General Equilibrium

Francis Ysidro Edgeworth, 1845-1926

Marie-Ésprit Léon Walras, 1834-1910

Exchange

- Two consumers, A and B.
- Their endowments of goods 1 and 2 are $\omega^A = (\omega_1^A, \omega_2^A)$ and $\omega^B = (\omega_1^B, \omega_2^B)$.
- E.g. $\omega^A = (6,4)$ and $\omega^B = (2,2)$.
- The total quantities available are $\omega_1^A + \omega_1^B = 6 + 2 = 8$ units of good 1 and $\omega_2^A + \omega_2^B = 4 + 2 = 6$ units of good 2.

Exchange

- Edgeworth and Bowley devised a diagram, called an **Edgeworth box**, to show all possible allocations of the available quantities of goods 1 and 2 between the two consumers.

Starting an Edgeworth Box

Starting an Edgeworth Box

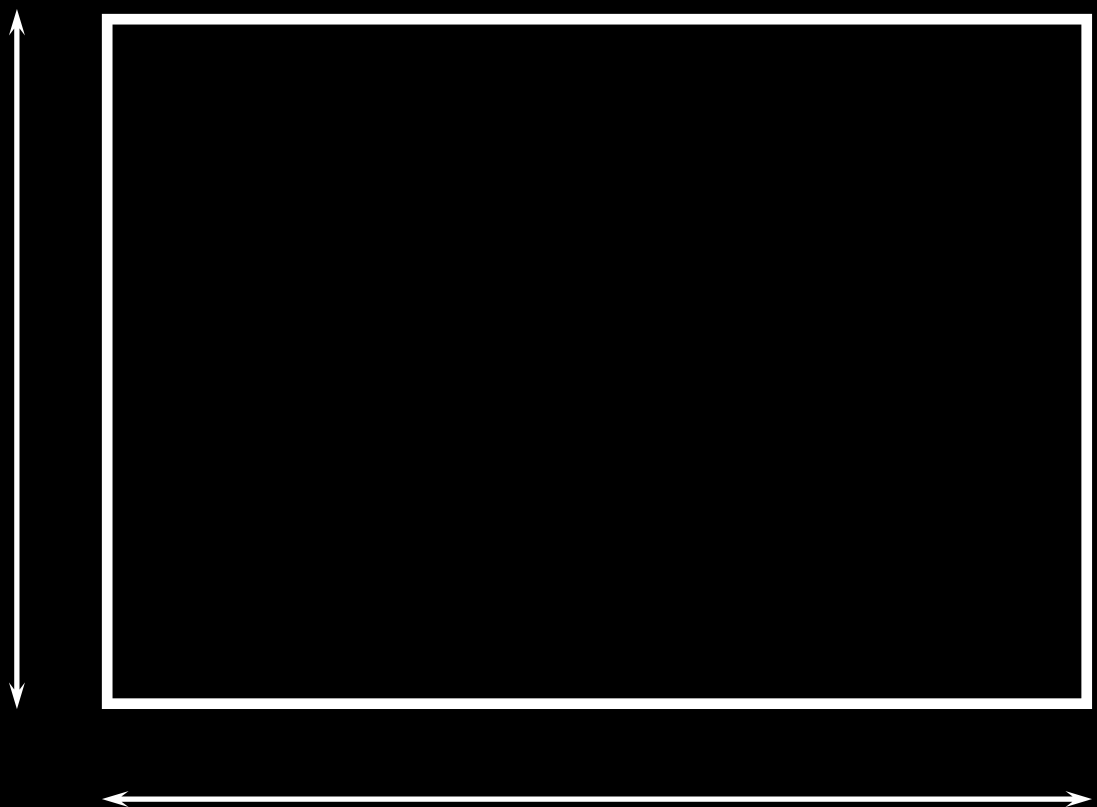
Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

Starting an Edgeworth Box

Height = $\omega_2^A + \omega_2^B$

$= 4 + 2$

$= 6$



Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

Starting an Edgeworth Box

Height = $\omega_2^A + \omega_2^B$

$= 4 + 2$

$= 6$

The dimensions of the box are the quantities available of the goods.

Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

Feasible Allocations

- What allocations of the 8 units of good 1 and the 6 units of good 2 are feasible?
- How can all of the feasible allocations be depicted by the Edgeworth box diagram?

Feasible Allocations

- What allocations of the 8 units of good 1 and the 6 units of good 2 are feasible?
- How can all of the feasible allocations be depicted by the Edgeworth box diagram?
- One feasible allocation is the before-trade allocation; i.e. the **endowment allocation**.

The Endowment Allocation

Height = $\omega_2^A + \omega_2^B$ $= 4 + 2$ $= 6$

The endowment allocation is

$\omega^A = (6,4)$

and

$\omega^B = (2,2).$

Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

The Endowment Allocation

Height = $\omega_2^A + \omega_2^B$

$= 4 + 2$

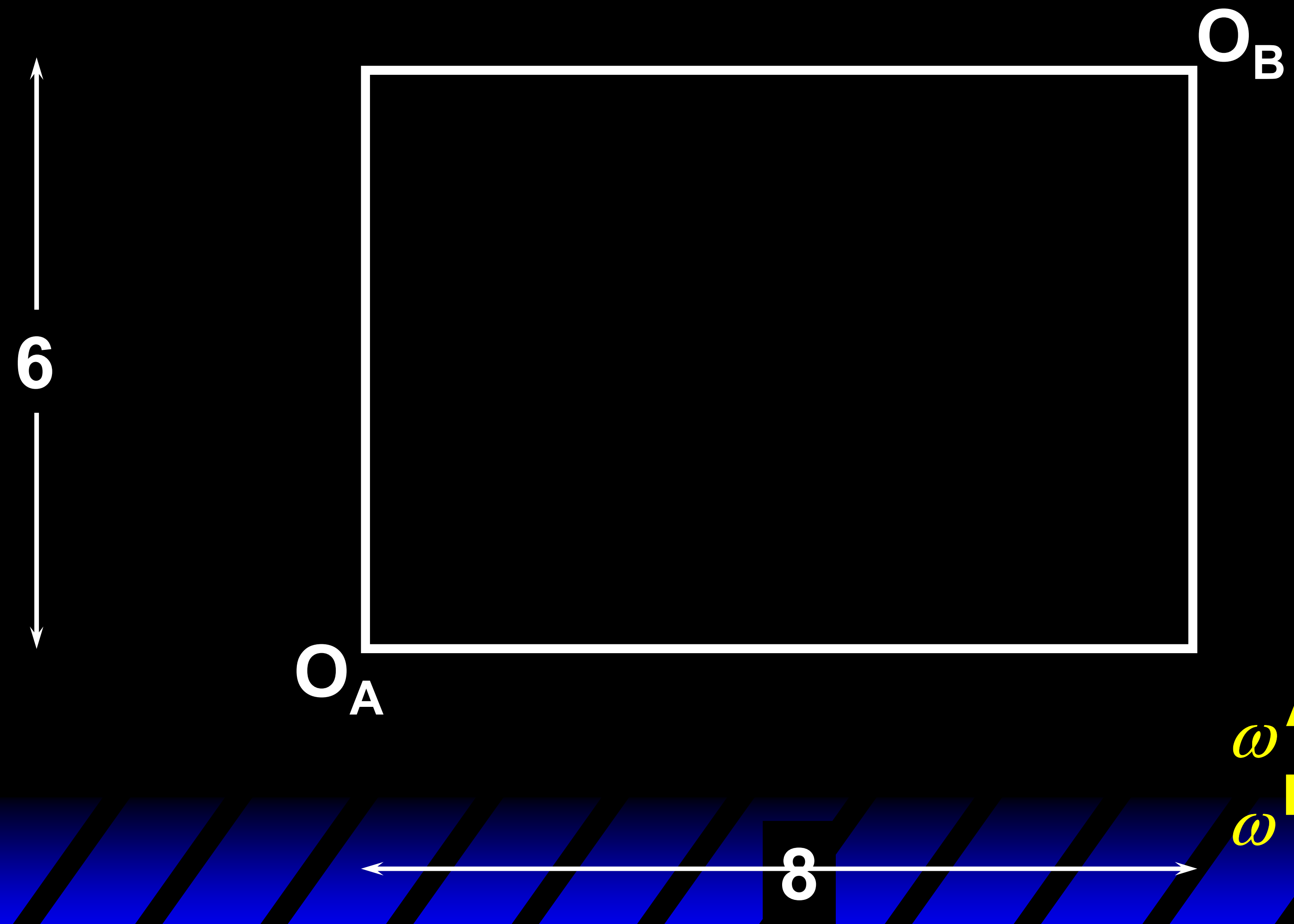
$= 6$

$\omega^A = (6,4)$

$\omega^B = (2,2)$

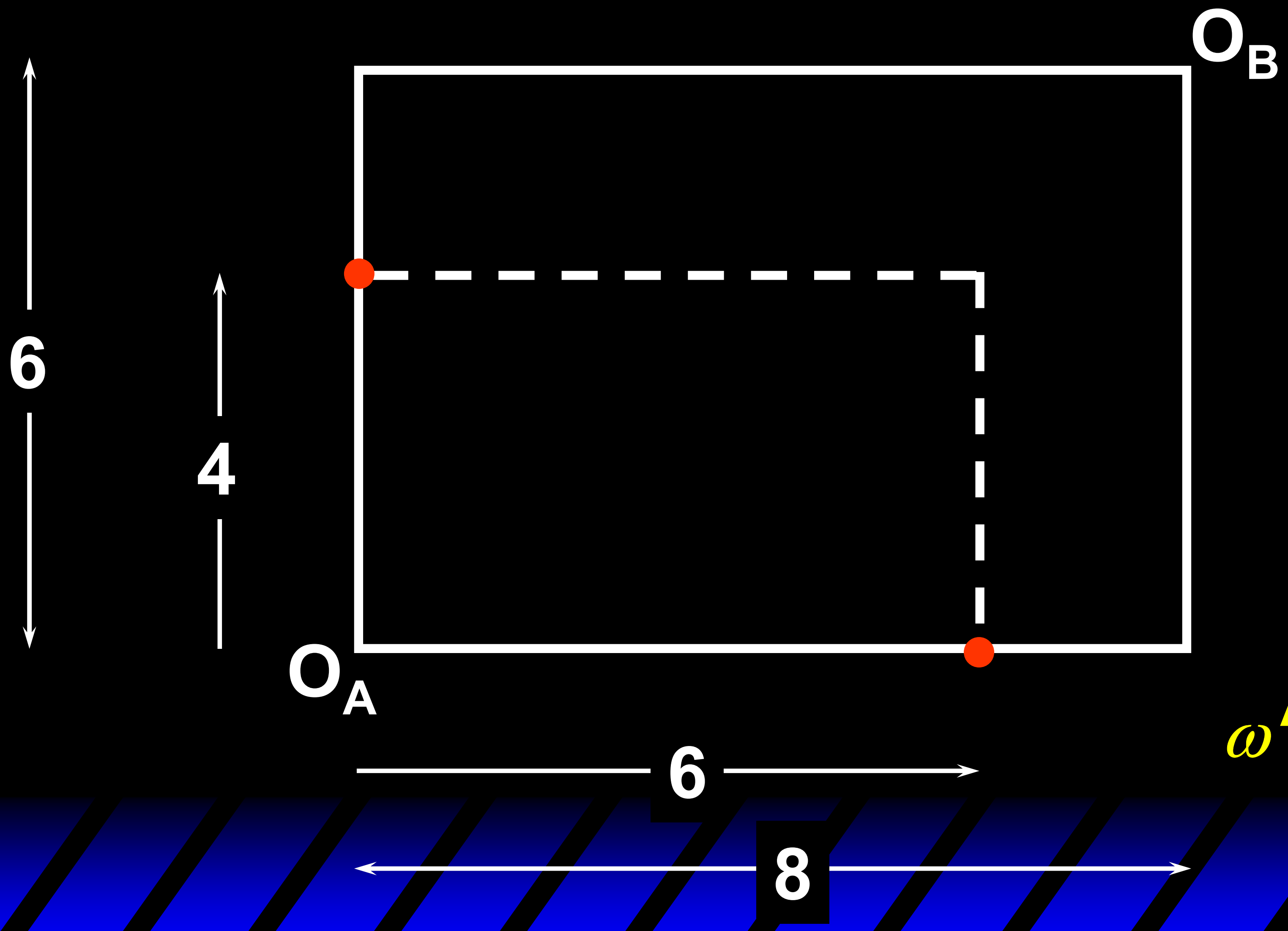
Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

The Endowment Allocation



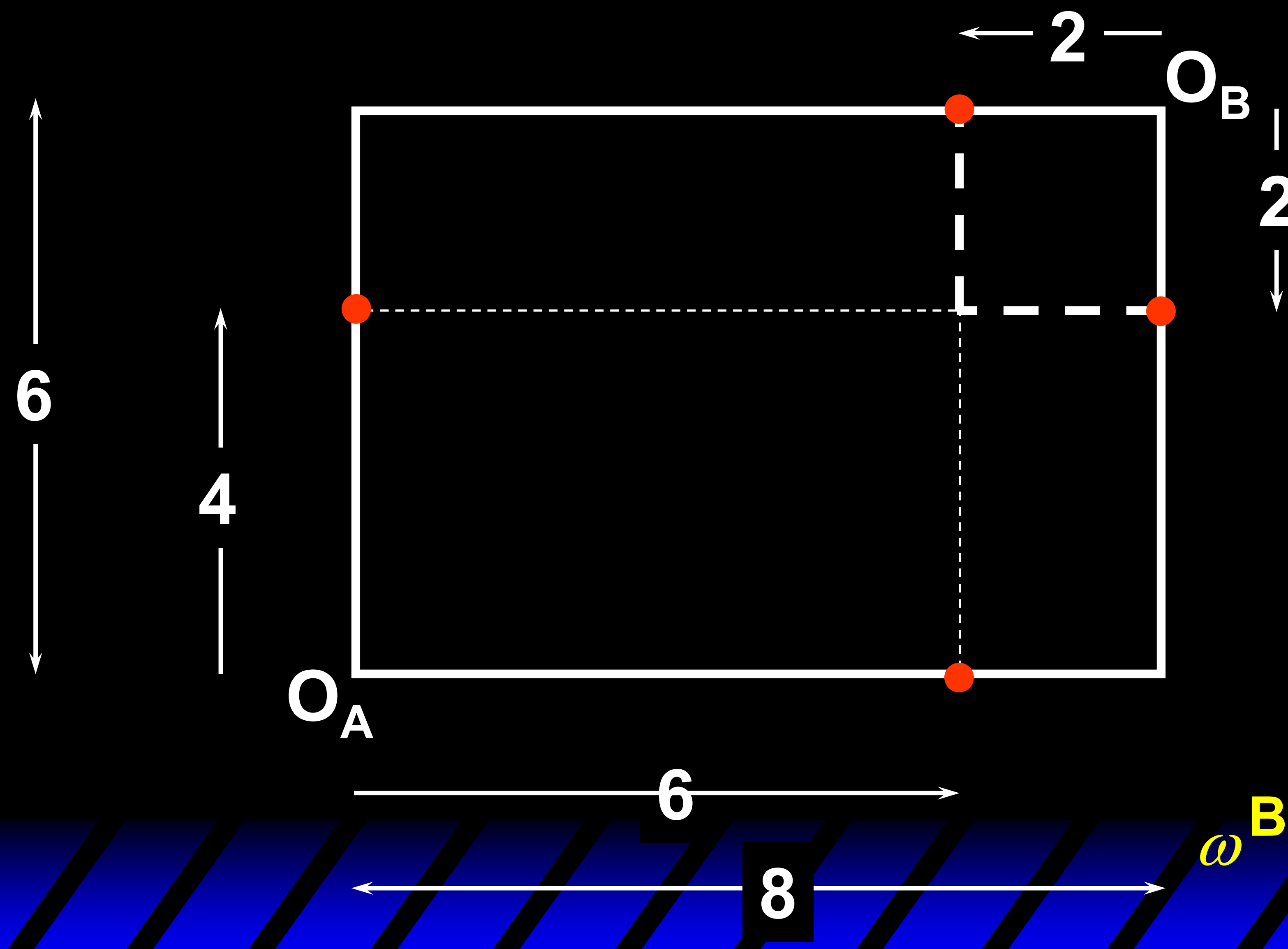
$$\omega^{A} = (6,4)$$
$$\omega^{B} = (2,2)$$

The Endowment Allocation



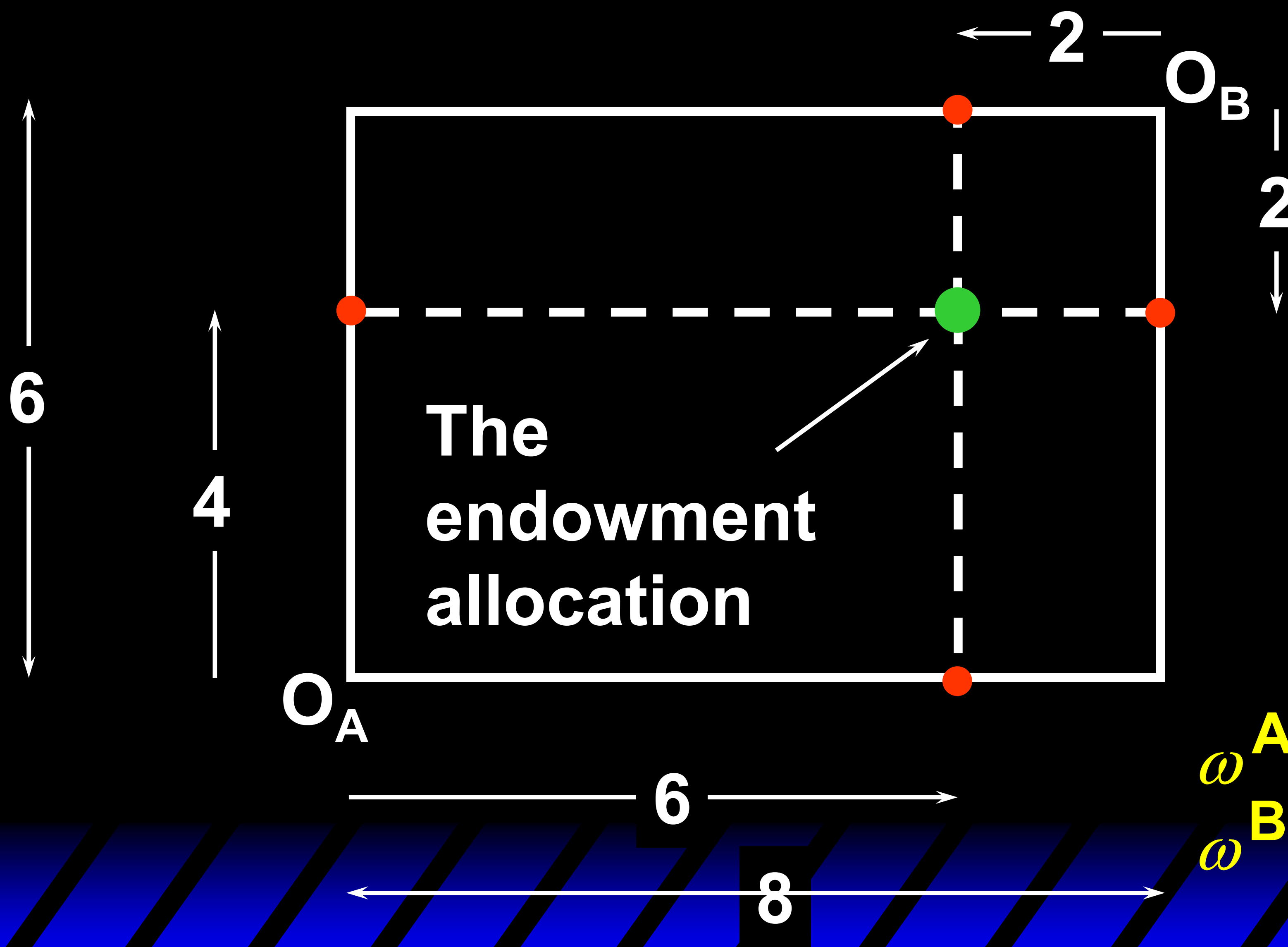
$\omega^A = (6,4)$

The Endowment Allocation



$\omega^B = (2,2)$

The Endowment Allocation

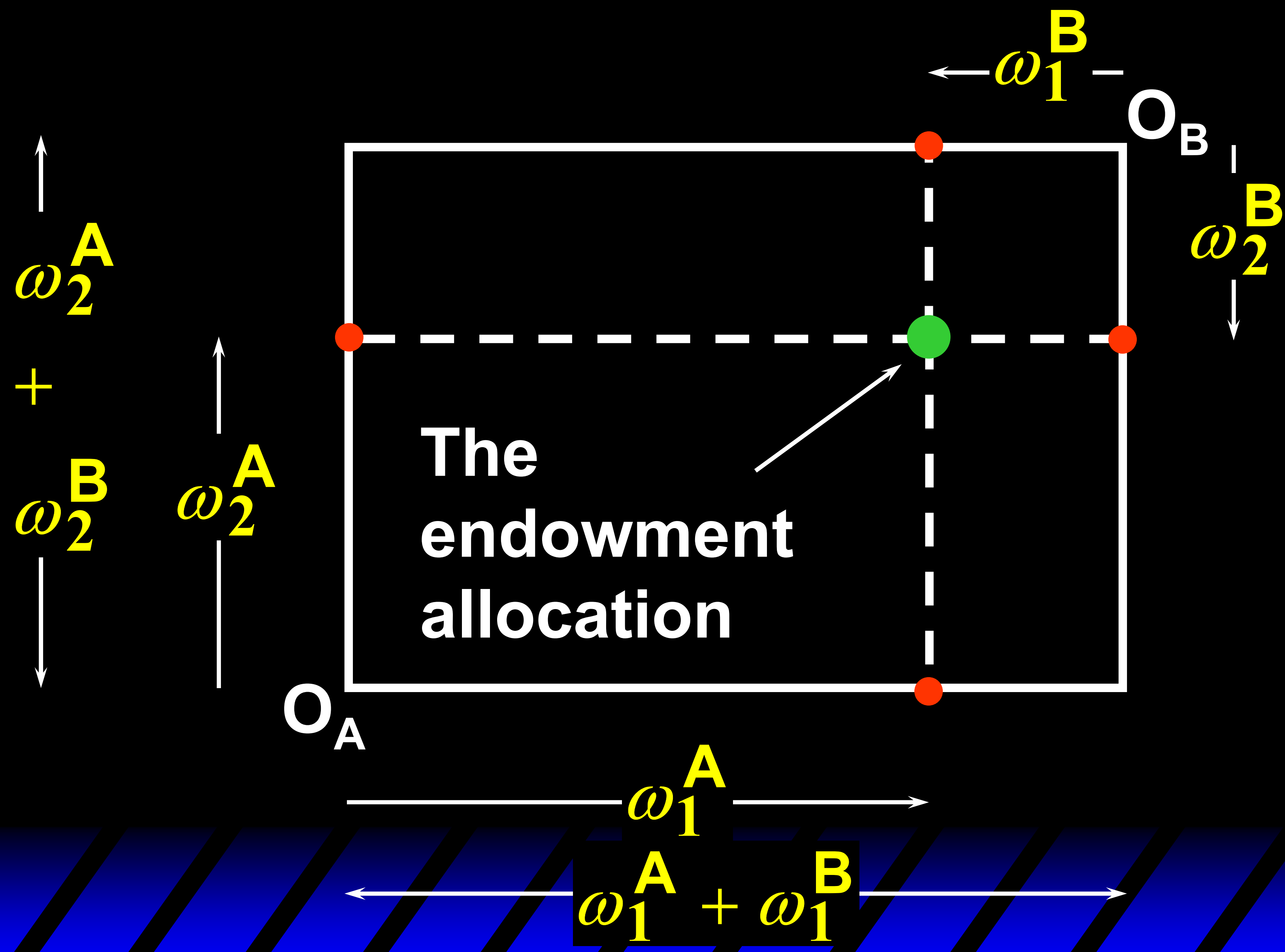


$$\omega^A = (6,4)$$
$$\omega^B = (2,2)$$

The Endowment Allocation

More generally, …

The Endowment Allocation



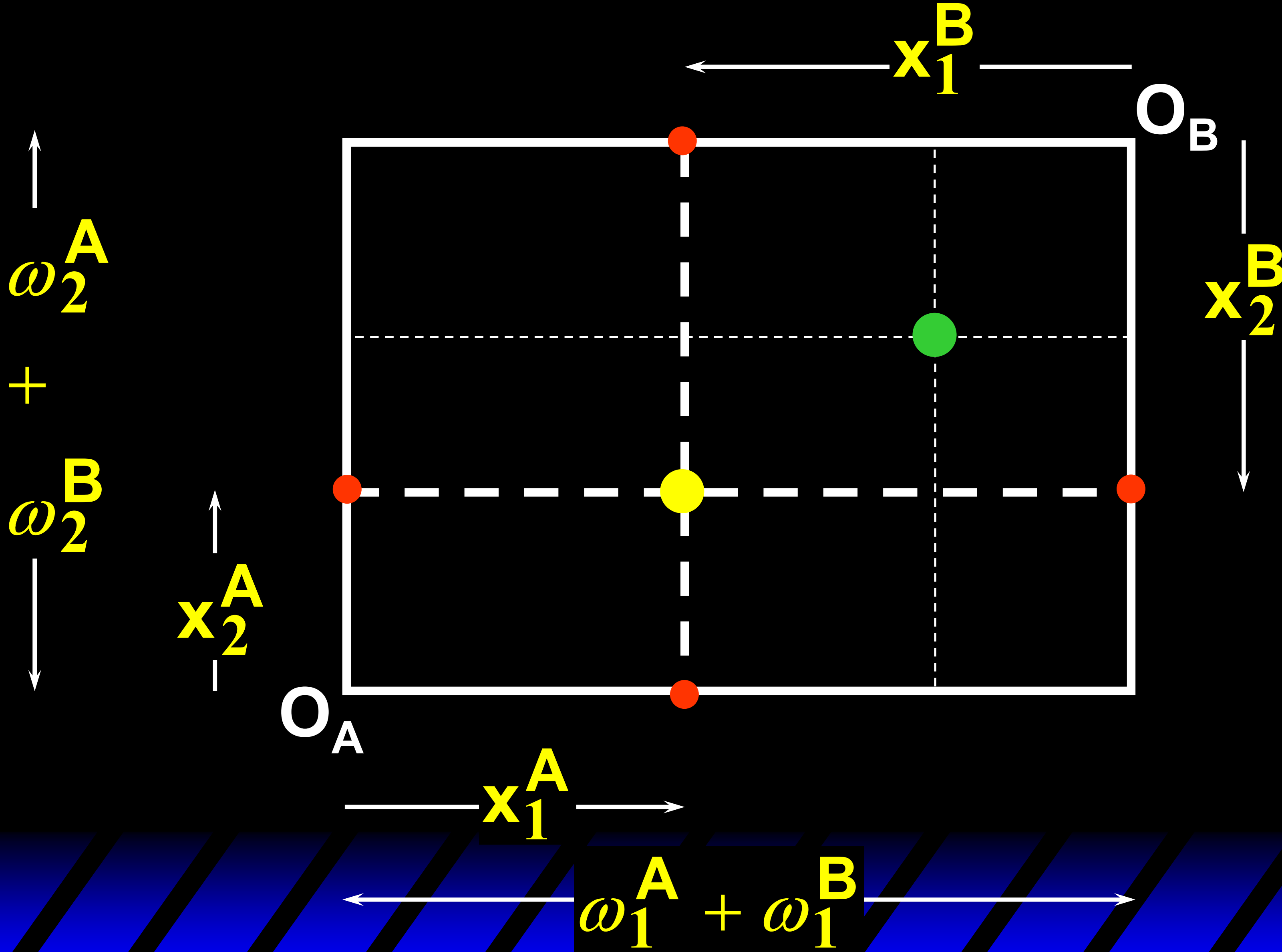
Other Feasible Allocations

- (x_1^A, x_2^A) denotes an allocation to consumer A.
- (x_1^B, x_2^B) denotes an allocation to consumer B.
- An allocation is **feasible** if and only if

