A geometric postscript to Section 4

Since the preceding analysis is rather technical and algebraic, the reader may have drawn the mistaken impression that the results depend on some complicated pathology. This is not at all the case, and we offer the following geometric treatment in order to explain why the decomposition theorems are really quite intuitive, at least for the case of two commodities. Let $\ell=2$, and let $D=\{(p_1, p_2) \in R^2_+ : p_1+p_2=1, 0<p_1<1\}$.

Proposition 10

If $F: D \to R$ is an arbitrary continuously differentiable function, then there exists a two-consumer, two-commodity exchange economy $\mathcal{E}=\{(\succsim_1, \omega_1), (\succsim_2, \omega_2)\}$ such that: (a) for $p \in \operatorname{int} D$, $F(p)$ is the market excess demand generated by $\mathcal{E}$ for the first commodity, and (b) for $p \in D$, $F(p)$ belongs to the market excess demand generated by $\mathcal{E}$ for the first commodity. Furthermore, both consumers can be chosen from the class of agents with homothetic and convex preferences. (The market excess demand for the second commodity is determined by Walras's Identity.)

Proof

Step 1: The line segment $(z_1, z_2)=z: D \to R^2$, defined by $p=(p_1, p_2) \to ((2p_1+p_2)/(p_1+p_2), p_1/(p_1+p_2))$ connecting (1,0) and (2,1), is the price consumption

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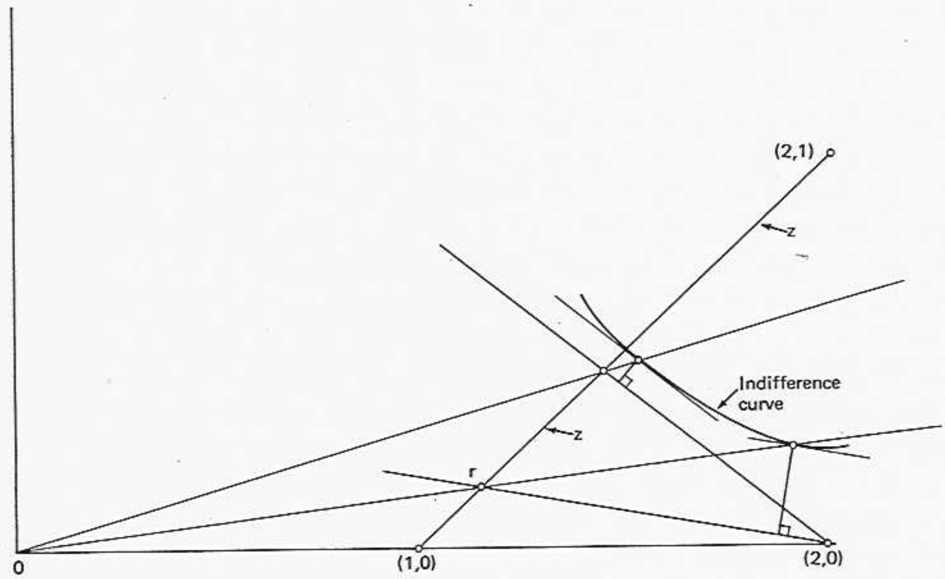

Figure 4.1.

curve of a consumer with a homogeneous utility function and initial endowment (2,0).[6]

Consult Figure 4.1. Preferences which generate the price consumption curve z are obtained by requiring that the marginal rate of substitution along rays such as $\overline{0r}$ (from 0 through r) be the negative of the slope of the line connecting (2,0) and the point where $\overline{0r}$ meets z: if $r=((2p_1+p_2)/(p_1+p_2), p_1/(p_1+p_2))$, then that marginal rate of substitution is p_1/p_2. (Let these preferences be represented by the homogeneous utility function u_1, and observe that they exhibit satiation with respect to the first commodity along the x_1 axis and with respect to the second commodity on and above the ray from the origin with slope one-half.)

Step 2: Sufficiently small smooth perturbations of z are also price consumption curves for consumers with homogeneous utility functions.

The critical feature of the previous construction is that as we move up and to the right along z, then the marginal rate of substitution associated with rays from

[6]For linguistic simplicity the wording will not apply to ∂D. The reader may either verify directly that our construction has the required property on ∂D, or note that this property follows from the upper hemicontinuity of excess demand functions which are generated by utility maximization.

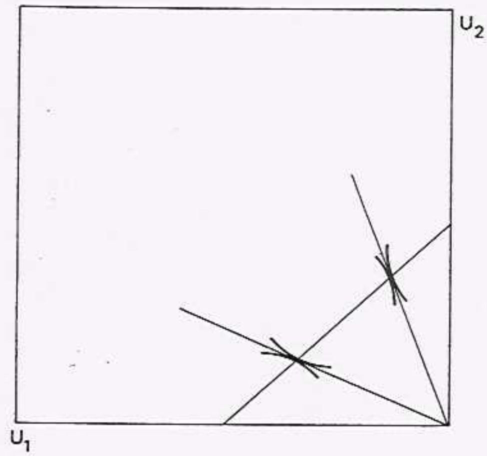

Figure 4.2.

the origin (parameterized by their height at $x_1=1$) is increasing with derivative bounded away from zero. This feature is preserved for sufficiently small smooth perturbations of z. (A smooth perturbation $z+\varepsilon G$ of z is defined by $(p_1, p_2)= ((2p_1+p_2)/(p_1+p_2)+\varepsilon G, p_1/(p_1+p_2)-(p_1/p_2)\varepsilon G)$, where G: $S^1 \to R$ is continuously differentiable; it is small if ε is small.

