c_{jj}/4 \end{pmatrix} \end{array}.$$

We would like this split of journal j not to influence the ranking of the articles published. In other words, the old and new relative rankings should be the same. We formalize this property next.

Let $R = \langle J, (a_j)_{j \in J}, (c_{ij})_{(i,j) \in J \times J} \rangle$ be a ranking problem. Each journal $j \in J$ will be split into $T_j \geq 1$ identical journal, denoted (j, t), for $t = 1, \ldots T_j$. With some abuse of notation, we shall denote by T_j both the number and the set of "types" of journal j. The resulting ranking problem is $R' = \langle J', (a_{j,t_j})_{j \in J, t_j \in T_j}, (c'_{(i,t_i)(j,t_j)})_{((i,t_i),(j,t_j)) \in J' \times J'} \rangle$ where $J' = \{(j, t_j) : j \in J, t_j \in T_j\}$, $a_{j,t_j} = a_j/T_j$ and $c'_{(i,t_i)(j,t_j)} = \frac{c_{ij}}{T_i T_j}$. We will call the problem R' a split of R, and we will denote its citation matrix $(c'_{(i,t_i)(j,t_j)})_{((i,t_i),(j,t_j)) \in J' \times J'}$ by C'.

As mentioned above, we would expect a split of a journal not to affect the relative valuations of the articles. This is the requirement imposed by the following property.

Definition 12 A ranking method f satisfies *invariance to splitting of journals* if for all ranking problems $R = \langle J, (a_j)_{j \in J}, (c_{ij})_{(i,j) \in J \times J} \rangle$, for all $i, j \in J$ and for all its splittings $R' = \langle J', (a_{j,t_j})_{j \in J, t_j \in T_j}, (c'_{(i,t_i)(j,t_j)})_{((i,t_i),(j,t_j)) \in J' \times J'} \rangle$, we have:

$$f_i(R)/f_j(R) = f_{i,t_i}(R')/f_{j,t_j}(R') \qquad \forall i, j \in J \text{ and } \forall t_i \in T_i \text{ and } t_j \in T_j.$$

We are now ready to state the main result of this section:

Theorem 2 There is a unique per manuscript ranking function that satisfies anonymity, invariance to citation intensity, homogeneity for two-journal problems, weak consistency, and invariance to splitting of journals. This is the per manuscript Invariant method.

Proof : Let $R = \langle J, (a_j)_{j \in J}, (c_{ij})_{(i,j) \in J \times J} \rangle$ be a ranking problem and let $A = \text{diag}(a_j)_{j \in J}$. Let f be a ranking method that satisfies all the foregoing axioms. We need to show that

$f(R)$ solves the equation $Av = CD^{-1}Av$. Let CD^{-1} be C's normalized matrix. Define $v^* \in \Delta_J$ to be the only solution to $CD^{-1}v = v$. That is, v^* would be the vector of relative valuations awarded by the Invariant method if all the journals in J had the same number of articles. Also, v^* can be interpreted as the vector of overall valuations of the journals in J. We need to show that if the method f is to satisfy all the axioms, and in particular invariance of splitting of journals, then $f(R) = \frac{A^{-1}v^*}{\|A^{-1}v^*\|}$. Let $\tilde{v} = A^{-1}v^*$. Let G be the greatest common divisor of $(a_j)_{j\in J}$ and let $T_j = a_j/G$. We will split each journal $j \in J$ into T_j identical journals. The set of journals will be $J' = \{(j, t_j) : j \in J, t_j \in T_j\}$. The number of articles of journal (j, t_j), for $j \in J$, $t_j \in T_j$, is given by $a_{(j,t_j)} = a_j/T_j = G$. The new matrix of citations is $C' = (c'_{(i,t_i)(j,t_j)})$ where $c'_{(i,t_i)(j,t_j)} = c_{ij}/(T_iT_j)$. Summarizing, $R' = \langle J', (a_{j,t_j})_{j\in J, t_j\in T_j}, (c'_{(i,t_i)(j,t_j)})_{((i,t_i),(j,t_j))\in J'\times J'}\rangle$.