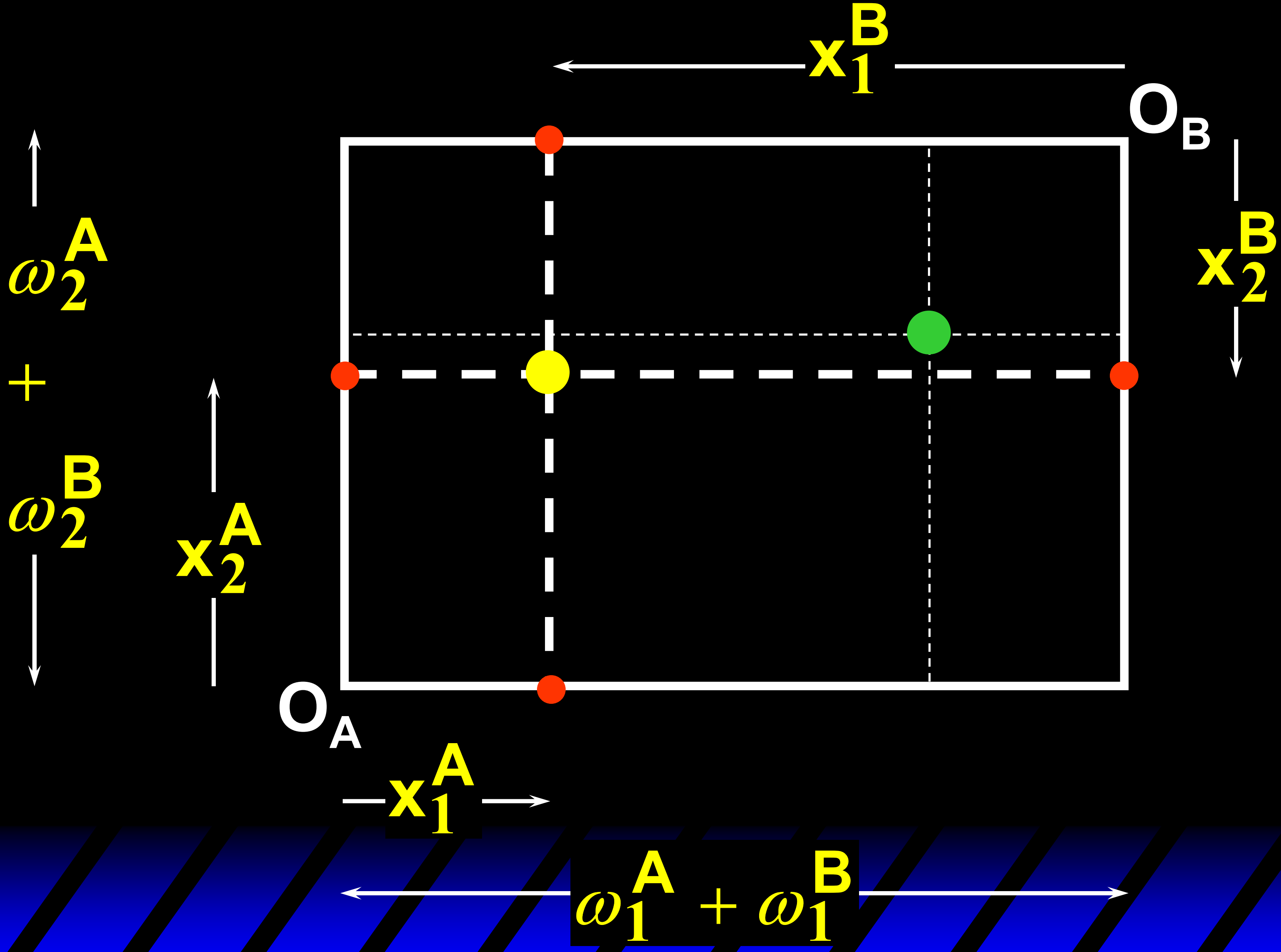
$$x_1^A + x_1^B \leq \omega_1^A + \omega_1^B$$

and $x_2^A + x_2^B \leq \omega_2^A + \omega_2^B.$

Feasible Reallocations



Feasible Reallocations



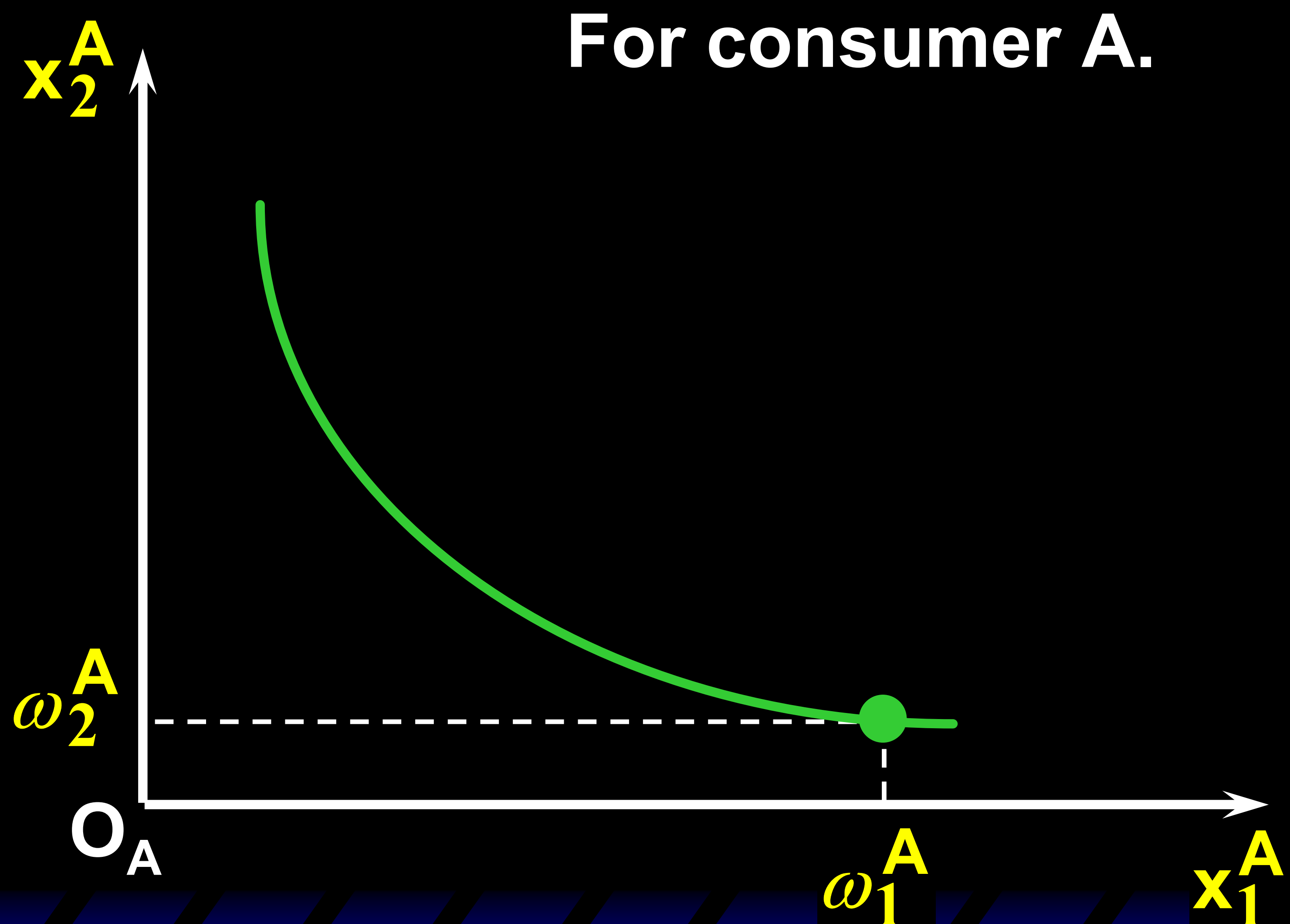
Feasible Reallocations

- All points in the box, including the boundary, represent feasible allocations of the combined endowments.

Feasible Reallocations

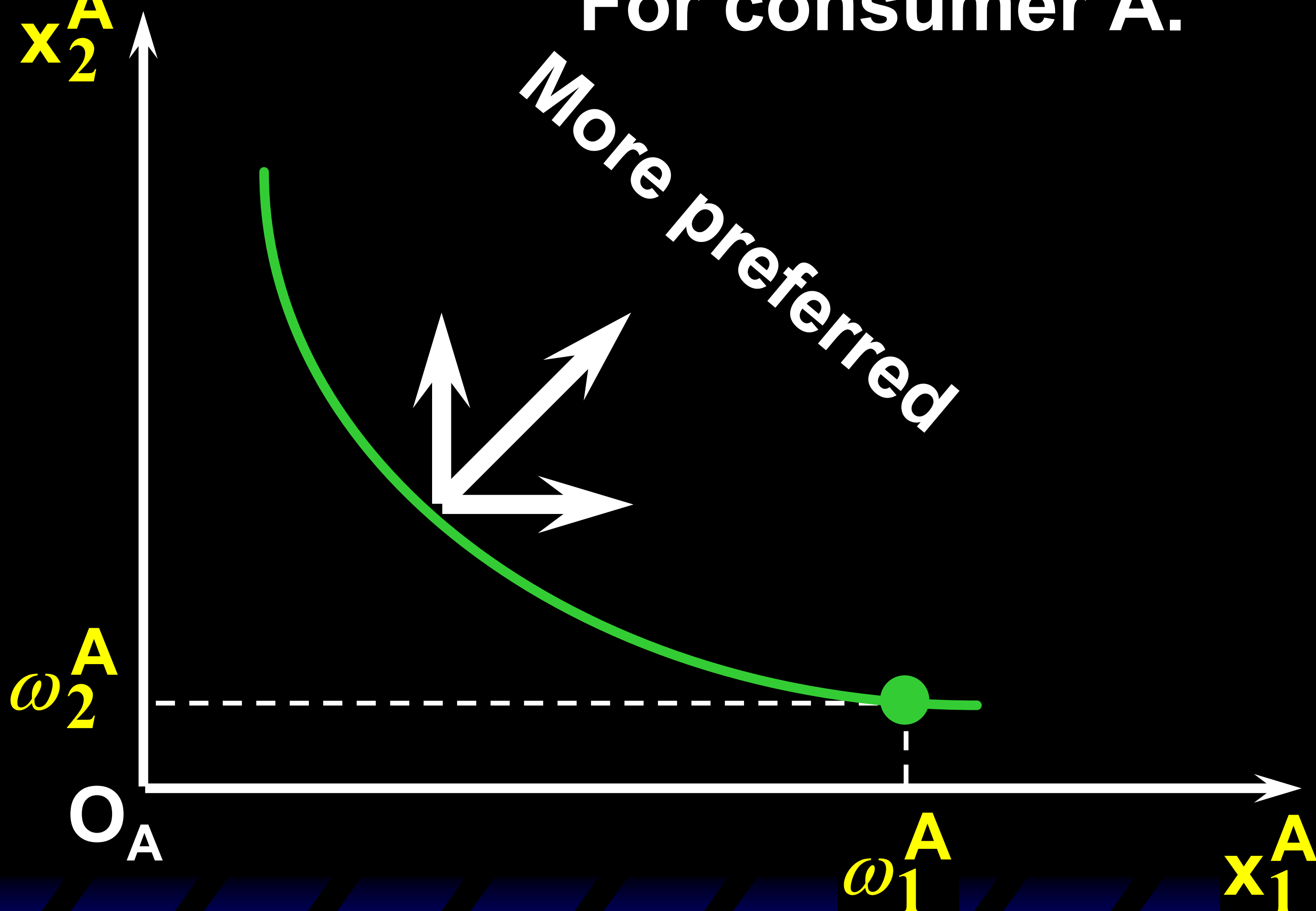
- All points in the box, including the boundary, represent feasible allocations of the combined endowments.
- Which allocations will be blocked by one or both consumers?
- Which allocations make both consumers better off?

Adding Preferences to the Box



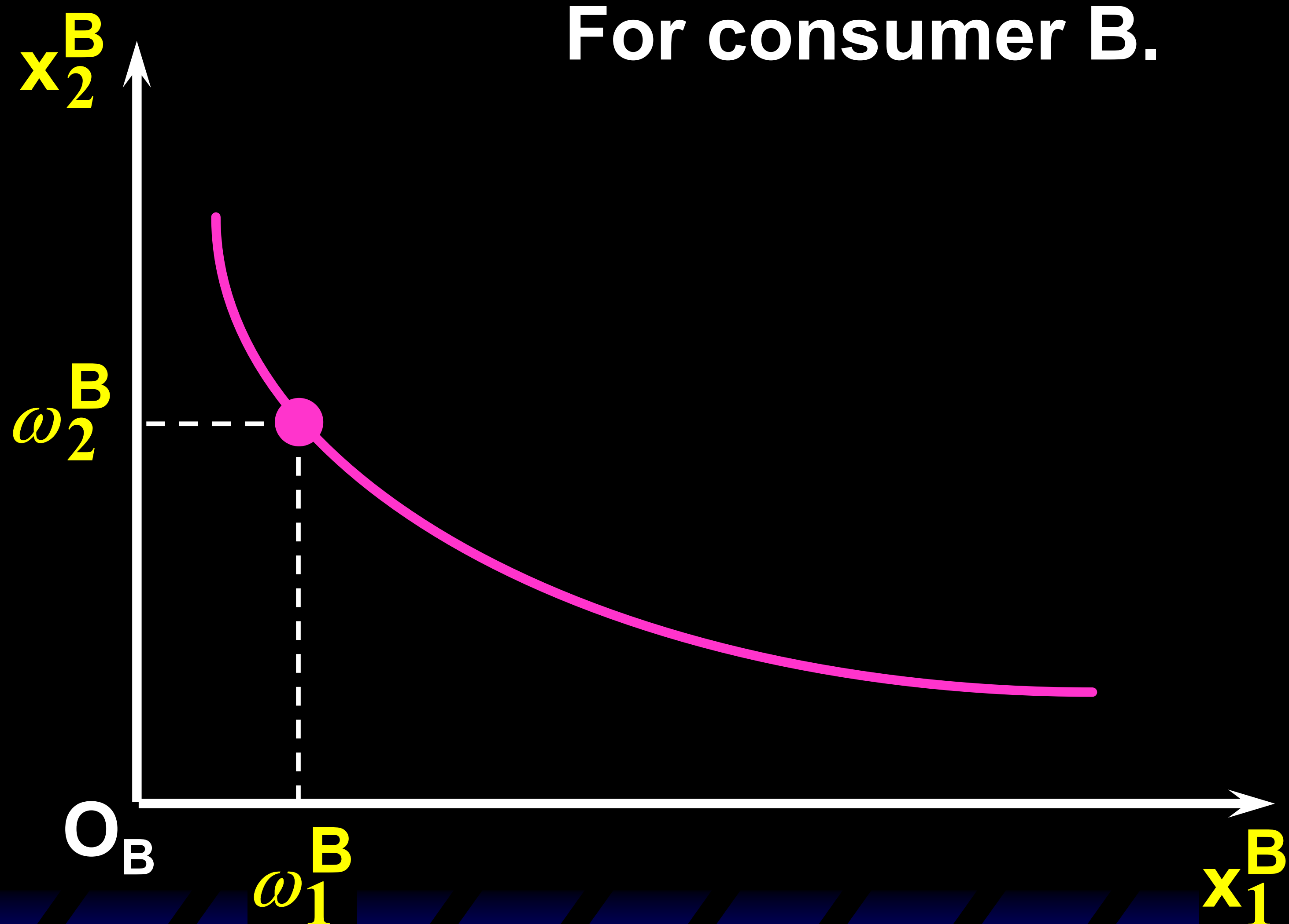
Adding Preferences to the Box

For consumer A.

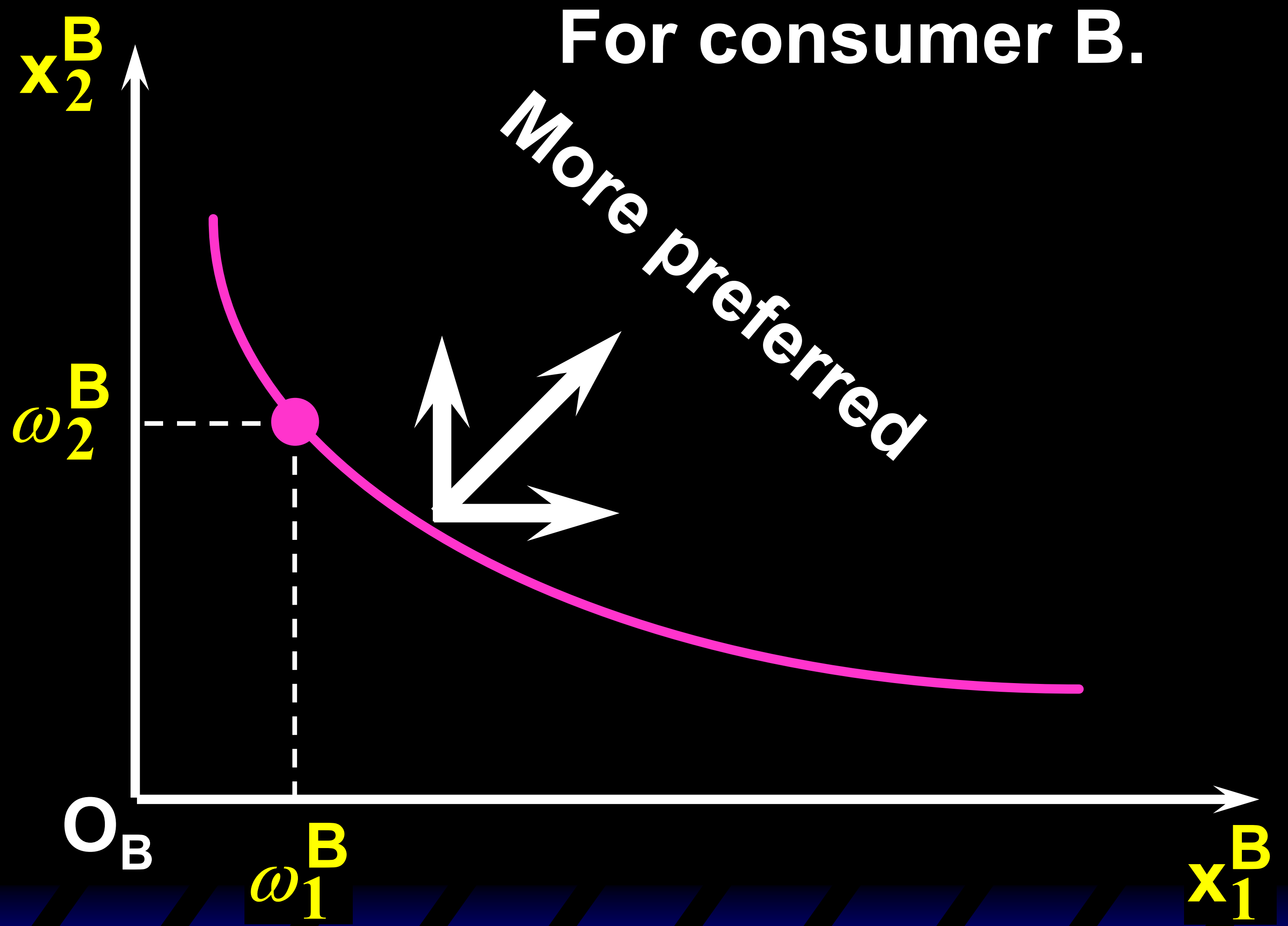


Adding Preferences to the Box

For consumer B.