Step 3: By adding a (symmetric) consumer with initial endowment (0,2) and preferences represented by $u_2(x_1, x_2)=u_1(x_2, x_1)$, we obtain a two-consumer, two-commodity (Edgeworth Box) economy in which every price is a Walras equilibrium price. (Consult Figure 4.2.)[7]

Step 4: Let F be given which satisfies the hypothesis of the proposition (see Figure 4.3). If the second individual's preferences are fixed at u_2 then the necessary choice of the other consumer [Consumer 1 with endowment (2,0)] for each p is defined by the condition that excess demand for the first commodity be $F(p)$. [For example, in Figure 4.3 the point s represents the choice by Consumer 1 necessary to make the excess demand for the first commodity at $(\frac{1}{2}, \frac{1}{2})$ equal to $F(\frac{1}{2}, \frac{1}{2})$. If F is replaced by $F/2$, the first consumer's choice must be $(s+t)/2$.] The fact that F is continuously differentiable means that the price consumption curve needed for Consumer 1 is a smooth perturbation of z. For ε sufficiently small, Step 2 tells us that $z+\varepsilon F$ is the price consumption curve for a consumer

[7]Some time ago A. Mas-Colell showed us a similar diagram. This, and a perturbation argument used by R. Mantel, are the basis for the current exposition.

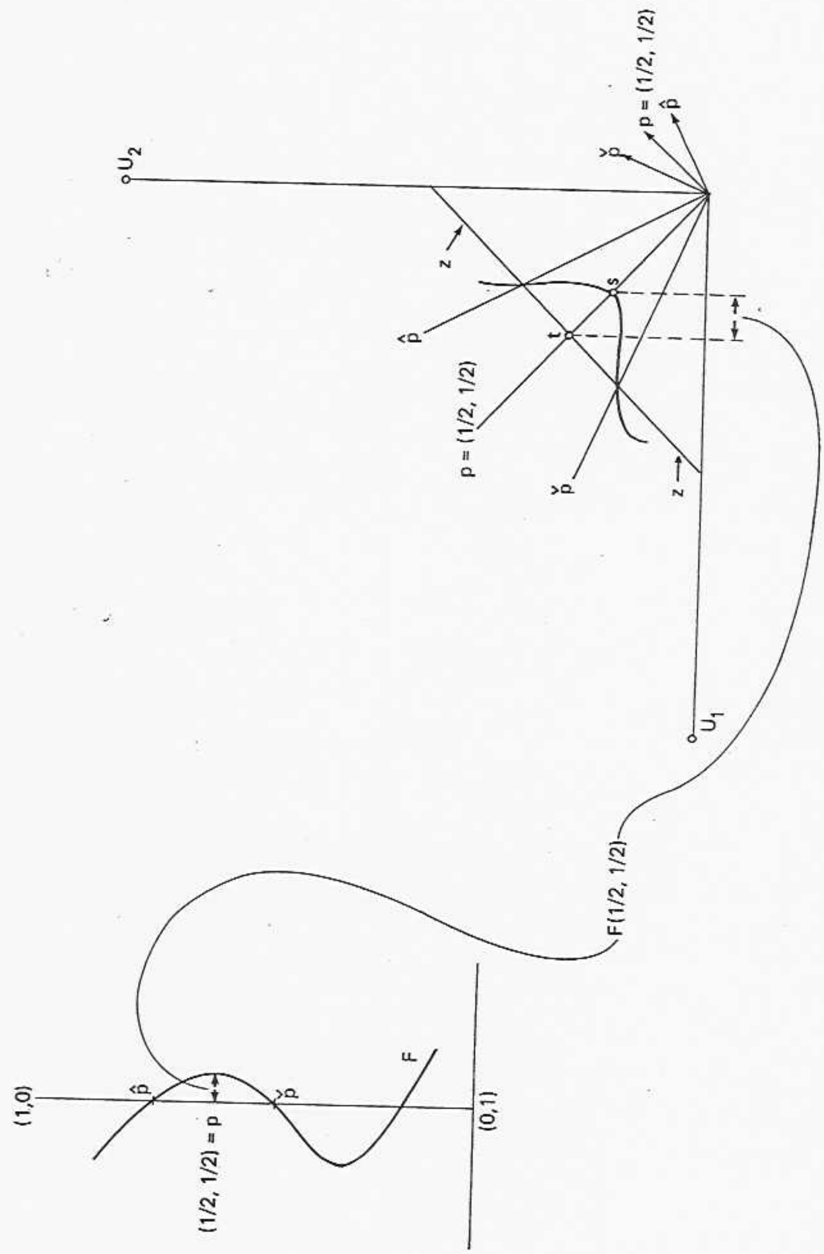

Figure 4.3.