Since R' is a ranking problem where all journals have the same number of articles, we know by Theorem 1 that $f(R')$ is the solution to $C'D'^{-1}v = v$, where $C'D'^{-1}$ is the normalization of C'. Denote this unique solution by $\bar{v}$. Note that $\bar{v}$ is a $|J'|$-dimensional vector. However, by anonymity of f, we know that $\bar{v}_{(i,t_i)} = \bar{v}_{(i,s_i)}$ for all $i \in J$ and for all $t_i, s_i \in T_i$. That is, an article published in sub-journal (i, t_i) has the same value as an article published in the sub-journal (i, s_i). Denote this common value by $\bar{v}_i$, $i \in J$. We shall show that, for all $i \in J$, $\bar{v}_iT_i = \sum_{j\in J} \bar{v}_jT_jc_{ij}/d_{jj}$.

To see this, note that

$$
\begin{aligned}
\bar{v}_i = \bar{v}_{(i,t_i)} &= \sum_{j\in J}\sum_{t_j\in T_j} \bar{v}_{(j,t_j)}c'_{(i,t_i)(j,t_j)}/d'_{(j,t_j)(j,t_j)} \\
&= \sum_{j\in J}\sum_{t_j\in T_j} \bar{v}_j\frac{c_{ij}}{T_iT_j}T_j/d_{jj} \\
&= \sum_{j\in J}\sum_{t_j\in T_j} \bar{v}_j\frac{c_{ij}}{T_i}/d_{jj} \\
&= \sum_{j\in J} \bar{v}_jT_j\frac{c_{ji}}{T_i}/d_{jj}.
\end{aligned}
$$

Therefore,

$$\bar{v}_iT_i = \sum_{j\in J} \bar{v}_jT_jc_{ij}/d_{jj},$$

which implies

$$\bar{v}_i T_i = v_i^*.$$

Dividing both sides by a_i we get

$$\frac{\bar{v}_i}{G} = \frac{v_i^*}{a_i} \qquad \text{for all } i \in J.$$

This means that the vectors $A^{-1}v^*$ and $(\bar{v}_i)_{i\in J}$ are proportional. But by the invariance of f to splitting of journals, we know that the vectors $f(R)$ and $(\bar{v}_i)_{i\in J}$ are proportional too, which implies that $A^{-1}v^*$ and $f(R)$ are proportional. Since $\|f(R)\| = 1$, we must have $f(R) = \frac{A^{-1}v^*}{\|A^{-1}v^*\|}$. □

4 Illustrations

In this section we offer two applications that illustrate the use of the overall and per-manuscript Invariant methods. We would like to stress that the only purpose of these applications is to illustrate the use of these methods and to compare them with the results obtained with other methods. Therefore, any similarities with the "true" rankings, that could be obtained only by considering the complete set of relevant journals, is pure coincidence.

We first report the Invariant rankings, both per journal and per manuscript in a sample of 36 journals in economics. We use data of citations during the year 2000 to articles published during the period 1993–1999. The data comes from the 2000 Social Sciences Edition of the Journal Citation Reports published by the Institute for Scientific Information.[13] The results are shown in Table 1 with and without the *Journal of Economics Literature* and the *Journal of Economic Perspectives*.

[13] The dataset considers all papers published by these journals including short papers, comments, and non-refereed articles.