Adding Preferences to the Box



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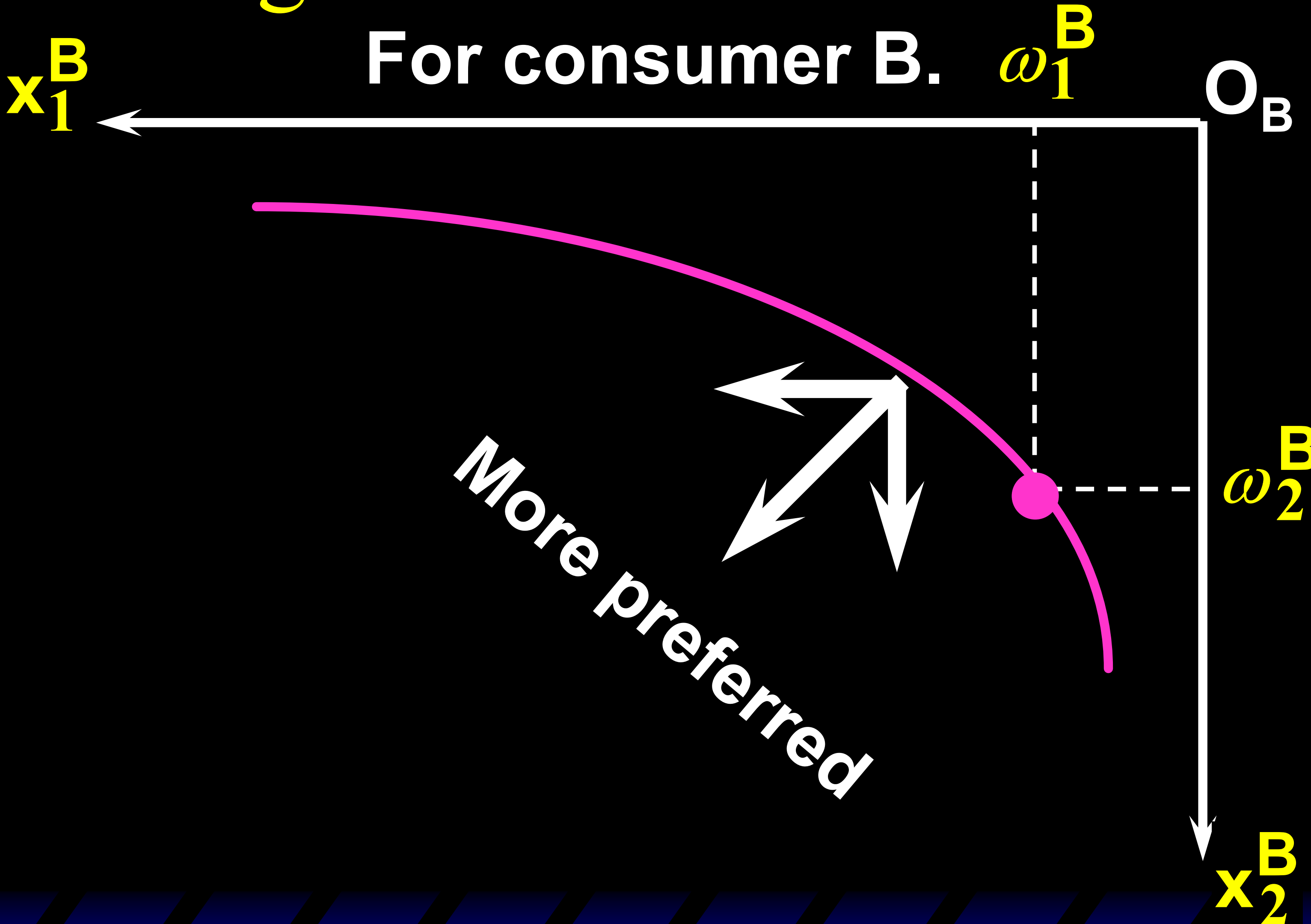

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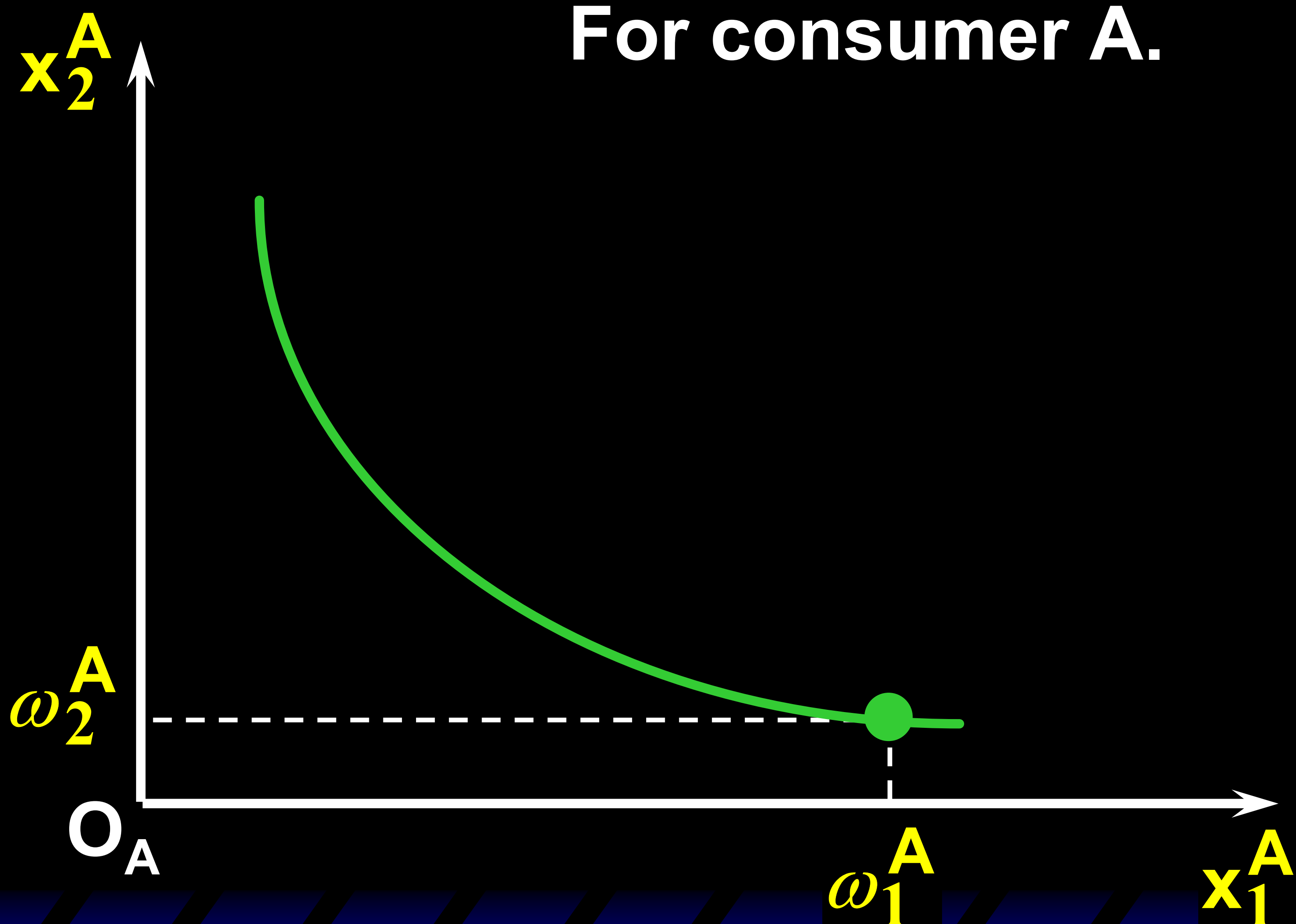

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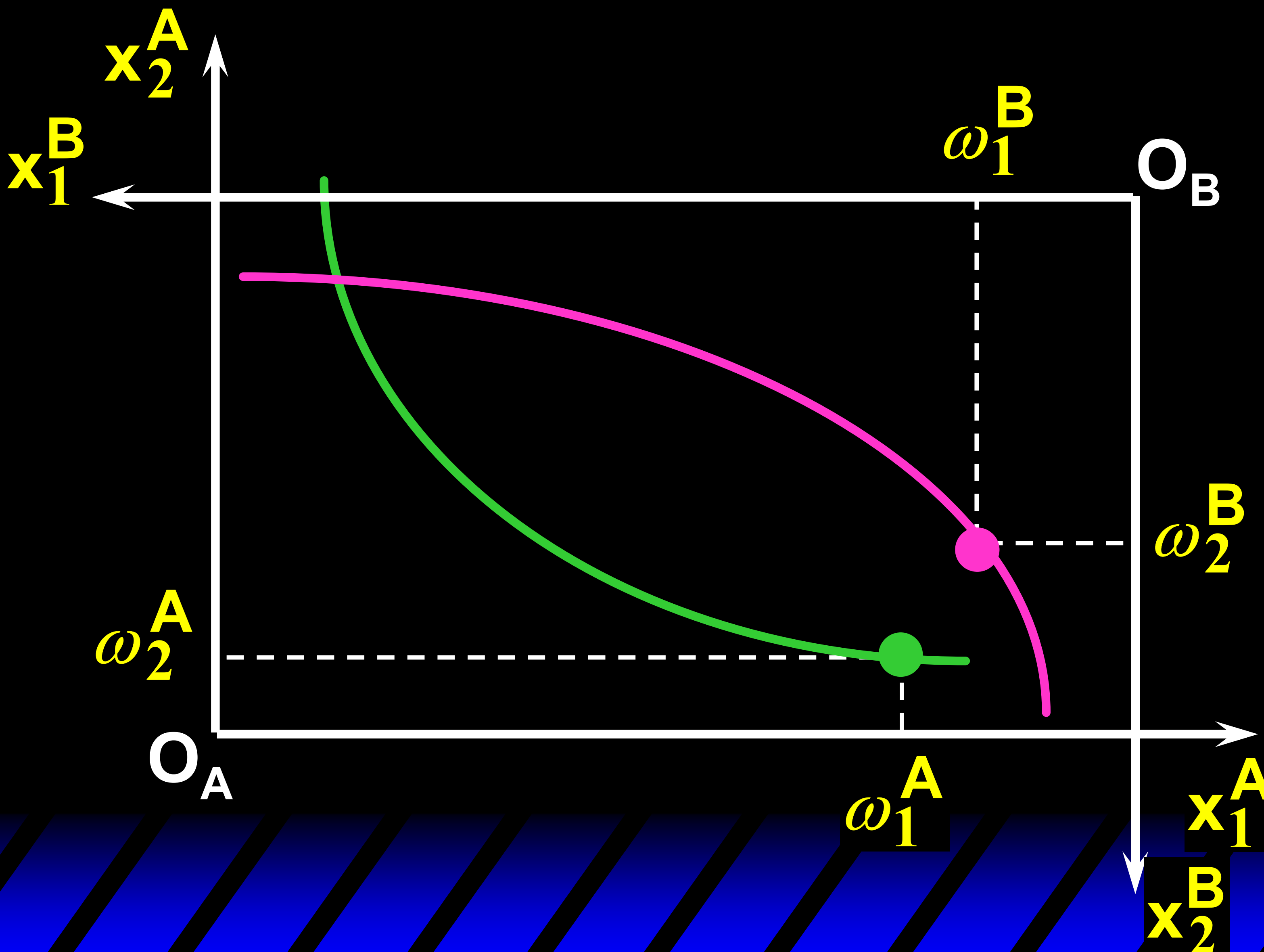
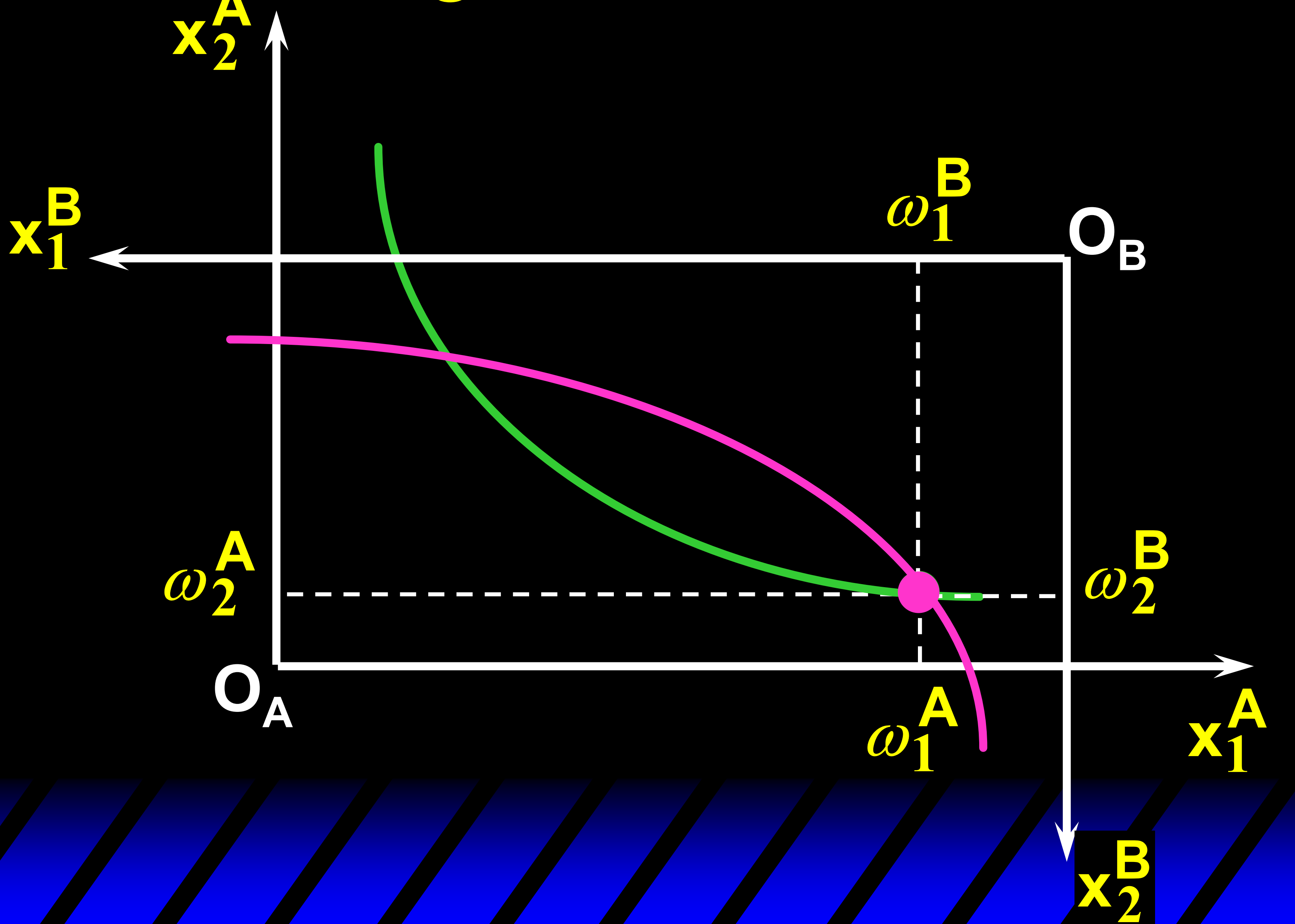


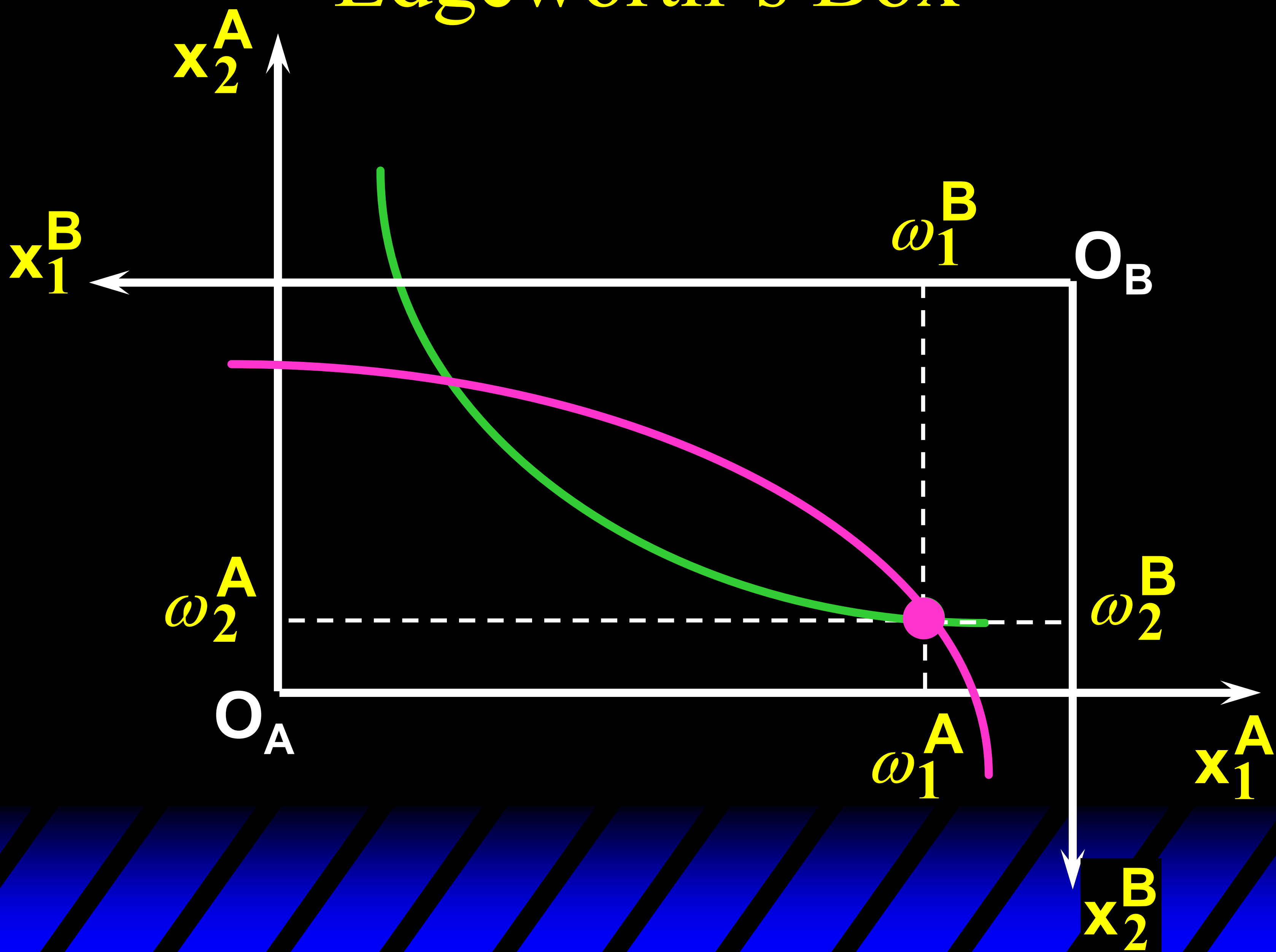
Edgeworth's Box



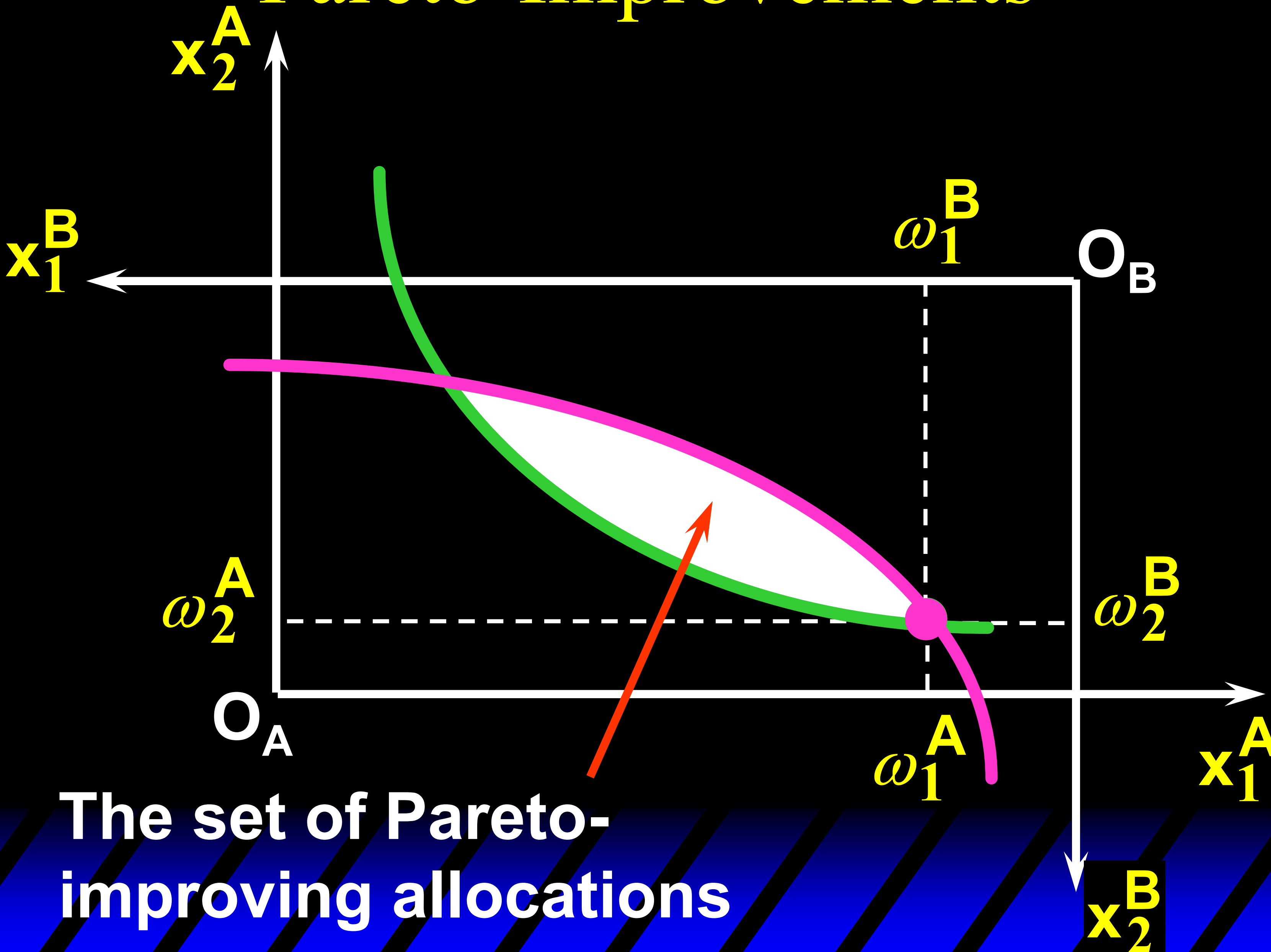
Pareto-Improvement

- An allocation of the endowment that improves the welfare of a consumer without reducing the welfare of another is a **Pareto-improving allocation**.
- Where are the Pareto-improving allocations?

Edgeworth's Box



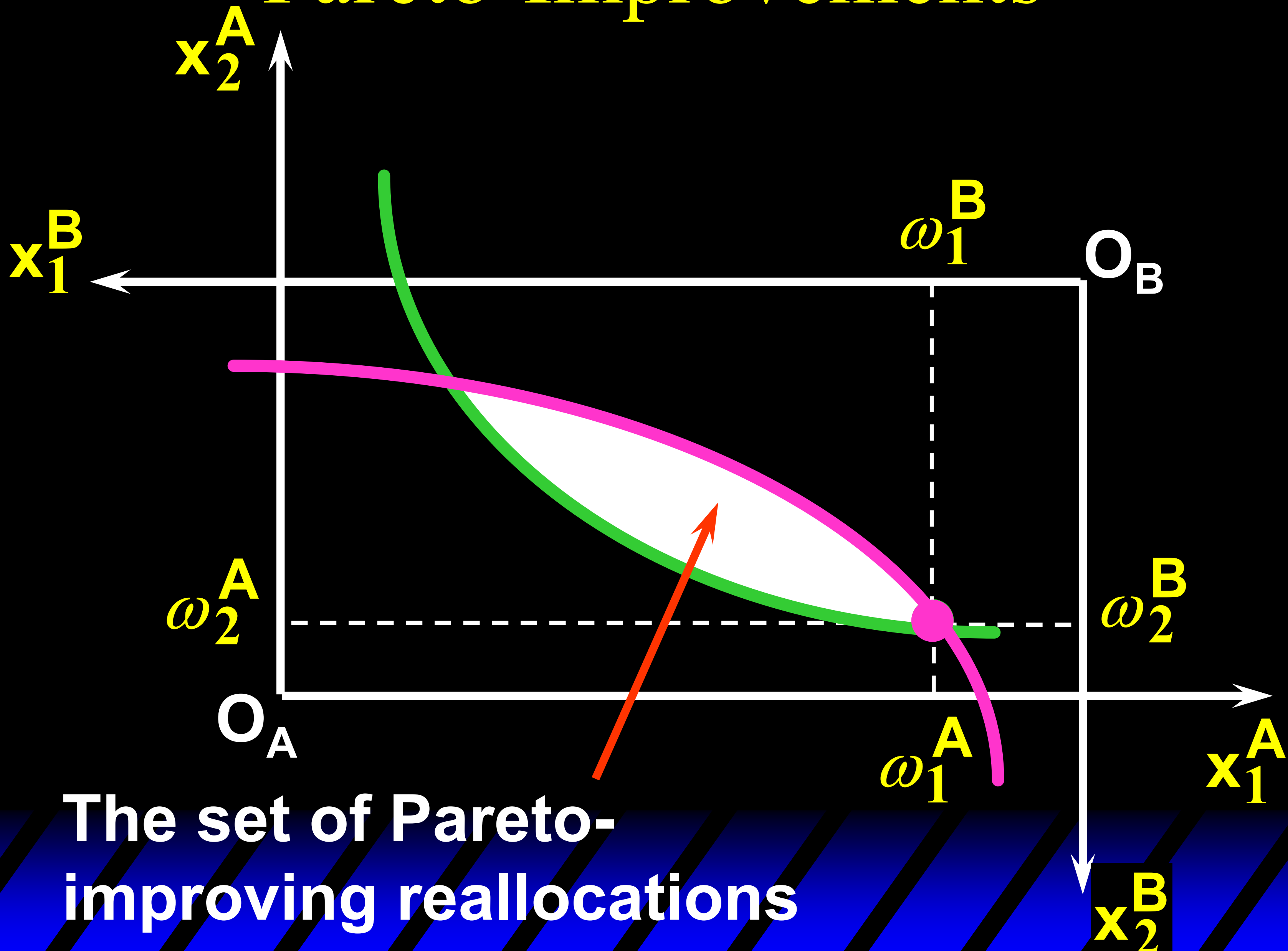
Pareto-Improvements



Pareto-Improvements

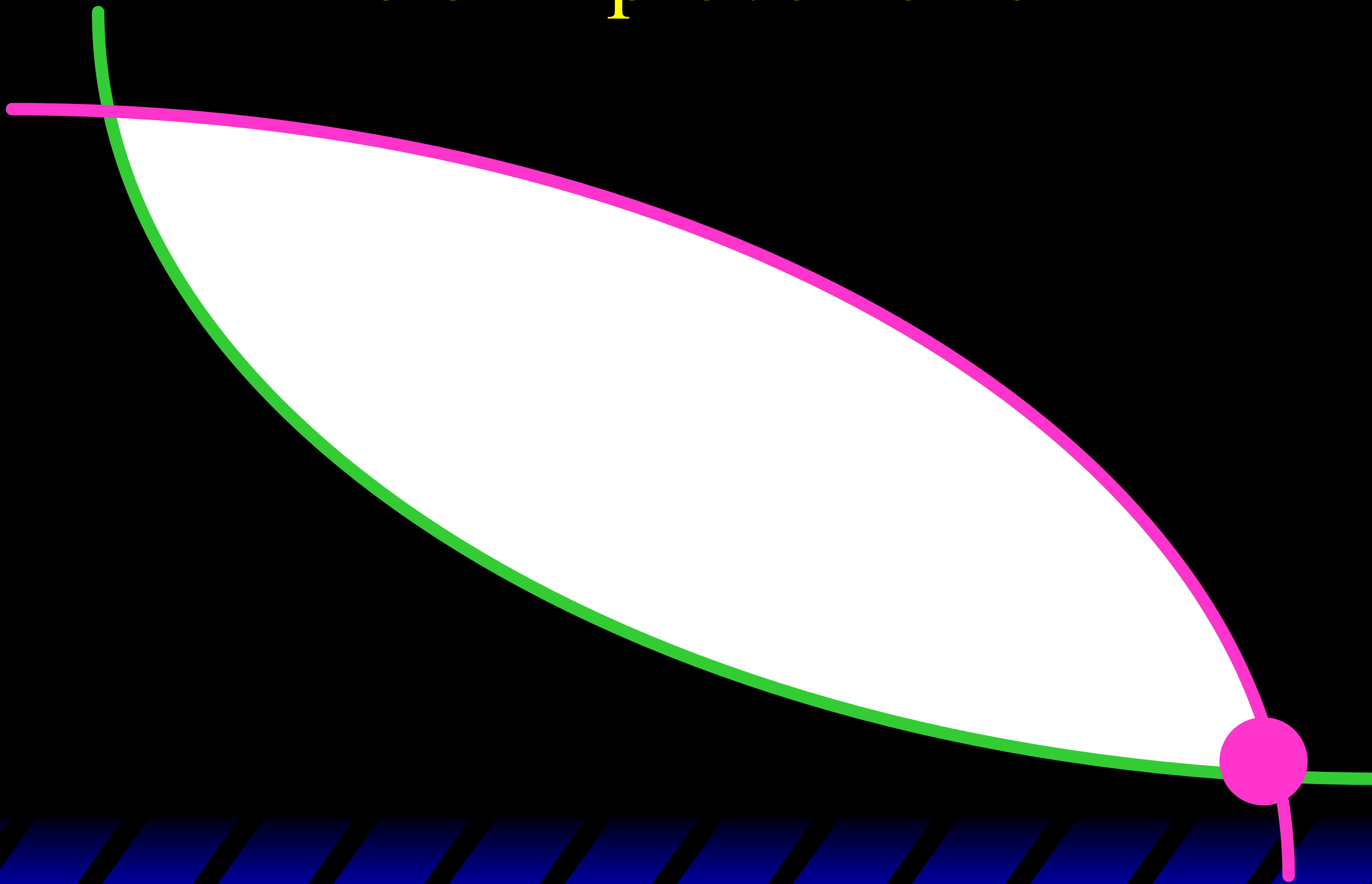
- Since each consumer can refuse to trade, the only possible outcomes from exchange are Pareto-improving allocations.
- But which particular Pareto-improving allocation will be the outcome of trade?

Pareto-Improvements

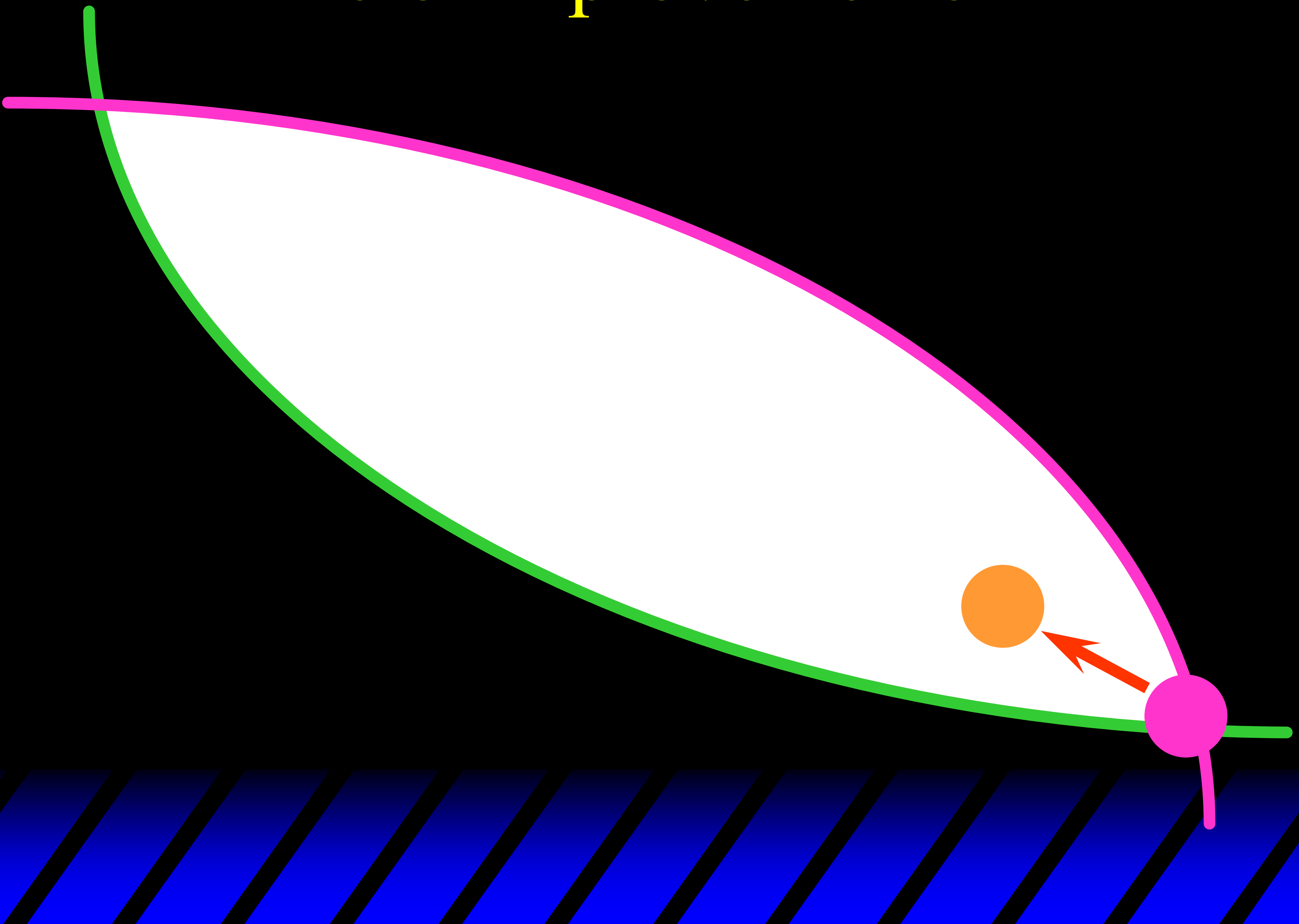


The set of Pareto-improving reallocations

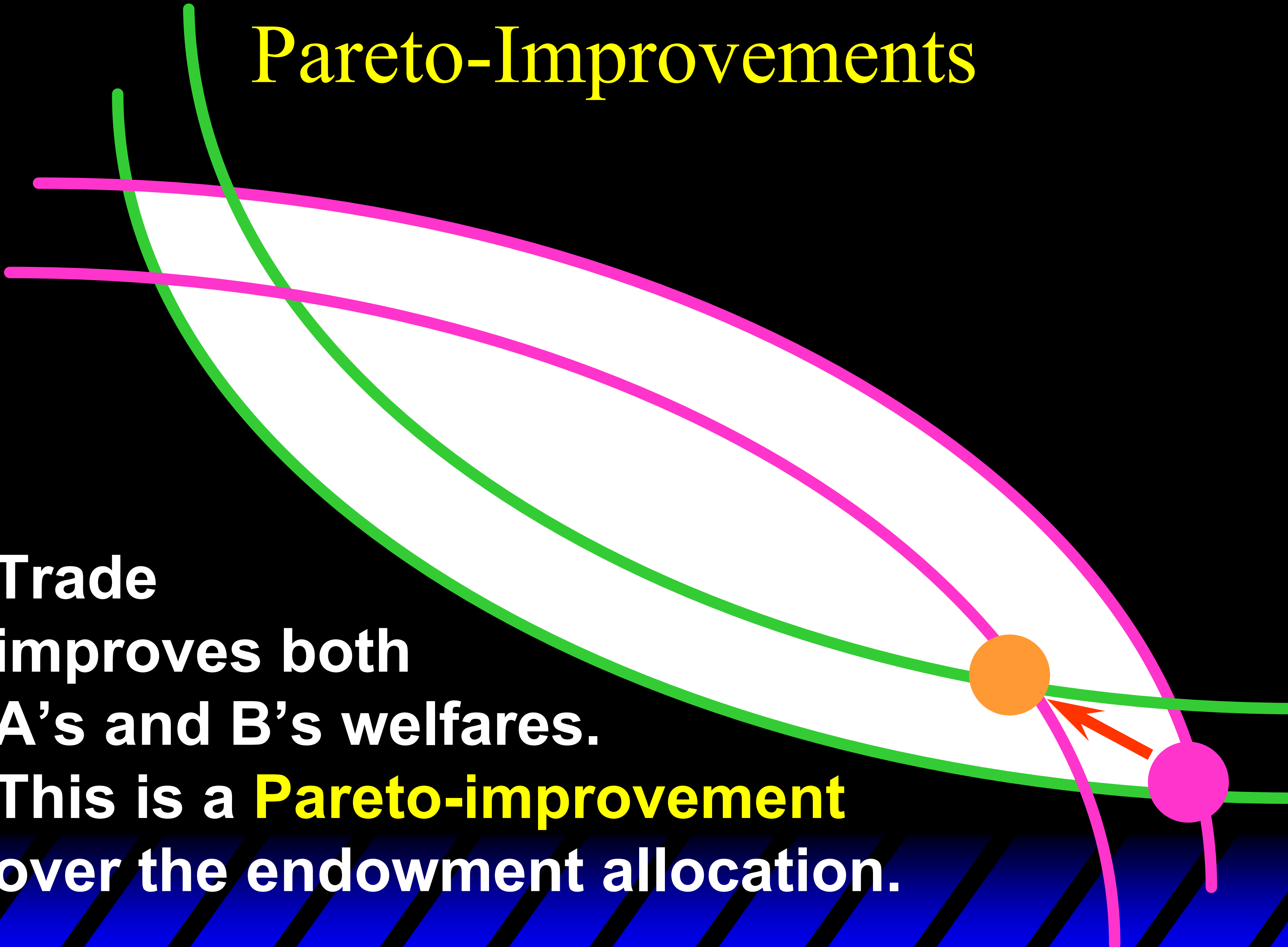
Pareto-Improvements



Pareto-Improvements

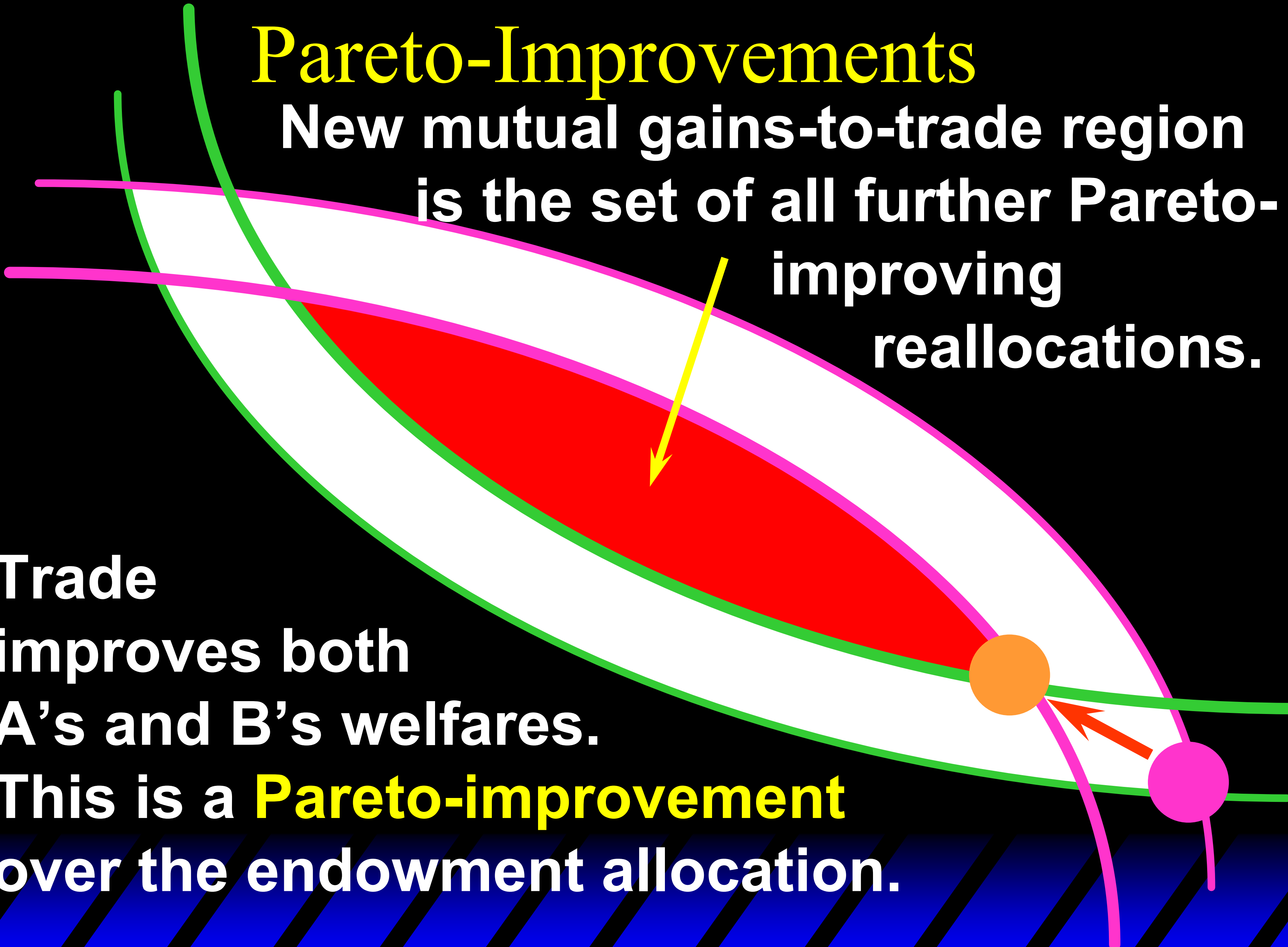


Pareto-Improvements



Trade improves both A’s and B’s welfares.
This is a Pareto-improvement over the endowment allocation.

Pareto-Improvements

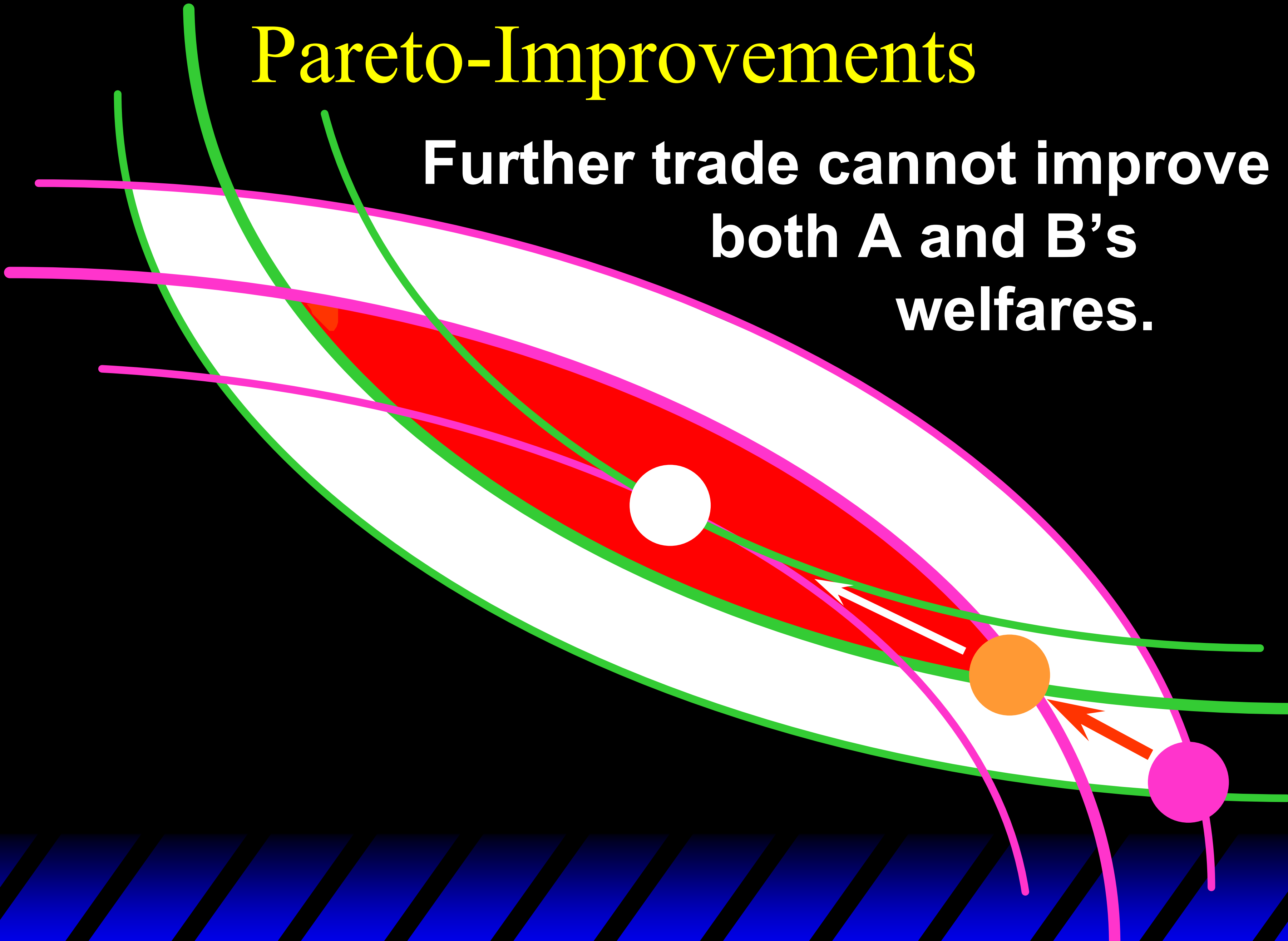


New mutual gains-to-trade region is the set of all further Pareto-improving reallocations.

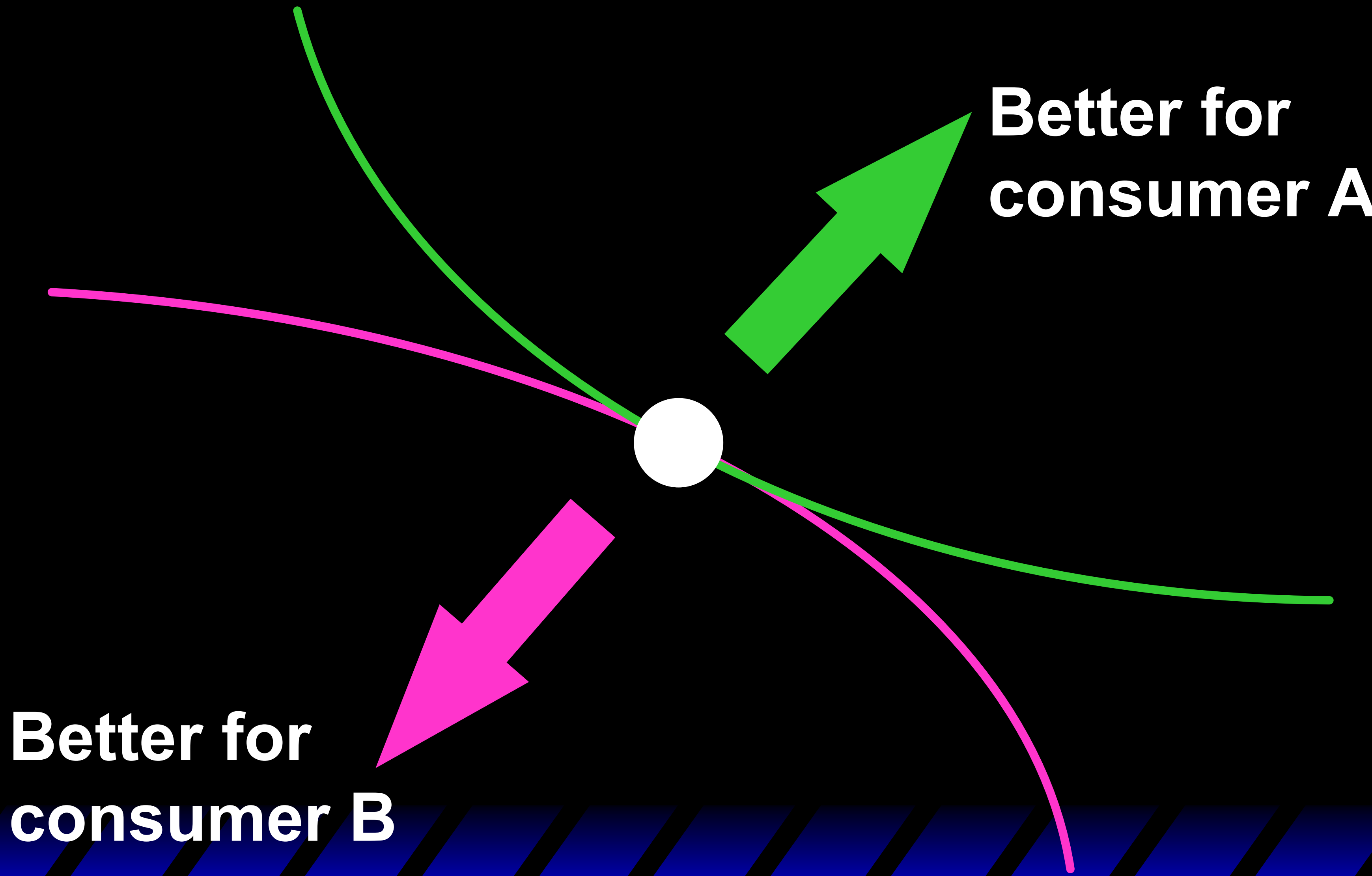
Trade improves both A's and B's welfares.
This is a Pareto-improvement over the endowment allocation.

Pareto-Improvements

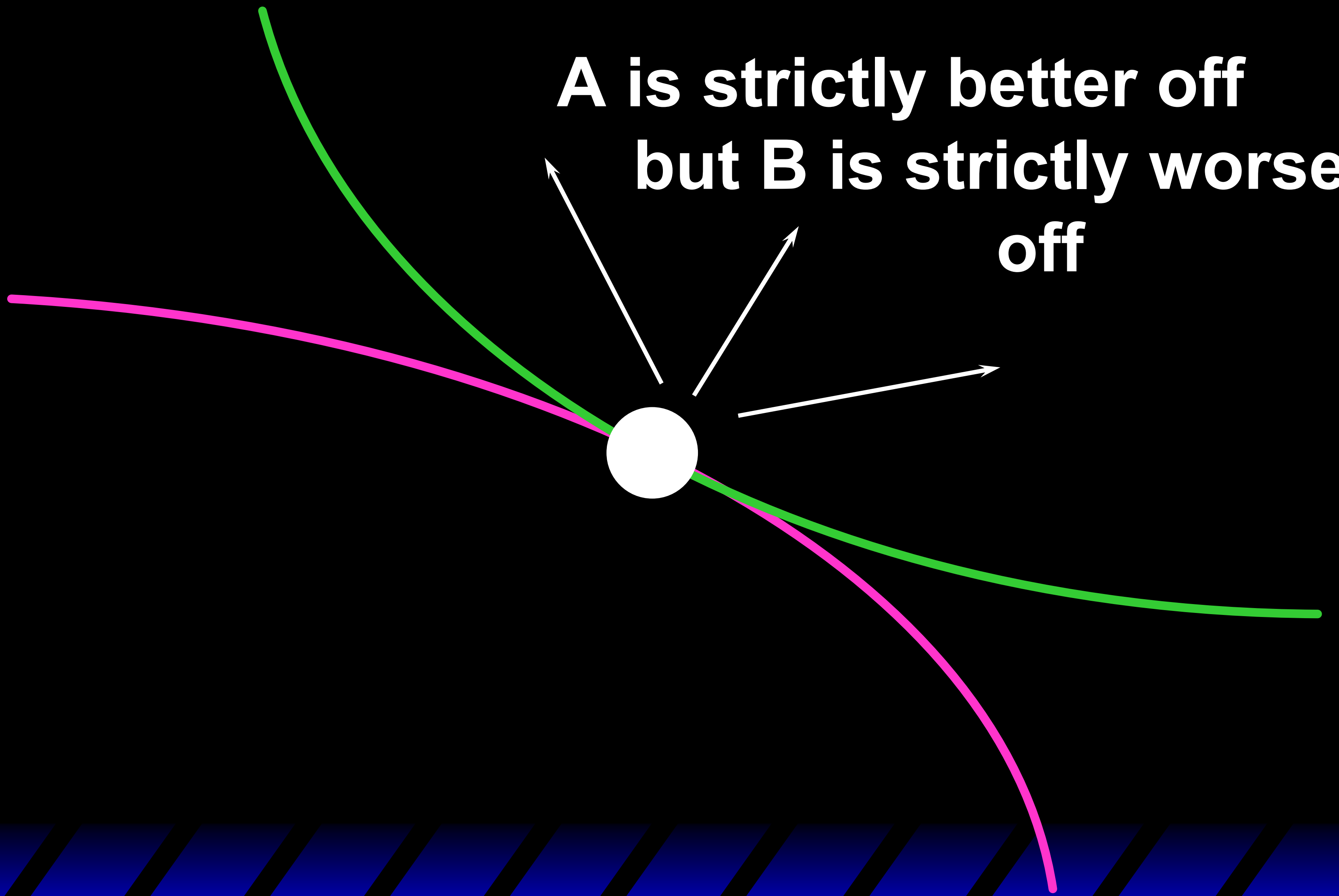
Further trade cannot improve both A and B's welfares.