Table 1
Impact Rankings Based on citations on 2000 – Invariant Method

Per-manuscript ranking			Overall Ranking		
Econometrica	100.00	100.00	American Economic Review	100.00	100.00
Quarterly Journal of Economics	98.72	94.29	Econometrica	87.48	92.26
Journal of Economic Literature	80.47		Quarterly Journal of Economics	69.13	69.65
Journal of Political Economy	67.02	65.26	Journal of Political Economy	54.95	56.43
Review of Economic Studies	64.10	64.64	Journal of Economic Theory	51.47	55.08
Journal of Monetary Economics	44.95	46.14	Review of Economic Studies	38.46	40.90
American Economic Review	37.06	35.13	Games and Economic Behavior	32.49	34.91
Journal of Economic Theory	34.40	34.90	Journal of Econometrics	31.13	32.48
Games and Economic Behavior	31.99	32.59	Journal of Monetary Economics	31.05	33.62
Journal of Economic Perspectives	31.65		Journal of Economics Perspectives	26.63	
Journal of Econometrics	21.38	21.16	Journal of Economic Literature	24.81	
Rand Journal of Economics	20.19	19.84	European Economic Review	22.20	23.27
Journal of Labor Economics	18.52	13.41	Journal of Public Economics	21.31	22.64
Economic Theory	18.18	18.91	Economic Theory	19.64	21.55
Journal of Human Resources	17.70	13.97	Review of Economics and Statistics	18.41	18.15
Review of Economics and Statistics	16.42	15.35	Economic Journal	17.26	17.04
Journal of Risk and Uncertainty	16.31	11.43	Econometric Theory	15.37	16.46
Econometric Theory	16.20	16.45	Rand Journal of Economics	13.66	14.15
Journal of Public Economics	15.95	16.07	International Economic Review	13.38	13.91
International Economics Review	15.63	15.42	Journal of Human Resources	11.80	9.82
Journal of Financial Economics	15.39	17.28	Journal of Bus. And Economics Statistics	11.61	12.51
Journal of Bus. and Economics Statistics	14.63	14.95	Journal of Financial Economics	11.29	13.38
European Economic Review	13.05	12.96	Journal of Economic Dynmcs. and Control	11.04	11.67
Journal of Applied Econometrics	12.79	13.23	Economics Letters	10.57	11.34
Social Choice and Welfare	12.45	13.01	Journal of Labor Economics	10.40	7.94
Journal of Environmental Ecs. and Mgmnt.	12.39	11.34	Journal of Environmental Ecs. and Mgmnt.	9.30	8.98
International Journal of Game Theory	12.18	12.57	Journal of International Economics	8.14	6.74
Economic Journal	12.07	11.30	Journal of Mathematical Economics	7.55	8.28
Journal of International Economics	11.43	8.97	Journal of Risk and Uncertainty	7.33	5.41
Journal of Economic Dynmcs. and Control	10.45	10.47	Social Choice and Welfare	6.73	7.41
Journal of Mathematical Economics	10.06	10.46	Journal of Applied Econometrics	6.60	7.20
Economic Inquiry	6.22	5.19	International Journal of Game Theory	6.09	6.62
Journal of Economic Behavior and Org.	5.04	5.10	Journal of Economic Behavior and Org.	6.04	6.45
Scandinavian Journal of Economics	4.03	4.12	Economic Inquiry	5.26	4.63
Economics Letters	3.08	3.13	Scandinavian Journal of Economics	2.45	2.65
Oxford Business and Economics Statistics	2.77	2.68	Oxford Business and Economics Statistics	1.56	1.59

We are particularly interested in comparing the Invariant methods to other methods, as well as in calculating the implied impact of individual rather than representative papers. We illustrate these applications using disaggregated data of the citations given by each paper published in the year 2000 to every paper published during a the period 1993–1999. Given the high cost involved in looking up and counting citations by hand from a large number of journals, we examine a small set of journals.[14] The journals we examine are the top 5 general interest journals obtained from the previous table: *Review of Economic Studies, American Economic Review, Econometrica, Journal of Political Economy* and *Quarterly Journal of Economics*.[15] For purposes of comparison, in addition to the overall

[14]The required data per article are not available in Journal Citation Reports or, to the best of our knowledge, in any other electronic database.

[15]The data we will use in what follows for the *American Economic Review* excludes the "Papers and

and per-manuscript Invariant methods, we also calculate the numerical rankings that result from the Liebowitz-Palmer (LP) overall method and the JEL per manuscript method. Table 2 first shows the aggregate data.

Table 2

Number of citations for five journals, 2000

Cited Journal	# of articles	Citing Journal: REStud	AER	Ecmta	JPE	QJE
Review of Economic Studies	252	21	24	15	13	10
American Economic Review	577	8	122	10	24	38
Econometrica	369	48	27	57	17	11
Journal of Political Economy	355	18	43	7	31	20
Quarterly Journal of Economics	294	13	73	10	24	53
Total		108	289	99	109	132

The following table shows the measure of overall impact for these journals according to the Invariant and the Liebowitz-Palmer (LP) methods.

Table 3

Overall impact of journal rankings based on citations on 2000

Invariant method		Liebowitz-Palmer method	
1. Econometrica	100.00	1. American Economic Review	100.00
2. American Economic Review	84.39	2. Quarterly Journal of Economics	79.22
3. Quarterly Journal of Economics	81.33	3. Econometrica	51.28
4. Journal of Political Economy	54.72	4. Journal of Political Economy	48.42
5. Review of Economic Studies	43.26	5. Review of Economic Studies	30.55

Note the substantial difference between the two numerical rankings. While according to the Invariant method, Econometrica has substantially more impact than the AER and the QJE, according to the LP method the overall impact of Econometrica is barely half that of the AER and some 65% of that of the QJE. This difference can be traced back to the failure of the LP method to satisfy invariance to citation intensity. Table 1 shows that the AER and QJE together have more cited references (longer list of references) than the other three journals. As a result, since the LP method gives more weight to the opinions of

Proceedings" issue published in May.

journals with greater citation intensity, this method yields a ranking more biased toward the opinions of the AER and QJE. Note that these two journals give more weight to AER and QJE than to the other three.

Table 4 shows the impact per manuscript of each of the five journals. Note again the great difference in the measurement of impact. While QJE and Econometrica have similar impact according to the Invariant method, according to the JEL method a paper in Econometrica has a 63% of the impact of a paper published in the QJE. Again, the differences in the numerical rankings are due to the failure of the JEL method to satisfy the invariance axiom.

Table 4
Per-Manuscript Impact of Journal Rankings Based on citations on 2000

Invariant method		JEL method	
1. *Quarterly Journal of Economics*	100.00	1. *Quarterly Journal of Economics*	100 .00
2. *Econometrica*	97.96	2. *Econometrica*	63.60
3. *Review of Economic Studies*	62.05	3. *Journal of Political Economy*	51.00
4. *Journal of Political Economy*	55.72	4. *American Economic Review*	55.06
5. *American Economic Review*	52.87	5. *Review of Economic Studies*	49.05

Lastly, a number of questions of interest require an analysis at the level of individual, rather than representative, manuscripts. For instance, in addition to the various issues mentioned in footnote 2, there is considerable discussion of the possible role of editors in steering disciplines, pushing or suppressing various lines of research, favoring friends and personal associates, and others. It also seems clear that articles may vary individually in quality and influence, and that not all articles appearing in the same journal, even in the same issue, are likely to be equally influential. A journal might, for example, provide the same impact by publishing all of its papers of similar quality while another journal may attain the same overall impact by publishing just a few articles of exceptional quality and many of low quality.[16] The Invariant method can also be used to calculate the impact of individual articles. The overall value of journal i according to the Invariant method is

[16]It would then be relevant to examine differences across journals in the degree of inequality in the distribution of influence of the articles in the journal, using inequality criteria base on normalized stochastic dominance. Any difference may reflect, at least in part, differences in risk-taking behavior or other aspects of the tastes of the editors.

$v_i = \sum_j \frac{c_{ij}}{c_j} v_j$. And the average value of an article in i according to the per-manuscript Invariant method is proportional to v_i/a_i. Therefore, the value of a particular paper p_i published in journal i is given by $v_{p_i} = \sum_j \frac{c_{p_i j}}{c_j} v_j$. Namely, the impact-adjusted sum of the citations of paper p_i. Alternatively, the value of paper p_i can be expressed as $v_{p_i} = \sum_j \frac{c_{p_i j}}{c_j/a_j} \frac{v_j}{a_j}$. Note that the average impact of a paper in journal i may be greater than that of a paper in journal j, while a citation from journal j may be more valuable than a citation from journal i. Formally, we may have $v_i/a_i > v_j/a_j$, while at the same time, $\frac{1}{c_j/a_j} > \frac{1}{c_i/a_i}$.