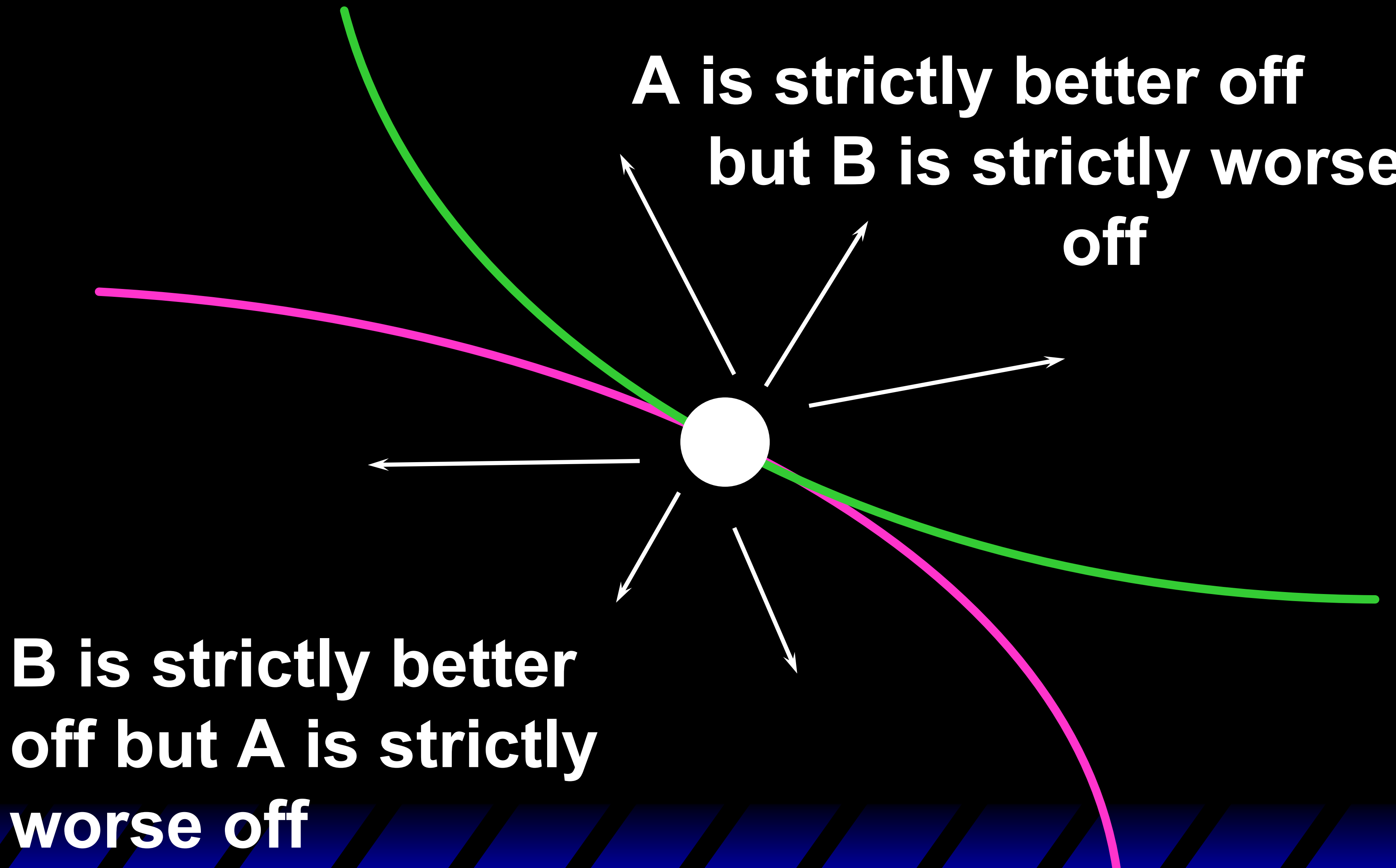
Pareto-Optimality



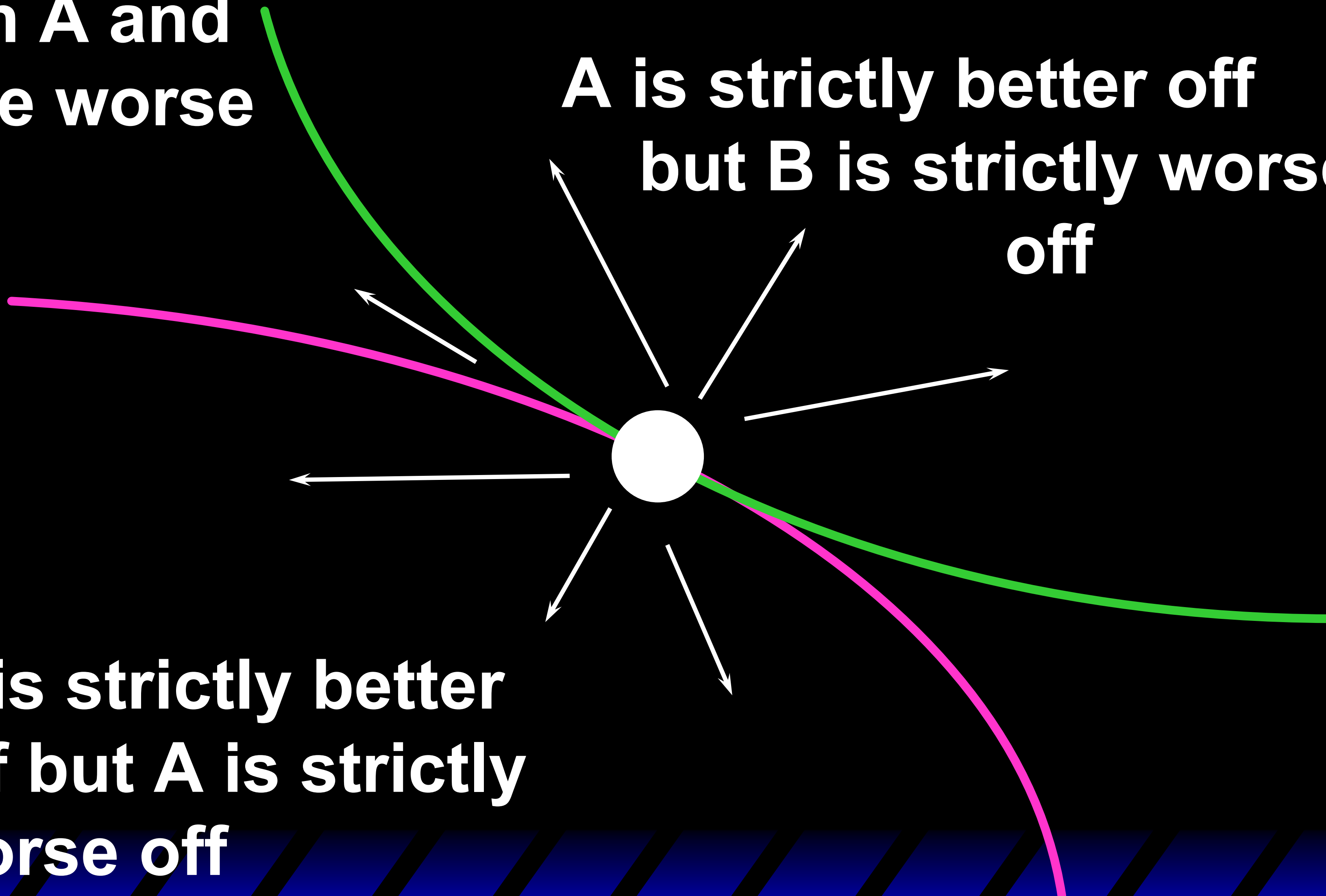
Pareto-Optimality



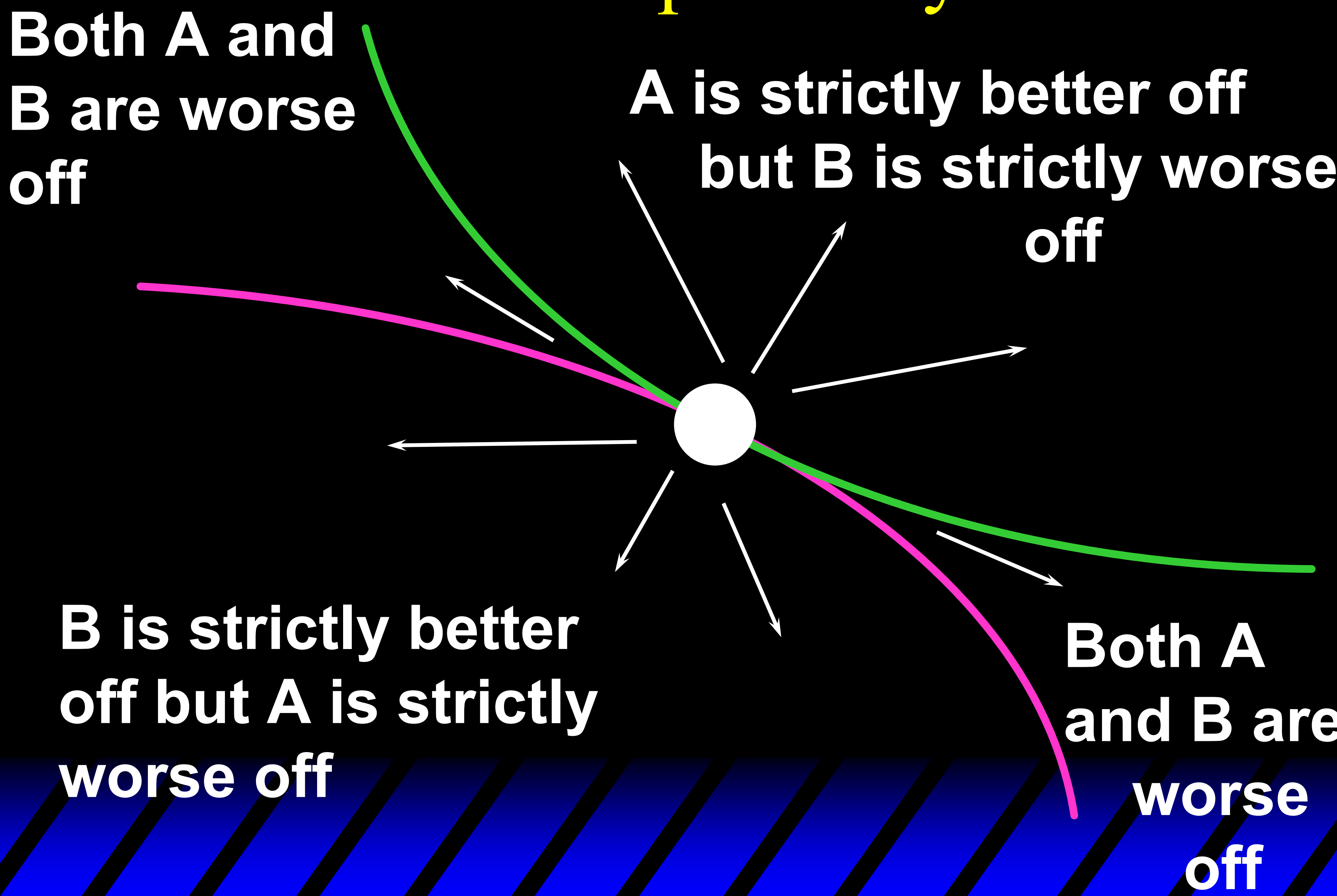
Pareto-Optimality



Pareto-Optimality



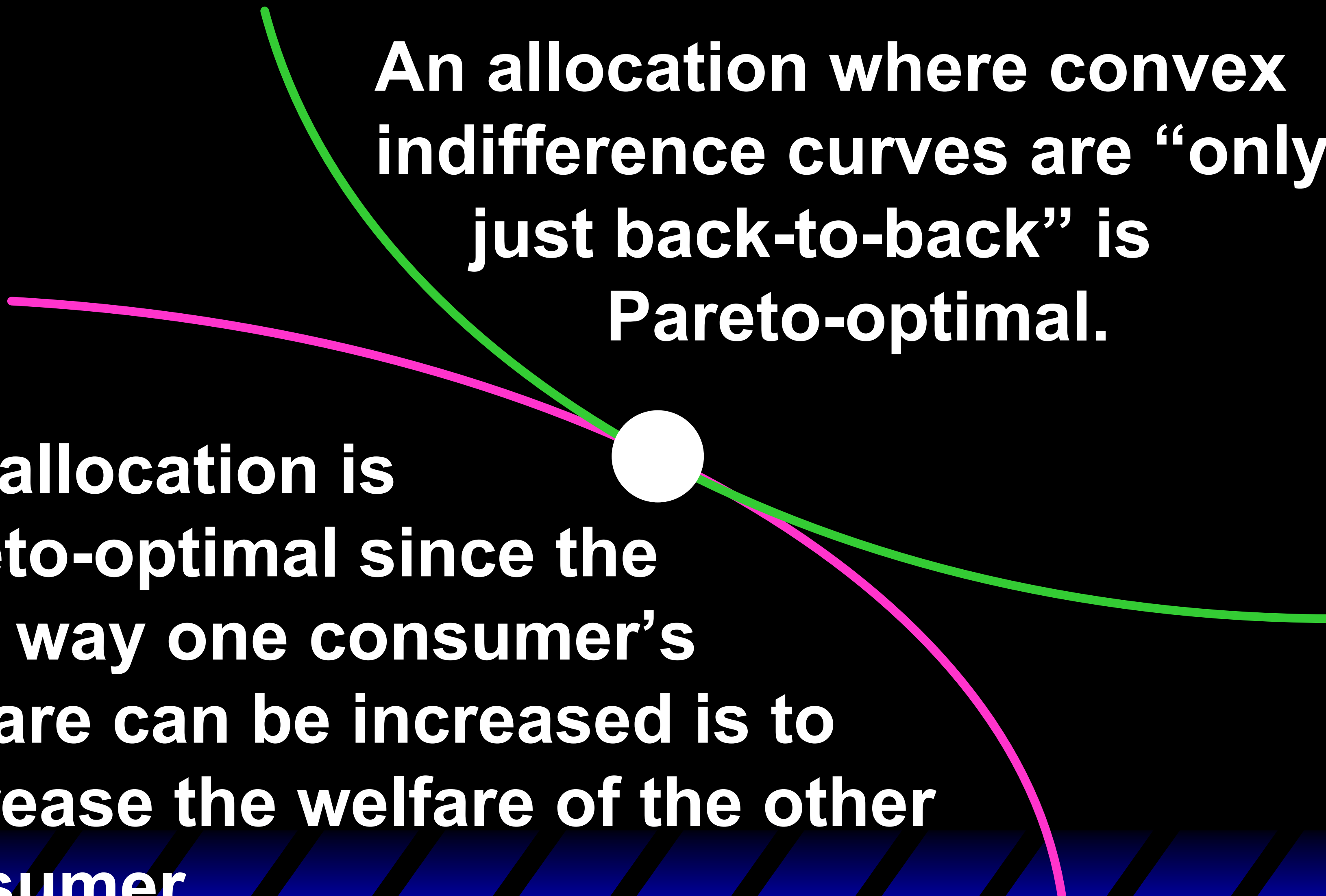
Pareto-Optimality



Pareto-Optimality

The allocation is **Pareto-optimal** since the only way one consumer's welfare can be increased is to decrease the welfare of the other consumer.

Pareto-Optimality



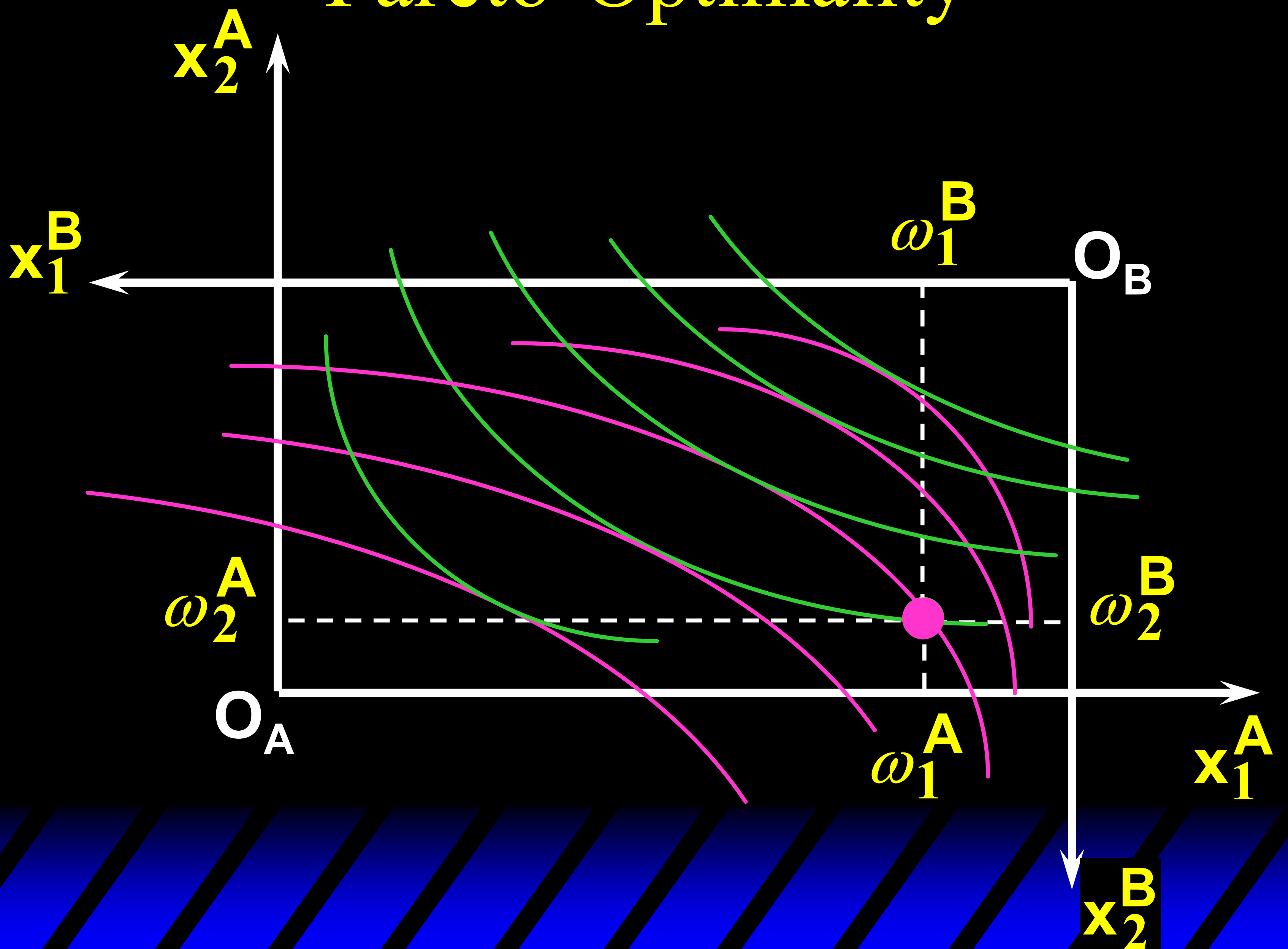
An allocation where convex indifference curves are "only just back-to-back" is Pareto-optimal.

The allocation is Pareto-optimal since the only way one consumer's welfare can be increased is to decrease the welfare of the other consumer.

Pareto-Optimality

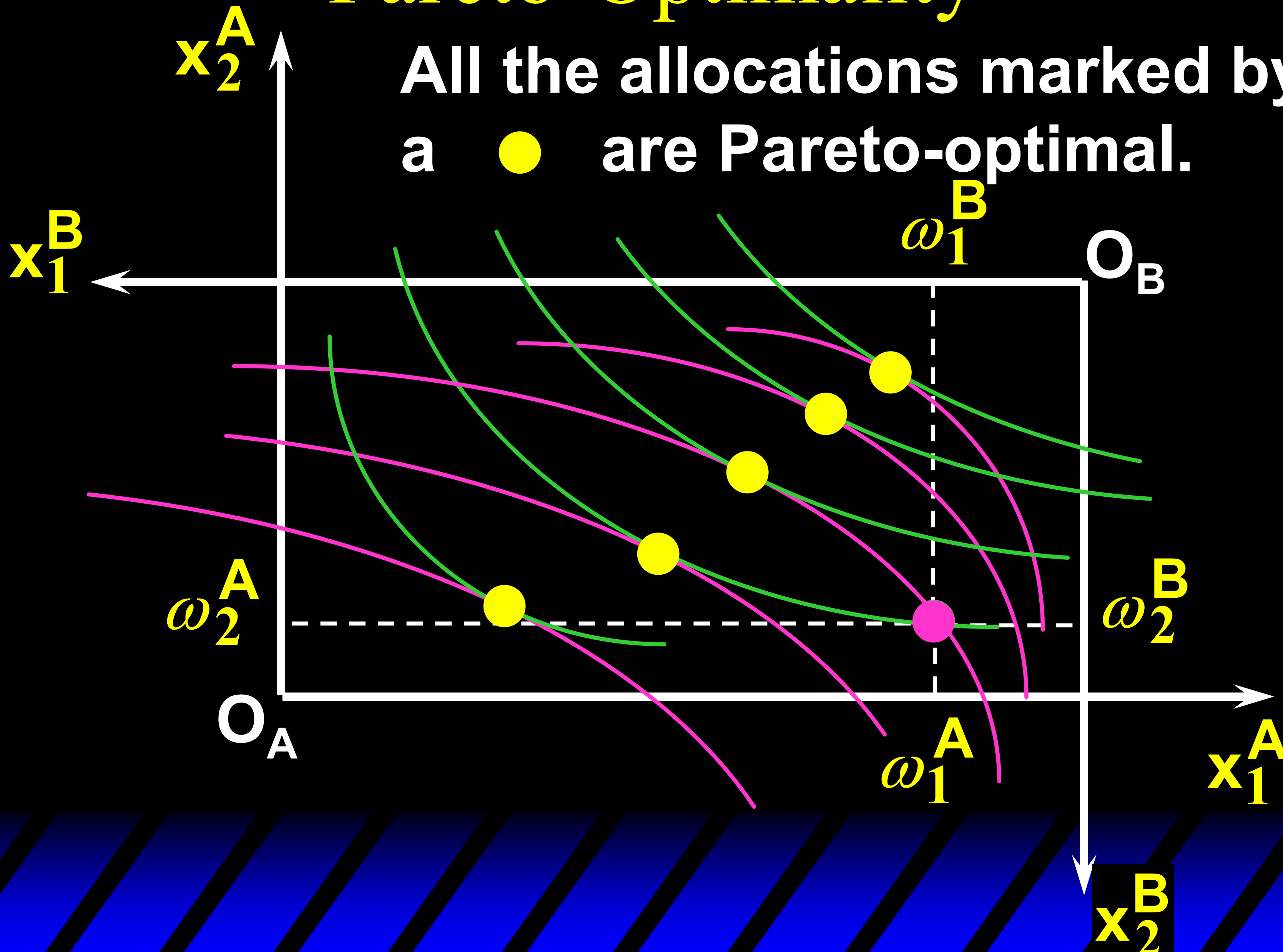
- Where are all of the Pareto-optimal allocations of the endowment?

Pareto-Optimality



Pareto-Optimality

All the allocations marked by a ● are Pareto-optimal.

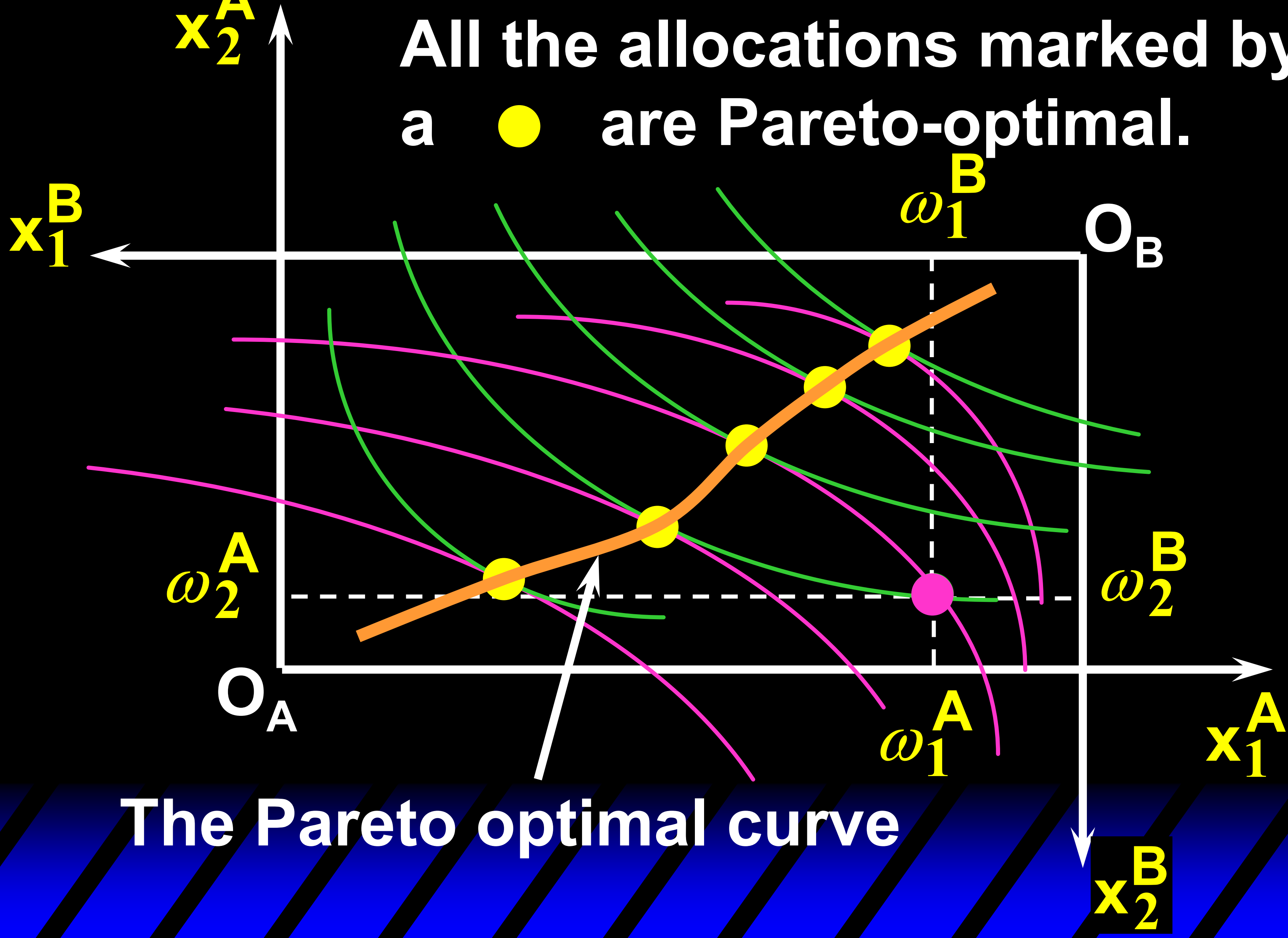


Pareto-Optimality

- **The Pareto optimal set is the set of all Pareto-optimal allocations.**

Pareto-Optimality

All the allocations marked by a ● are Pareto-optimal.