The following are the top ten papers in our sample according to the Invariant method, along with their citations.

Table 5
Top ten impact-adjusted cited articles, 2000

Cited Article	Impact
Benabou (1996)	3.02
Milgrom and Shannon (1994)	3.01
Ellison (1993)	2.90
Juhn, Murphy and Pierce (1993)	2.90
Autor, Katz and Krueger (1998)	2.83
Rabin (1993)	2.83
Galor and Tsiddon (1997)	2.79
Glaeser, Sacerdote and Scheinkman (1996)	2.76
Bolton and Dewatripont (1994)	2.74
Piketty (1995)	2.71

Note that although, on average, a paper in the Quarterly Journal of Economics has more impact than a paper in the American Economic Review, the paper with most impact in our sample appeared in the AER. Also note that although a paper published in the QJE has more impact that one published in Econometrica, a citation from Econometrica is more valuable than a citation from QJE. This is so because the average paper in QJE has a longer reference list, thus awarding each of its cited references a small proportion of its value.[17]

[17]The references for these papers are: R. Benabou (1996), "Heterogeneity, Stratification, and Growth: Macroeconomic Implications of Community Structure and School Finance," AER, pp. 584-609; P. Milgrom and C. Shannon (1994), "Monotone Comparative Statics," Econometrica, pp. 157-180; G. Ellison (1993),

5 Concluding Remarks

We have examined the problem of measuring influence using the information contained in the communication between scholarly publications. The same problem arises in the analysis of the basing of judicial decisions on previous decisions, the analysis of the dynamics of innovation where patent records contain citation references to previous patents and discoveries, in the World Wide Web where search engines rank web pages according to their importance, and in other areas. The information provided by these communications is important. It allows us to make rigorous quantitative analyses of elusive but important phenomena such as reputation, influence, the diffusion of scientific knowledge, the quality of journals, the productivity of scholars, changes in the publication process and in the returns and incentives for publication, several aspects of the labor market for scientists, personnel decisions in academic administration, and many others. Yet, the actual use of this information has been conducted without any use of theoretical methodologies that could help us understand the content in these communications. In particular, simple citation counts and a number of arbitrary methods have been widely employed in the measurement of influence and in applications.

The result of these arbitrary practices is that the information content of the network communications that play a paramount role in the exchange, dissemination, and certification of knowledge and information is little understood. This, in turn, has important implications for all the empirical applications mentioned above.

In this paper we apply the axiomatic methodology to bear on the problem of measuring influence based on communication data, and characterize different measurement methods according to the properties that they satisfy and those that they do not satisfy. We obtain the result that a unique ranking method can be characterized by certain basic properties.

"Learning, Local Interaction, and Coordination," Econometrica, pp. 1047-1071; C. Juhn, K. M. Murphy, and B. Pierce (1993), "Wage Inequality and the Rise in Returns to Skill," JPE, pp. 410-442; D. H. Autor, L. F. Katz, and A. B. Krueger (1998), "Computing Inequality: Have Computers Changed the Labor Market?," QJE, pp. 1169-1213; M. Rabin (1993), "Incorporating Fairness into Game Theory and Economics," AER, pp. 1281-1302; O. Galor and D. Tsiddon (1997), "Technological Progress, Mobility, and Economic Growth," AER, pp. 363-382; E. L. Glaeser, B. Sacerdote, and J. A. Scheinkman (1996), "Crime and Social Interactions," QJE, pp. 507-548; P. Bolton and M. Dewatripont (1994), "The Firm as a Communication Network," QJE, pp. 809-839; T. Piketty (1995), "Social Mobility and Redistributive Politics," QJE, pp. 551-584.

Lastly, we would like to stress that the analysis of communication data cannot be a substitute for critical reading and expert judging, but rather a complement. The communication between scholarly journals, legal opinions, patents, web pages, and other publications contains valuable information that can be used to address several empirical questions of interest. We have argued that we should ask why a given methodology to measure influence is reasonable and informative.