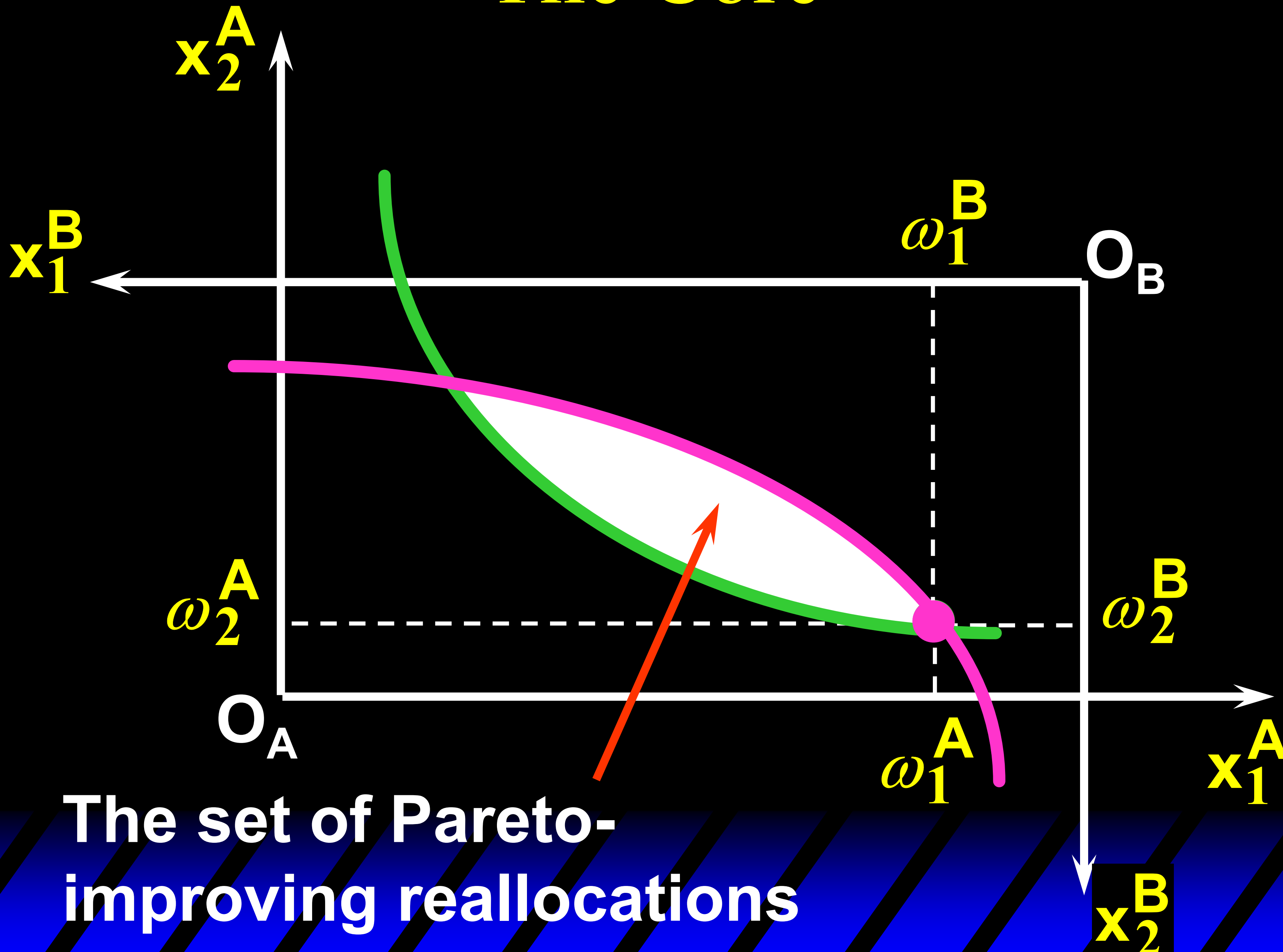


The Pareto optimal curve

Pareto-Optimality

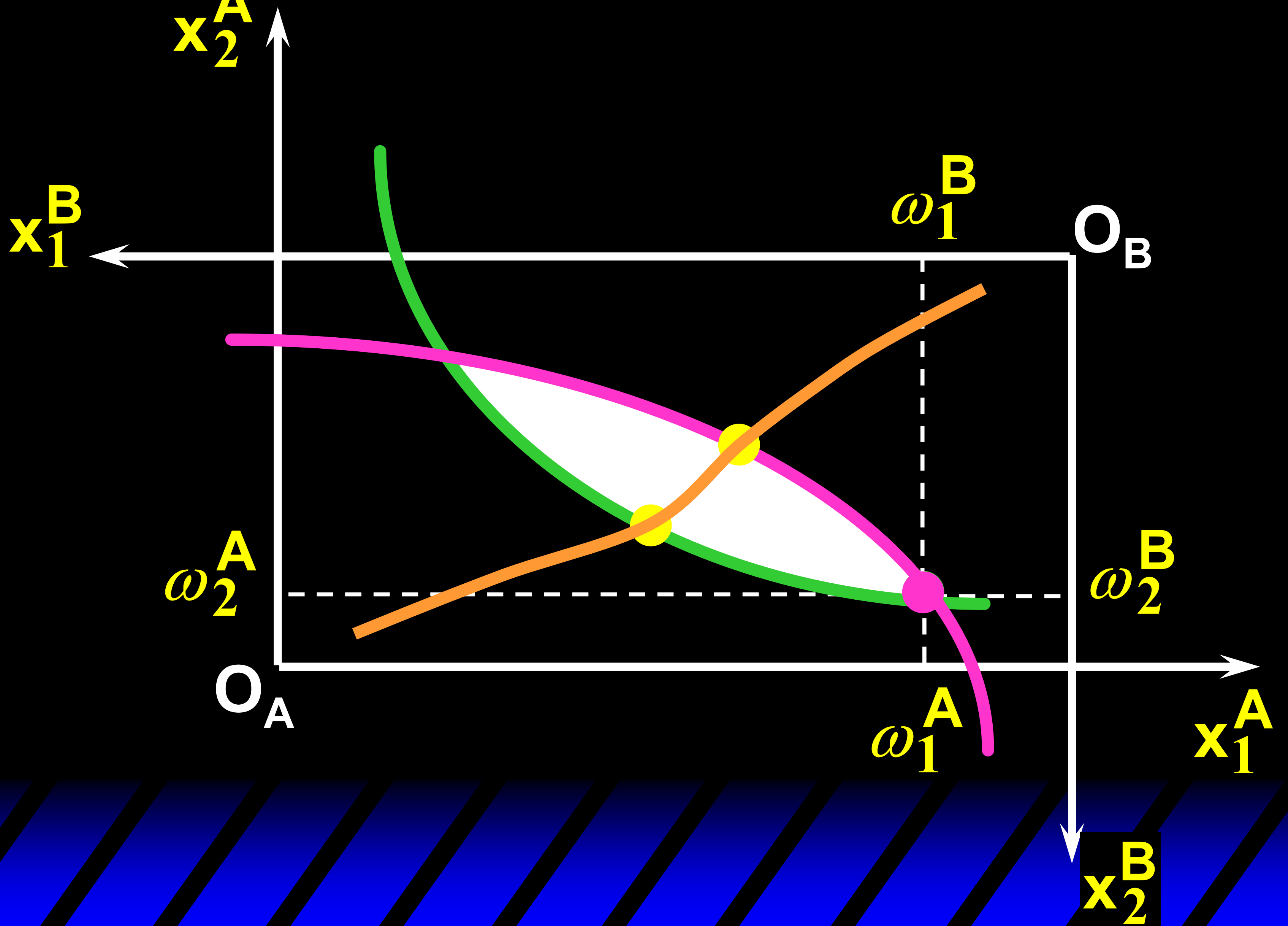
- But to which of the many allocations on the Pareto optimal curve will consumers trade?
- That depends upon how trade is conducted.
- In perfectly competitive markets? By one-on-one bargaining?

The Core

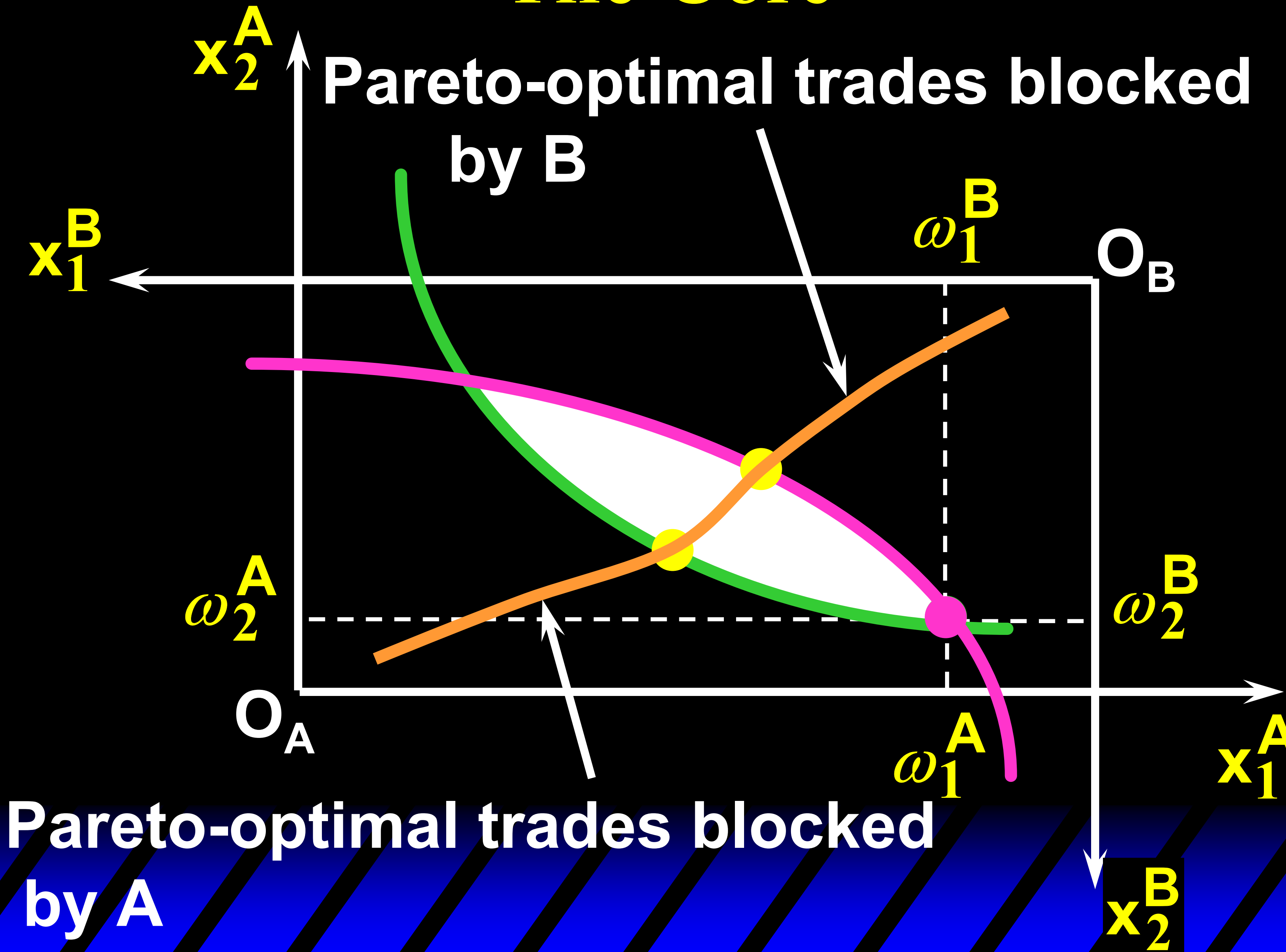


The set of Pareto-improving reallocations

The Core

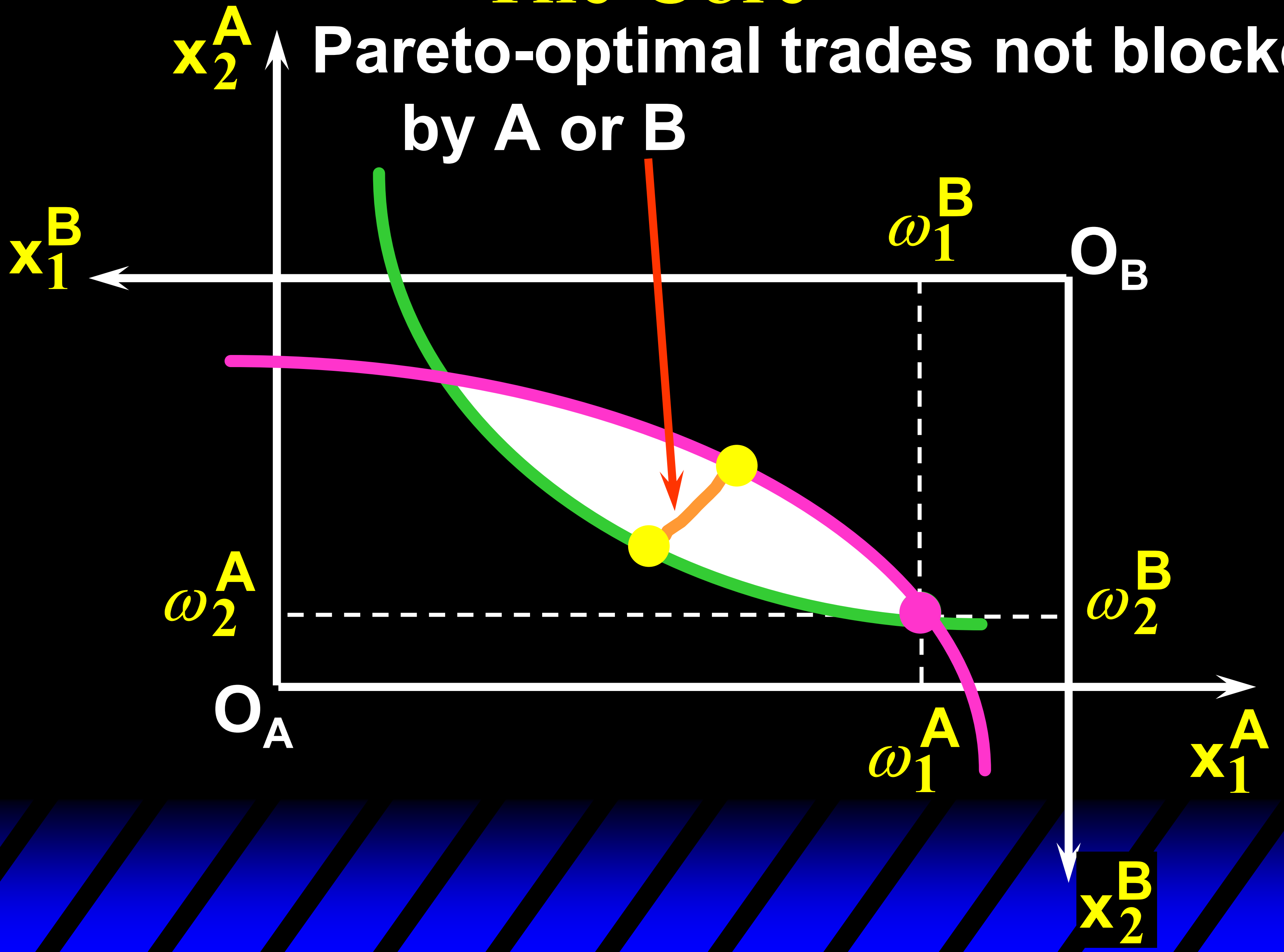


The Core



x_2^A
Pareto-optimal trades blocked by B
x_1^B
ω_1^B
O_B
ω_2^A
ω_2^B
O_A
ω_1^A
x_1^A
Pareto-optimal trades blocked by A
x_2^B

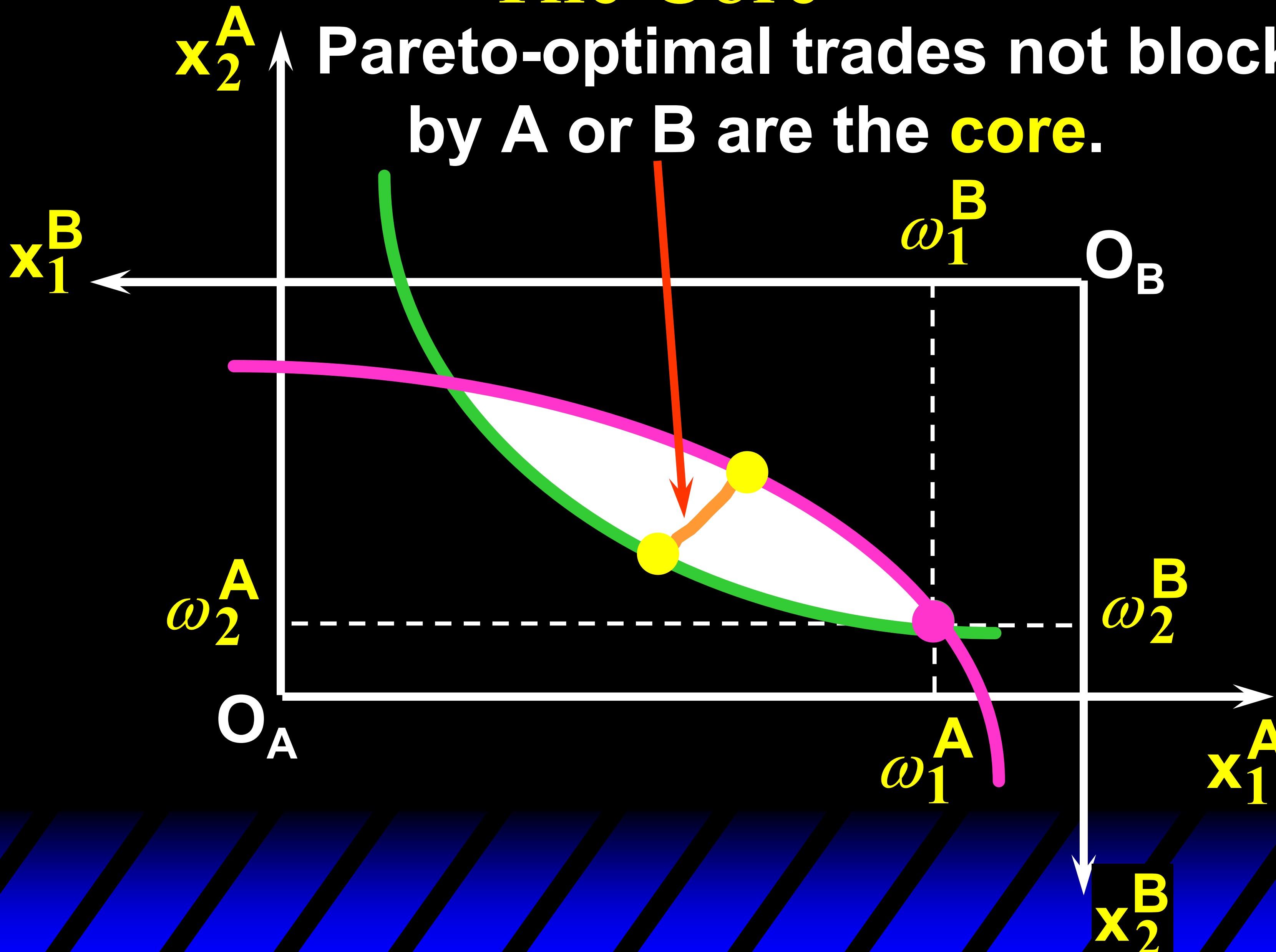
The Core



x_2^A
Pareto-optimal trades not blocked by A or B
x_1^B
ω_1^B
O_B
ω_2^A
ω_2^B
O_A
ω_1^A
x_1^A
x_2^B

The Core

Pareto-optimal trades not blocked by A or B are the core.



The Core

- The **core** is the set of all Pareto-optimal allocations that are welfare-improving for both consumers relative to their own endowments.
- Rational trade should achieve a core allocation.

The Core

- But which core allocation?
- Again, that depends upon the manner in which trade is conducted.

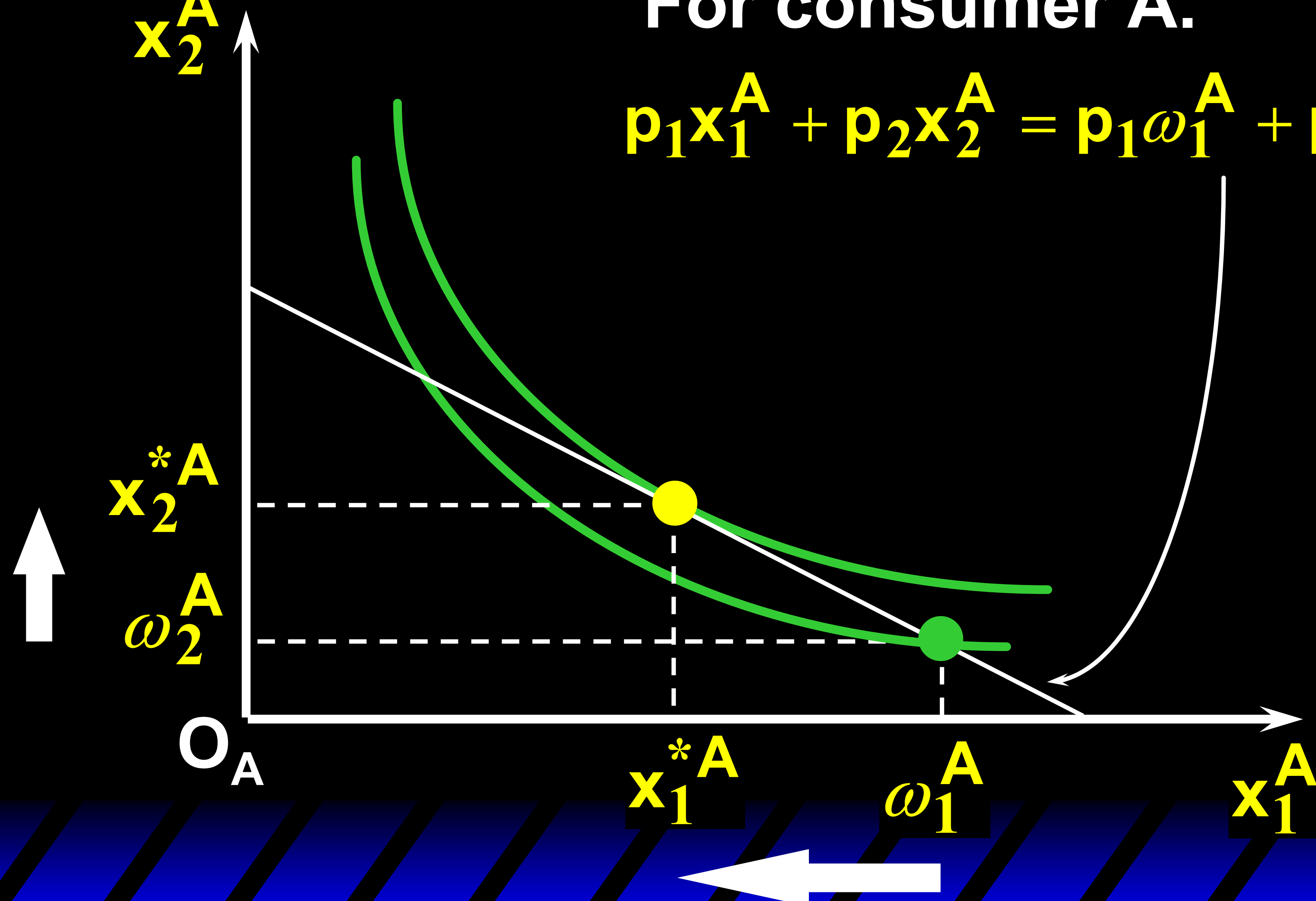
Trade in Competitive Markets

- Consider trade in perfectly competitive markets.
- Each consumer is a price-taker trying to maximize her own utility given p_1, p_2 and her own endowment. That is, ...

Trade in Competitive Markets

For consumer A.

$$p_1x_1^A + p_2x_2^A = p_1\omega_1^A + p_2\omega_2^A$$



Trade in Competitive Markets

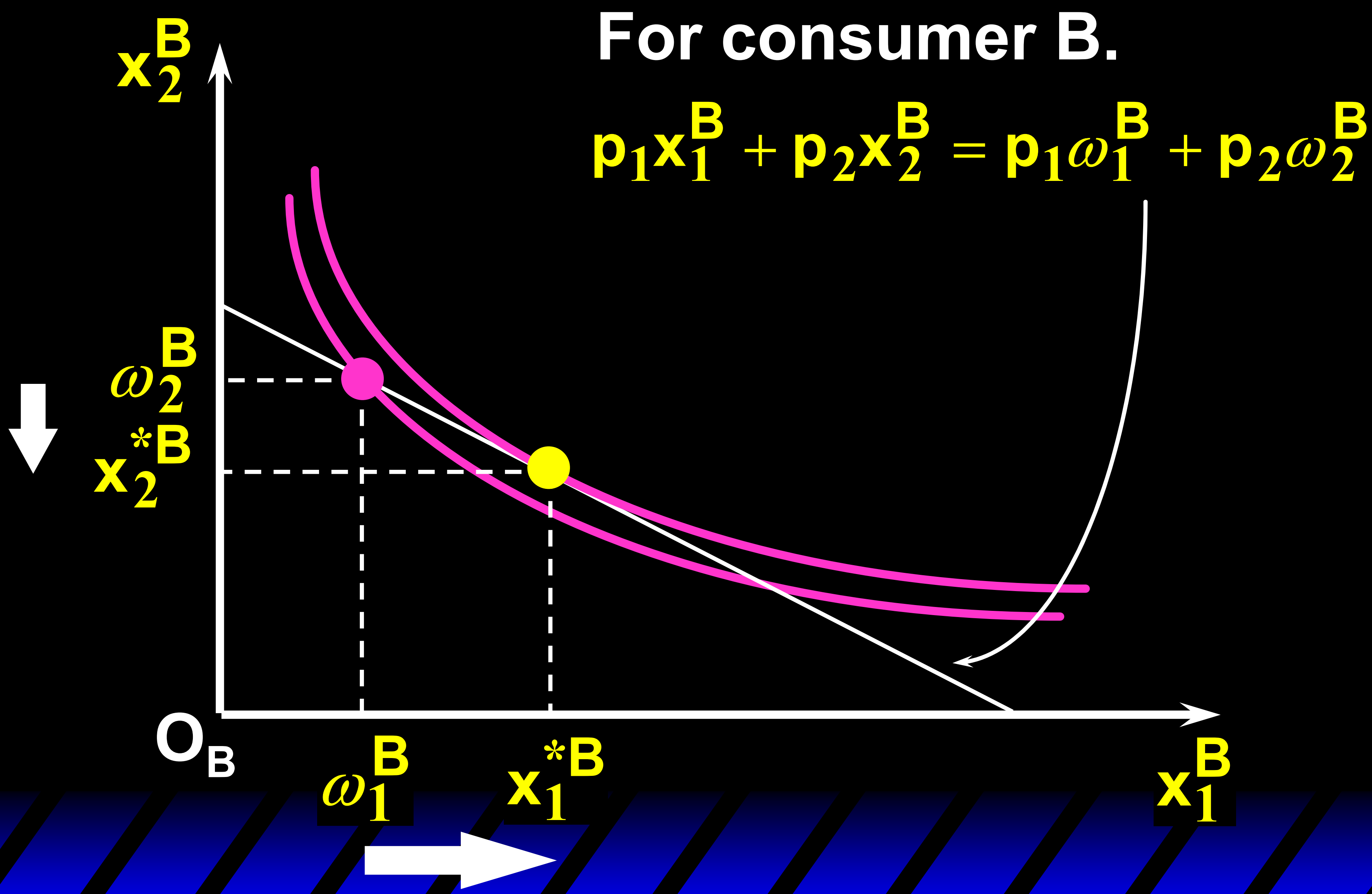
- So given p_1 and p_2, consumer A's net demands for commodities 1 and 2 are

$$x_1^{*A} - \omega_1^A \quad \text{and} \quad x_2^{*A} - \omega_2^A.$$

Trade in Competitive Markets

- And, similarly, for consumer B …

Trade in Competitive Markets



Trade in Competitive Markets

- So given p_1 and p_2, consumer B's net demands for commodities 1 and 2 are

$$x_1^{*B} - \omega_1^B \quad \text{and} \quad x_2^{*B} - \omega_2^B.$$

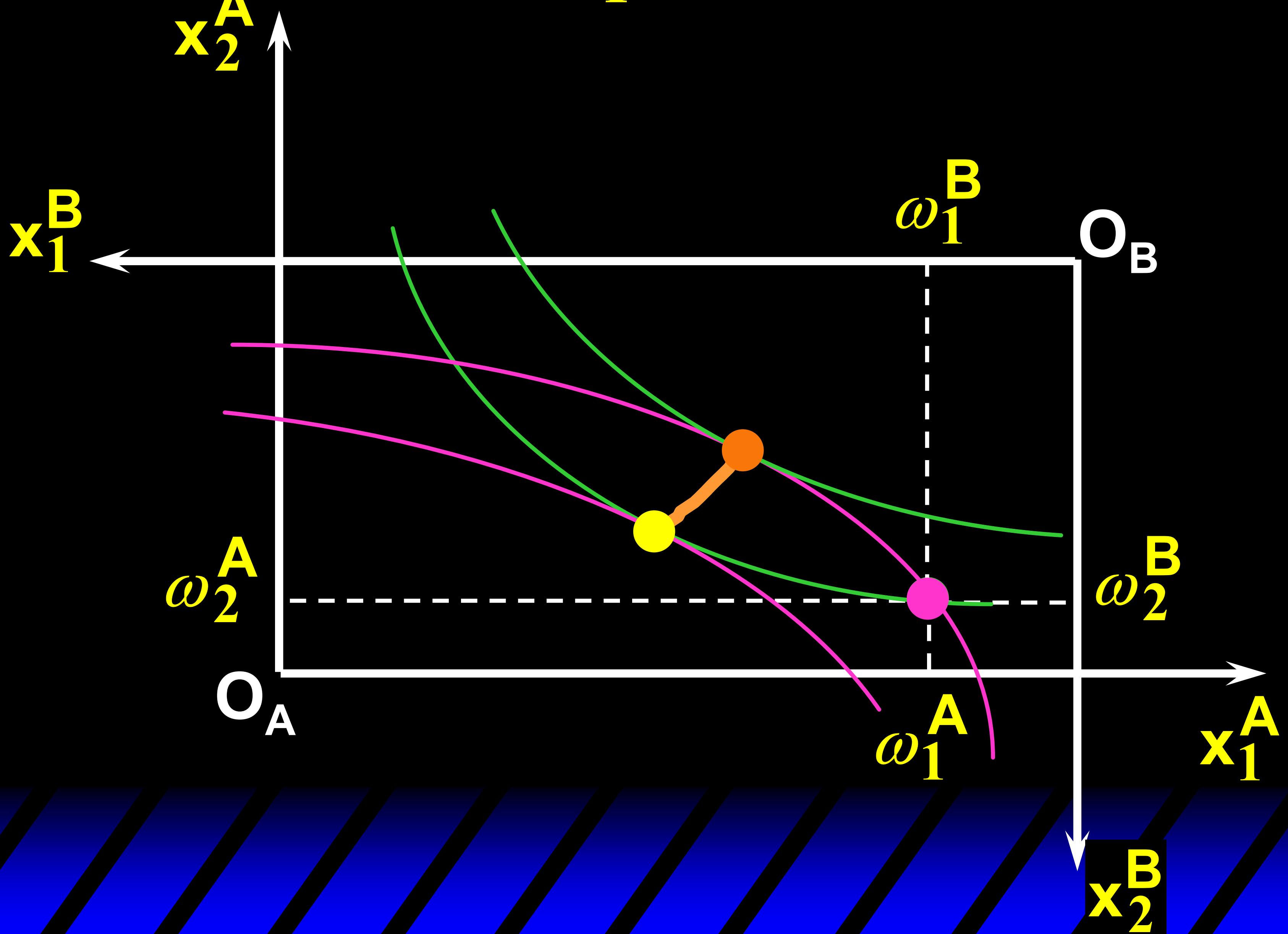
Trade in Competitive Markets

- A **general equilibrium** occurs when prices p_1 and p_2 cause both the markets for commodities 1 and 2 to clear; i.e.

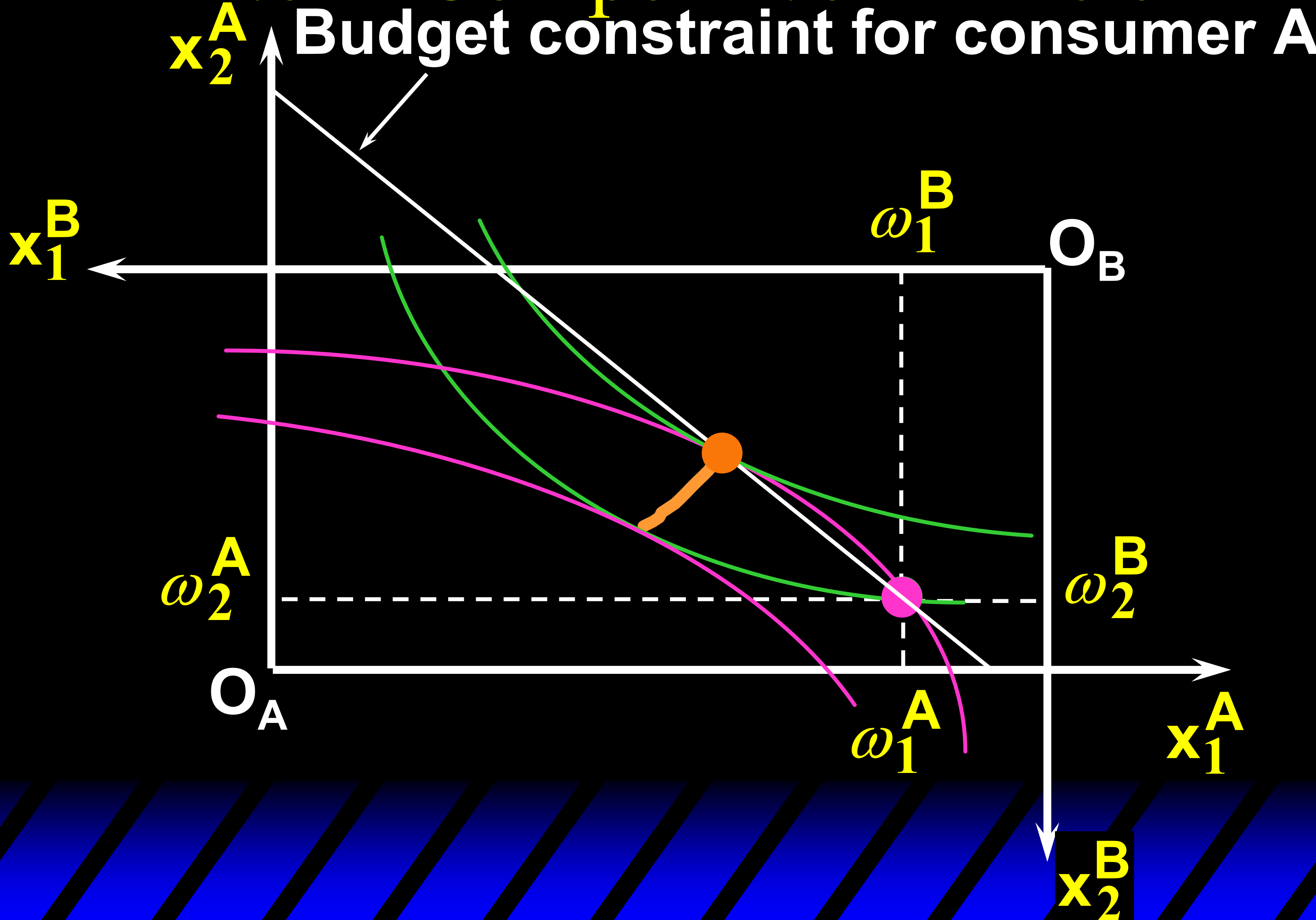
$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$$