Appendix

Claim 1 The method ϕ_{SC} satisfies anonymity.

Proof For any matrix $M = (m_{ij})$, let $\mathrm{dg}M = \mathrm{diag}(m_{ii})$. With this notation, for each ranking problem $\langle J, C \rangle$ we have

$$\phi_{SC}(J, C) = \frac{(CD^{-1} - \mathrm{dg}(CD^{-1}))\mathbf{1}_{|J|\times 1}}{\|(CD^{-1} - \mathrm{dg}(CD^{-1}))\mathbf{1}_{|J|\times 1}\|},$$

where $\mathbf{1}_{|J|\times 1}$ is the $|J| \times 1$ vector whose entries are all 1. Therefore, for any permutation matrix P we have

$$\begin{aligned} \phi_{SC}(J, PCP^T) &= \frac{(PCD^{-1}P^T - \mathrm{dg}(PCD^{-1}P^T))\mathbf{1}_{|J|\times 1}}{\|(PCD^{-1}P^T - \mathrm{dg}(PCD^{-1}P^T))\mathbf{1}_{|J|\times 1}\|} \\ &= \frac{P(CD^{-1} - \mathrm{dg}(CD^{-1}))P^T\mathbf{1}_{|J|\times 1}}{\|P(CD^{-1} - \mathrm{dg}(CD^{-1}))P^T\mathbf{1}_{|J|\times 1}\|} \\ &= \frac{P(CD^{-1} - \mathrm{dg}(CD^{-1}))\mathbf{1}_{|J|\times 1}}{\|(CD^{-1} - \mathrm{dg}(CD^{-1}))\mathbf{1}_{|J|\times 1}\|} \\ &= P\phi(J, C). \qquad \square \end{aligned}$$

Claim 2 The Liebowitz-Palmer overall ranking method ϕ_{LP} satisfies weak-consistency.

Proof: Let $R = \langle J, C \rangle$ be a ranking problem where all the columns of the citation matrix C add up to the same constant λ, and let $(v_i)_{i \in J} = \phi_{LP}(R)$, and let $k \in J$. It is enough to show that

$$\sum_{j \in J\setminus\{k\}} c_{ij}^k v_j = \lambda v_i.$$

By definition of c_{ij}^k,

$$\begin{aligned} \sum_{j \in J\setminus\{k\}} c_{ij}^k v_j &= \sum_{j \in J\setminus\{k\}} (c_{ij} + c_{kj} \frac{c_{ik}}{\sum_{t \in J\setminus\{k\}} c_{tk}}) v_j \\ &= \sum_{j \in J\setminus\{k\}} c_{ij} v_j + \frac{c_{ik}}{\sum_{t \in J\setminus\{k\}} c_{tk}} \sum_{j \in J\setminus\{k\}} c_{kj} v_j. \end{aligned}$$

Since $\lambda v_i = \sum_{j \in J} c_{ij} v_j$, we have $\sum_{j \in J\setminus\{k\}} c_{ij} v_j = \lambda v_i - c_{ik} v_k$. Therefore,

$$\begin{aligned} \sum_{j \in J\setminus\{k\}} c_{ij}^k v_j &= \lambda v_i - c_{ik} v_k + \frac{c_{ik}}{\lambda - c_{kk}} (\lambda v_k - c_{kk} v_k) \\ &= \lambda v_i - c_{ik} v_k + \frac{c_{ik}}{\lambda - c_{kk}} v_k (\lambda - c_{kk}) \\ &= \lambda v_i. \qquad \square \end{aligned}$$

References

Borokhovich, K., R. J. Bricker, K. R. Brunarski, and B. J. Simkins (1995). Finance research productivity and influence. *Journal of Finance 50*, 1691–1717.

Bush, W. C., P. W. Hamelman, and R. J. Staaf (1974). A quality index for economic journals. *Review of Economics and Statistics 56*, 123–125.

Dagan, N. (1994). Consistency, decentralization and the Walrasian allocations correspondence. Mimeo, The Hebrew University of Jerusalem.

Dusansky, R. and C. J. Vernon (Winter 1998). Rankings of u.s. economics departments. *Journal of Economic Perspectives 12(1)*, 157–170.

Ellison, G. (2000a). Evolving standards for academic publishing: A *q-r* theory. Mimeo, MIT.

Ellison, G. (2000b). The slowdown of the economics publishing process. Mimeo, MIT.

Hammermesh, D. S., G. E. Johnson, and B. A. Weisbrod (October 1982). Scholarship, citations and salaries: Economics rewards in economics. *Southern Economic Journal 21*, 200–215.

Hart, S. and A. Mas-Colell (1989). Potential, value and consistency. *Econometrica 57*, 589–614.

Jaffe, A. B. and M. Trajtenberg (2002). *Patents, Citations and Innovations.* M.I.T. Press.

Kalaitzidakis, P., T. P. Mamuneas, and T. Stengos (2001). Ranking of academic journals and institutions in economics. Working Paper 2001–10, Dept. of Economics, University of Cyprus.

Laband, D. N. and M. J. Piette (1994a). The relative impacts of economics journals: 1970–1990. *Journal of Economic Literature 32*, 640–666.

Laband, D. N. and M. T. Piette (February 1994b). Favoritism versus search for good papers: Empirical evidence regarding the behavior of journal editors. *Journal of Political Economy 102(1)*, 194–203.

Laband, D. N. and R. D. Tollison (June 2000). Intellectual collaboration. *Journal of Political Economy 108(3)*, 632–662.

Landes, W. M., L. Lessig, and M. E. Solimine (1998). Judicial influence: A citation analysis of federal courts of appeal judges. *Journal of Legal Studies 27*, 271–332.

Landes, W. M. and R. A. Posner (1976). Legal precedent: A theoretical and empirical analysis. *Journal of Law and Economics 19*, 249–307.

Lensberg, T. (1988). Stability and the Nash solution. *Journal of Economic Theory 45*, 330–341.

Liebowitz, S. and J. Palmer (1984). Assessing the relative impacts of economic journals. *Journal of Economic Literature 22*, 77–88.

Minc, H. (1988). *Nonnegative Matrices*. John Wiley & Sons.

Peleg, B. (1985). An axiomatization of the core of cooperative games without side payments. *Journal of Mathematical Economics 14*, 203–214.

Peleg, B. (1986). On the reduced game property and its converse. *International Journal of Game Theory 15*, 187–200.

Peleg, B. and S. Tijs (1996). The consistency principle for games in strategic form. *International Journal of Game Theory 25*, 13–34.

Posner, R. A. (2000). An economic analysis of the use of citations in the law. *American Law and Economics Review 2(2)*, 381–406.

Posner, R. A. (2001). *Public Intellectuals*. Harvard University Press, Cambridge, Mass.

Sauer, R. D. (August 1988). Estimates of the returns to quality and coauthorship in economic academia. *Journal of Political Economy 96(4)*, 855–866.

Segal, U. (June 2000). Let's agree that all dictators are equally bad. *Journal of Political Economy 108(3)*, 569–589.

Stigler, G. J., S. M. Stigler, and C. Friedland (1995). The journals of economics. *Journal of Political Economy 103*, 331–359.

Stigler, S. M. (1994). Citation patterns in the journals of statistics and probability. *Statistical Science 9(1)*, 94–108.

Thomson, W. (1990). The consistency principle. In T. Ichiishi, A. Neyman, and Y. Tauman (Eds.), *Game Theory and Applications*, pp. 187 – 215. Academic Press.

Thomson, W. (1995, March). Consistent allocation rules. Mimeo, University of Rochester.

Tuckman, H. P. and J. Leahey (October 1975). What is an article worth? *Journal of Political Economy 88(5)*, 951–967.

Wilkinson, J. H. (1965). *The Algebraic Eigenvalue Problem*. Oxford University Press.