and $$x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B.$$

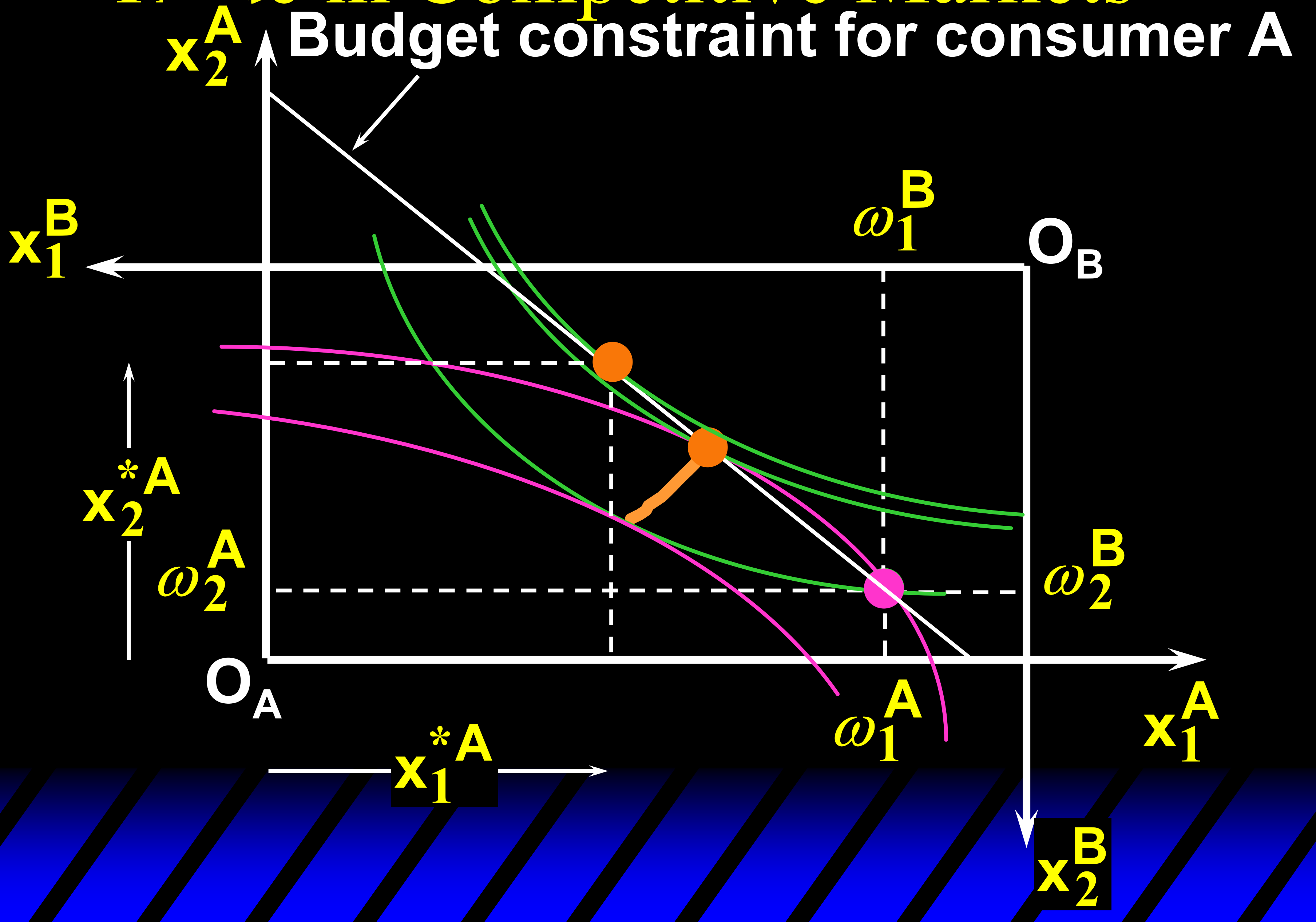
Trade in Competitive Markets



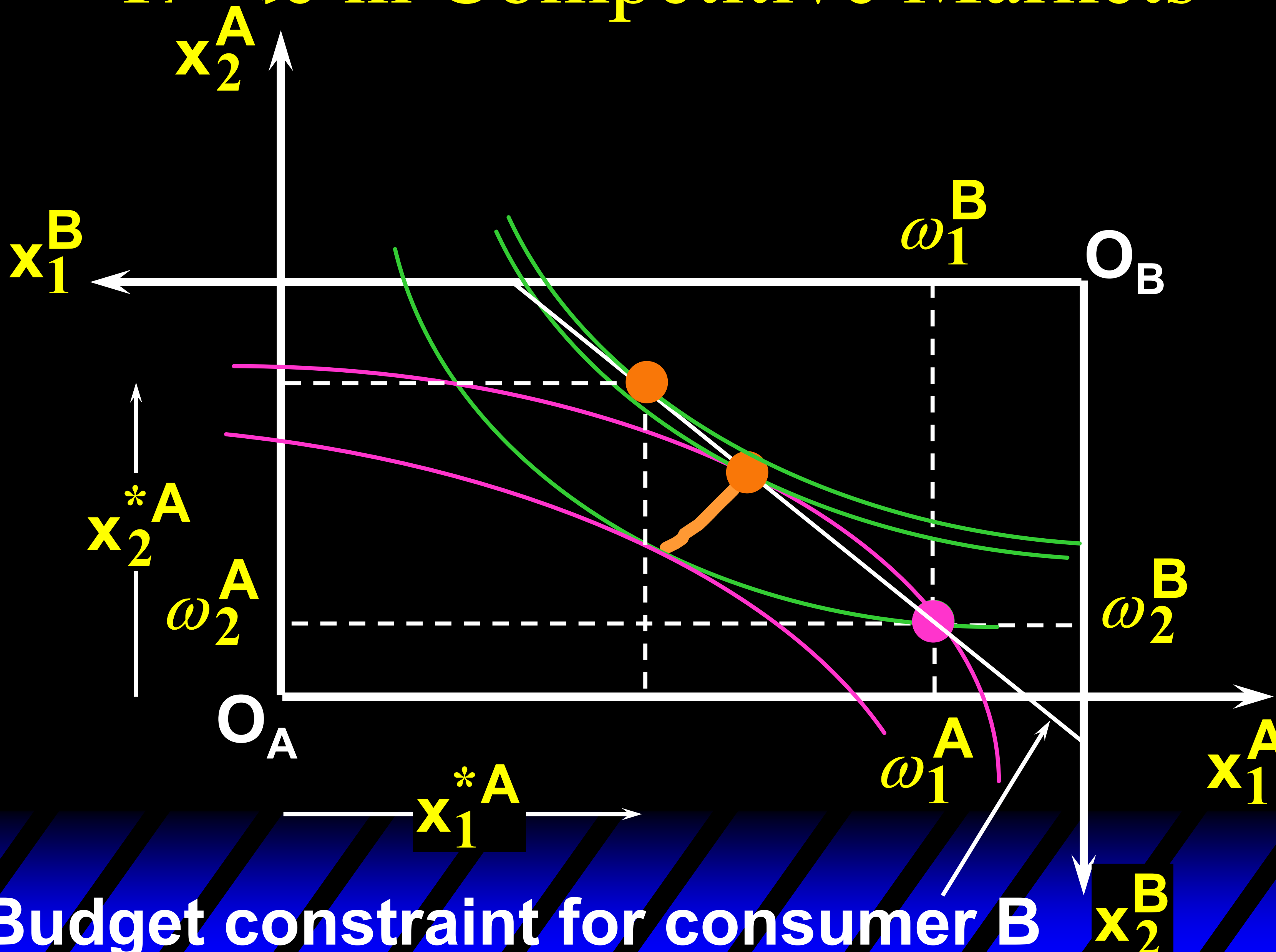
Trade in Competitive Markets



Trade in Competitive Markets

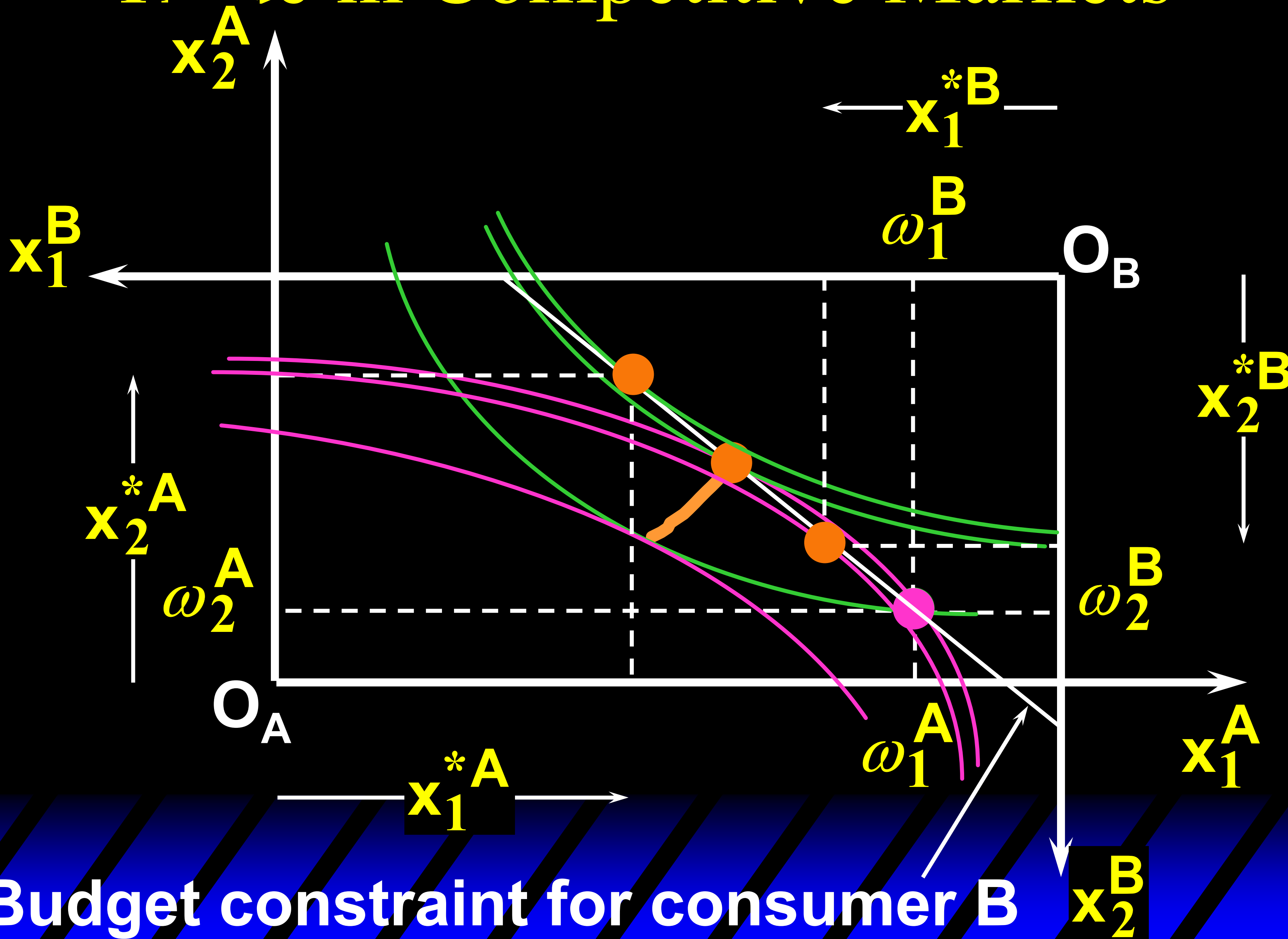


Trade in Competitive Markets



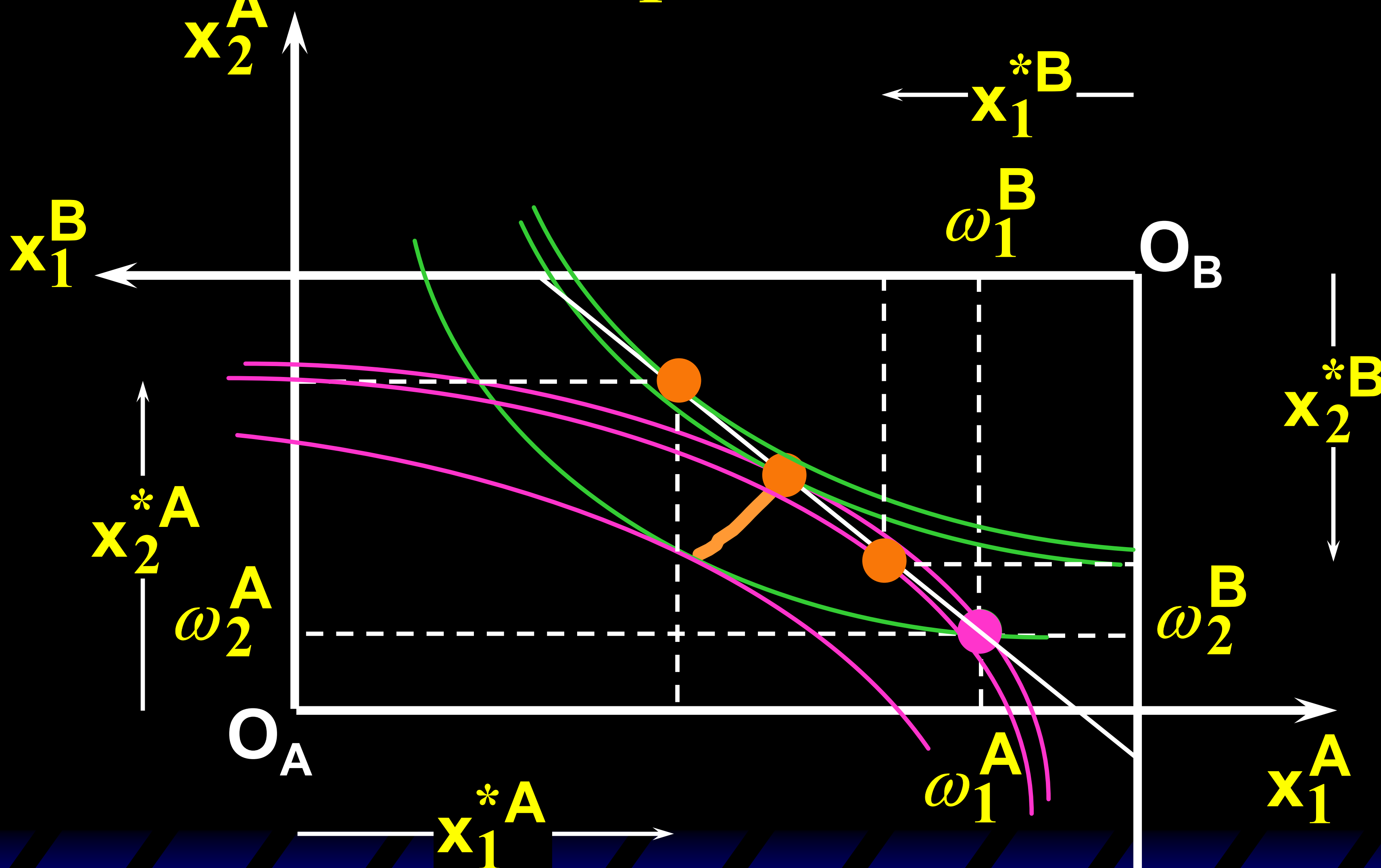
Budget constraint for consumer B

Trade in Competitive Markets



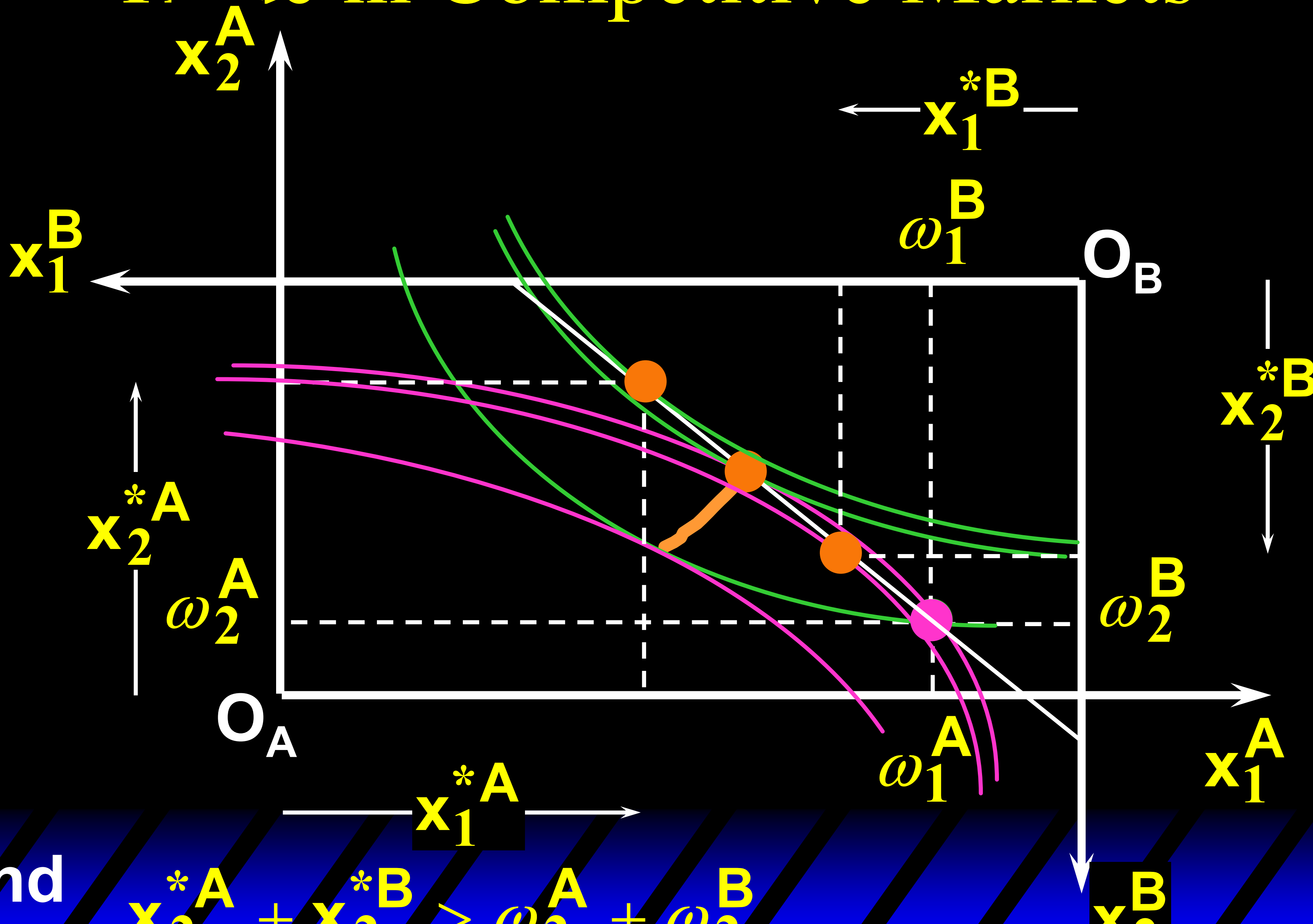
Budget constraint for consumer B

Trade in Competitive Markets



But $x_1^{*A} + x_1^{*B} < \omega_1^A + \omega_1^B$

Trade in Competitive Markets



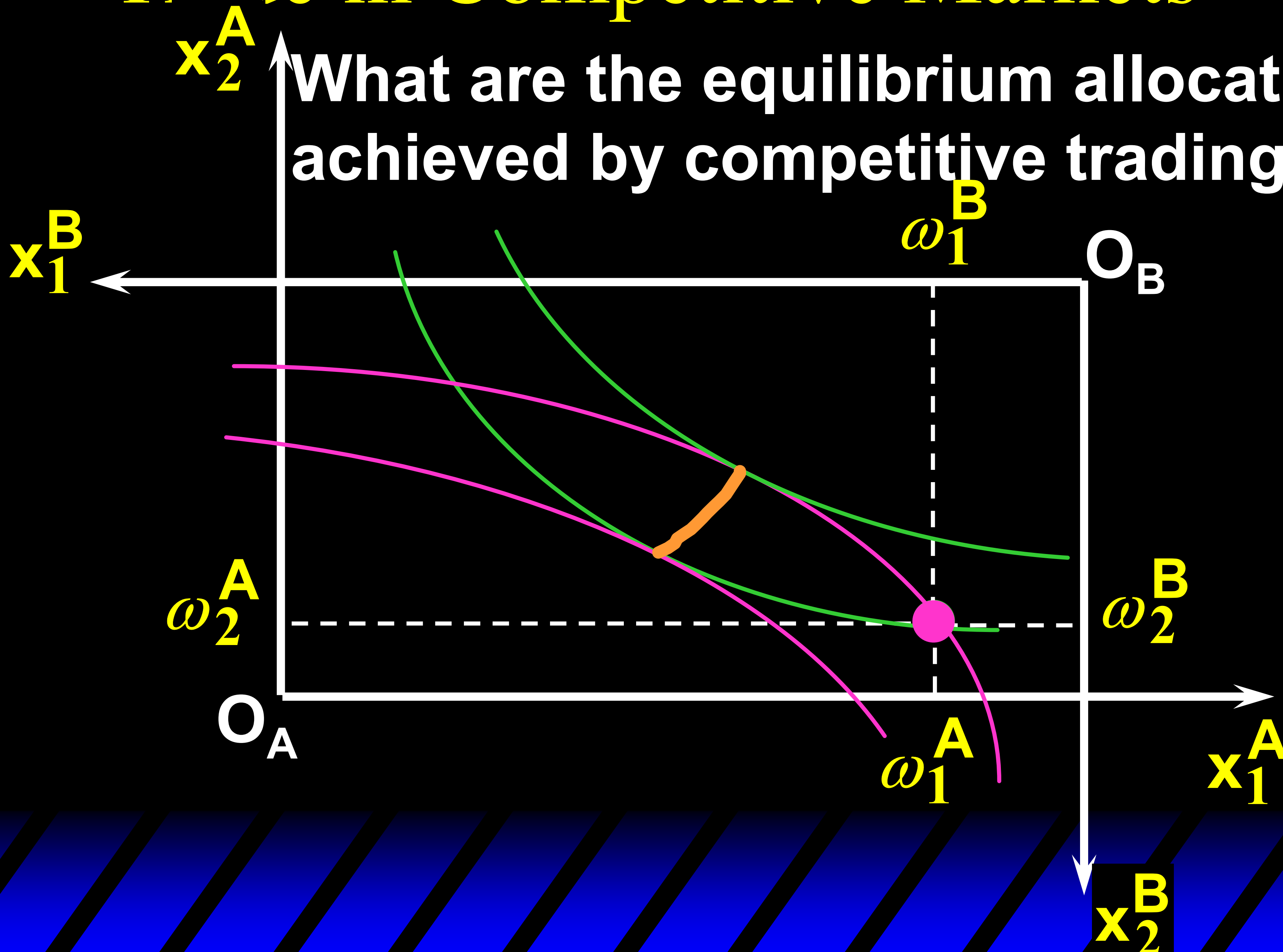
and $x_2^{*A} + x_2^{*B} > \omega_2^A + \omega_2^B$

Trade in Competitive Markets

- So at the given prices p_1 and p_2 there is an
  – excess supply of commodity 1
  – excess demand for commodity 2.
- Neither market clears so the prices p_1 and p_2 do not cause a general equilibrium.

Trade in Competitive Markets

What are the equilibrium allocations achieved by competitive trading?

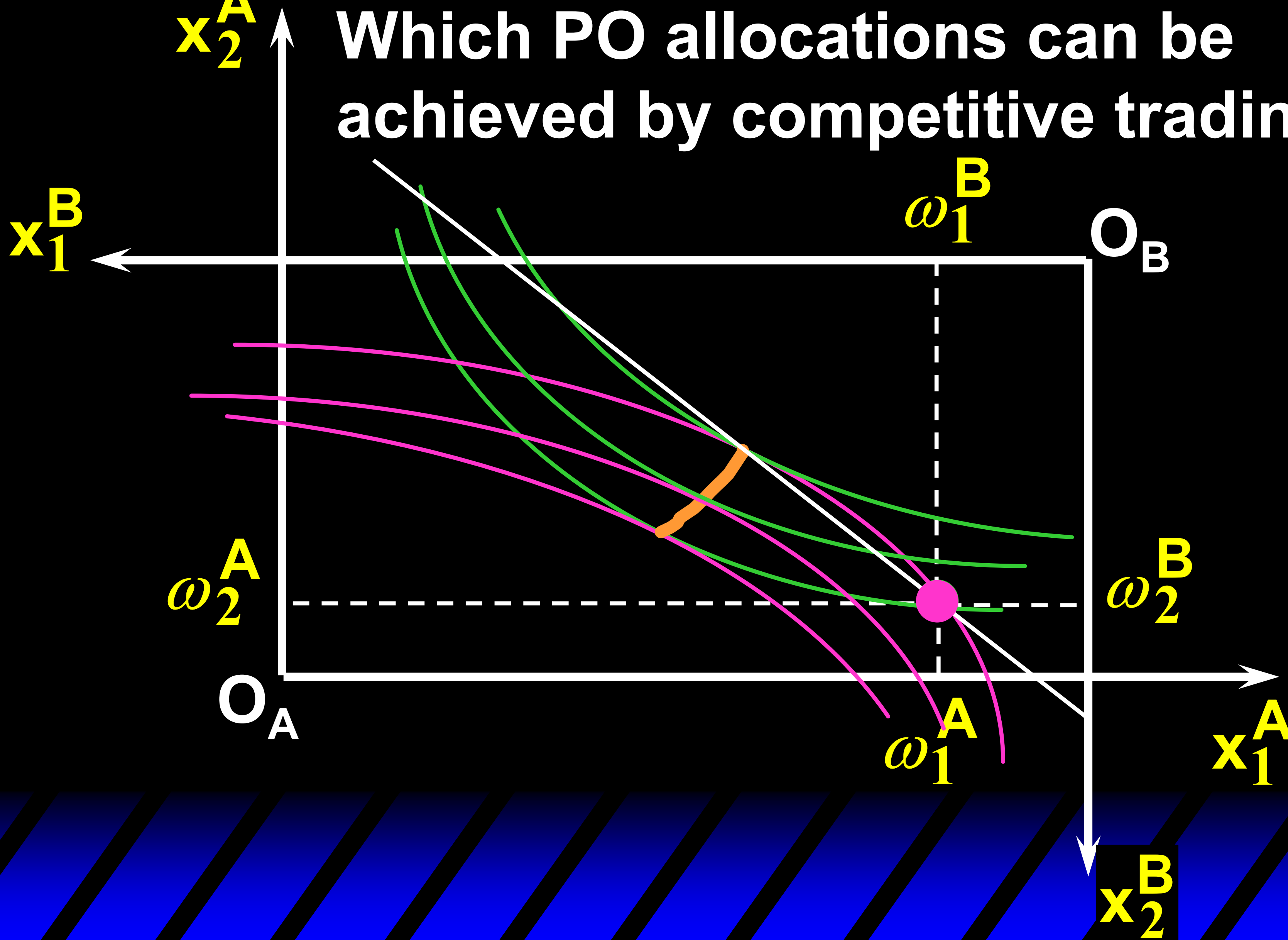


Trade in Competitive Markets

- Since there is an excess demand for commodity 2, p_2 will rise.
- Since there is an excess supply of commodity 1, p_1 will fall.
- The slope of the budget constraints is $- p_1/p_2$ so the budget constraints will pivot about the endowment point and become less steep.

Trade in Competitive Markets

Which PO allocations can be achieved by competitive trading?



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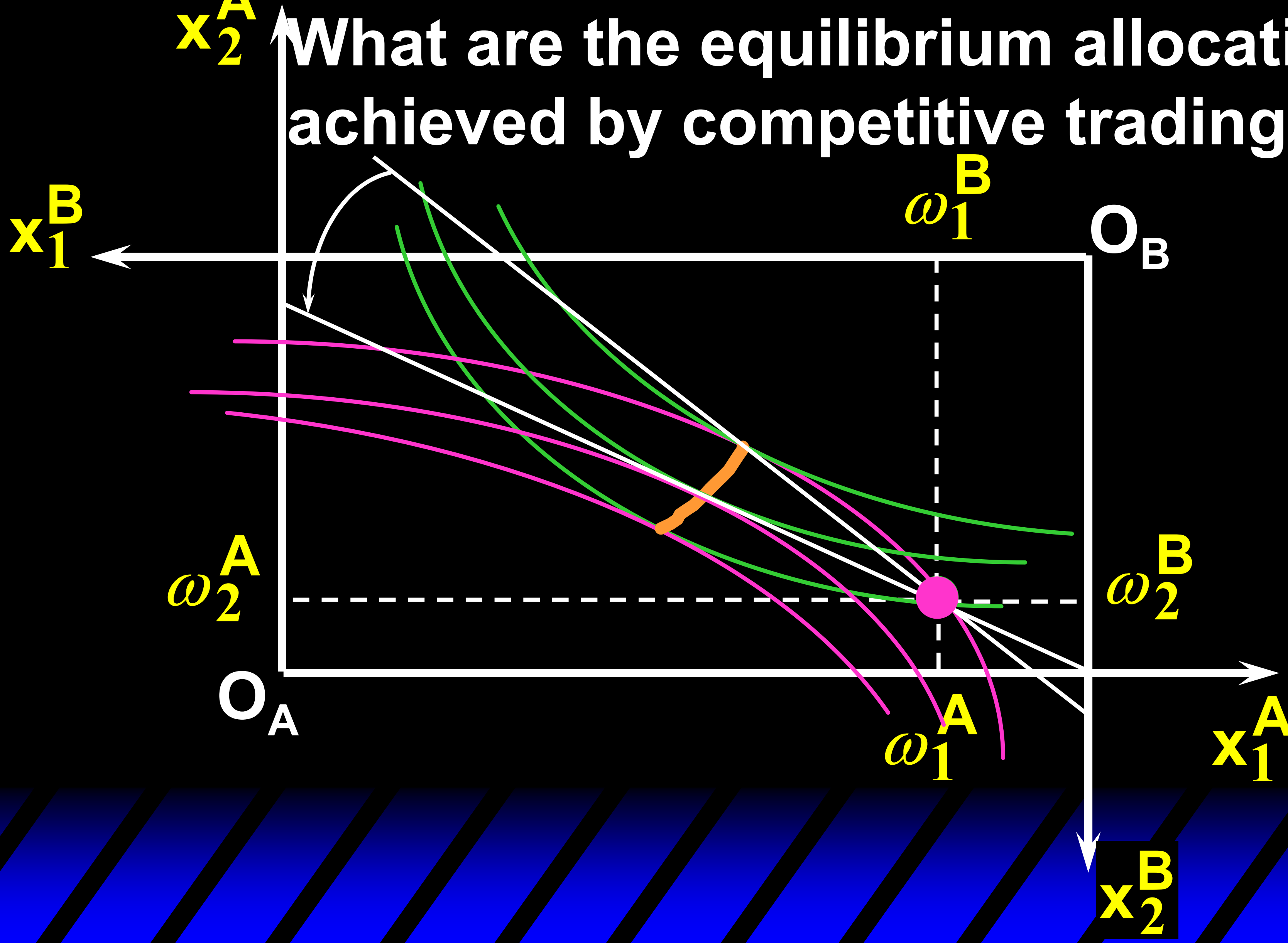

Trade in Competitive Markets

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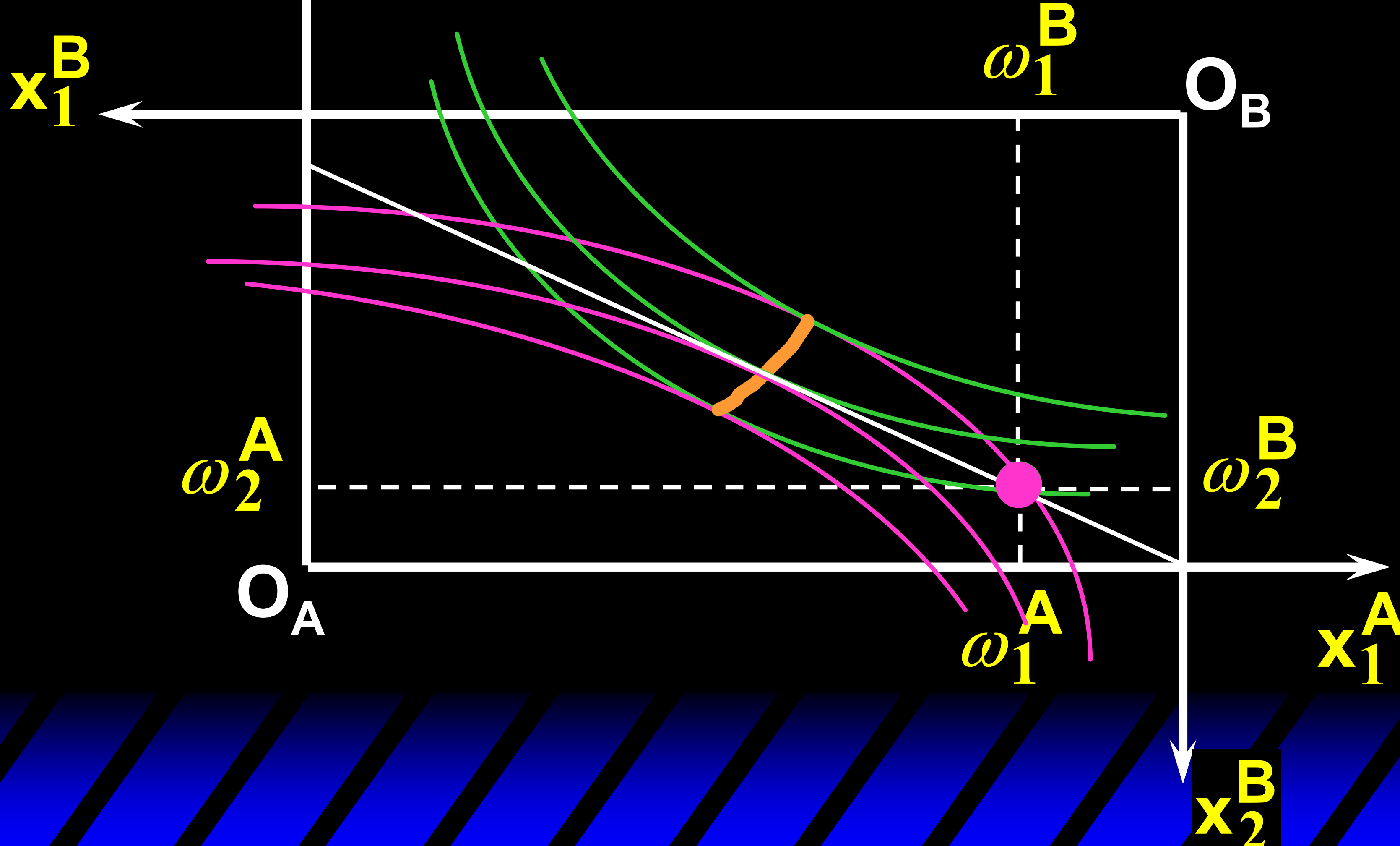
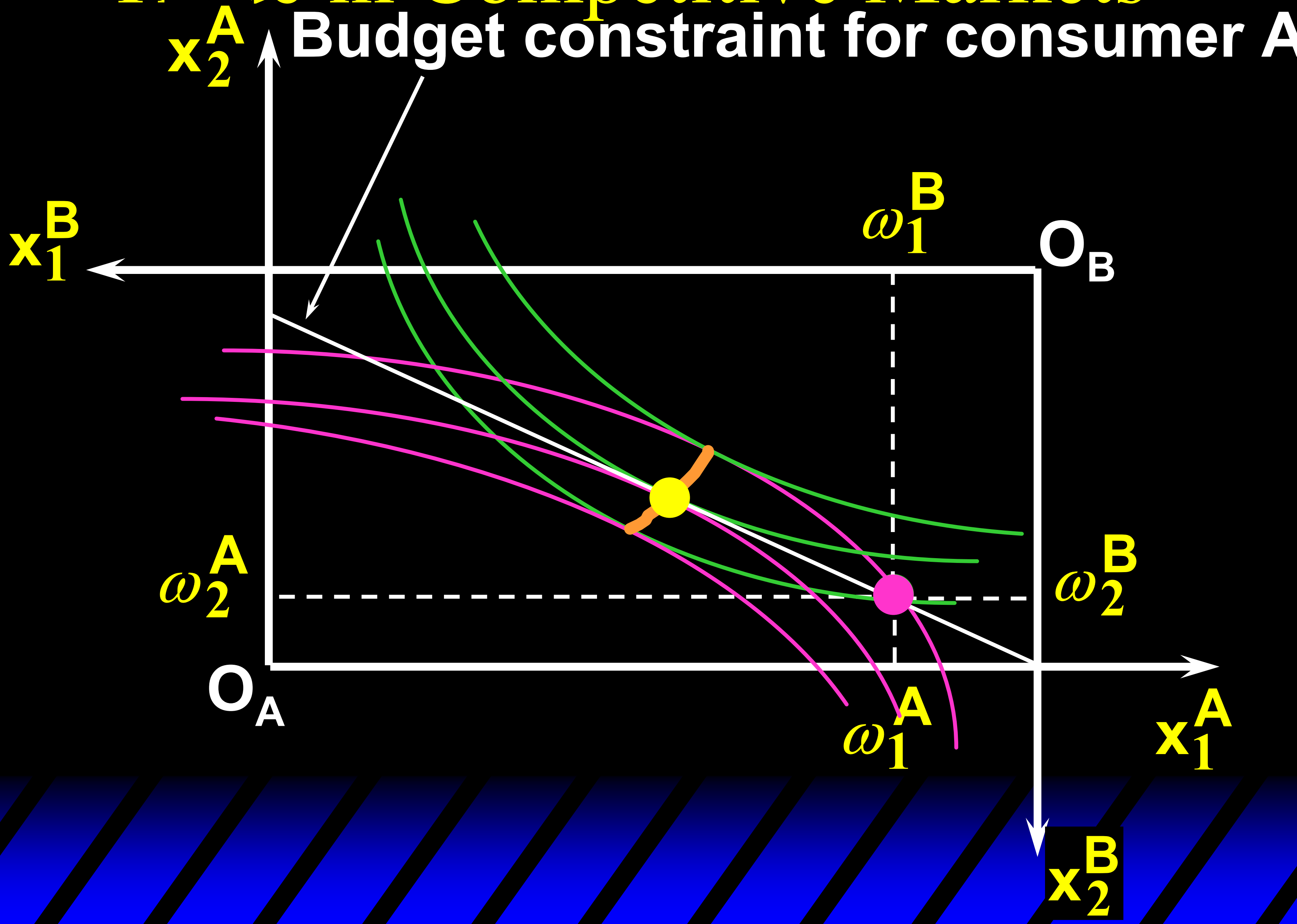
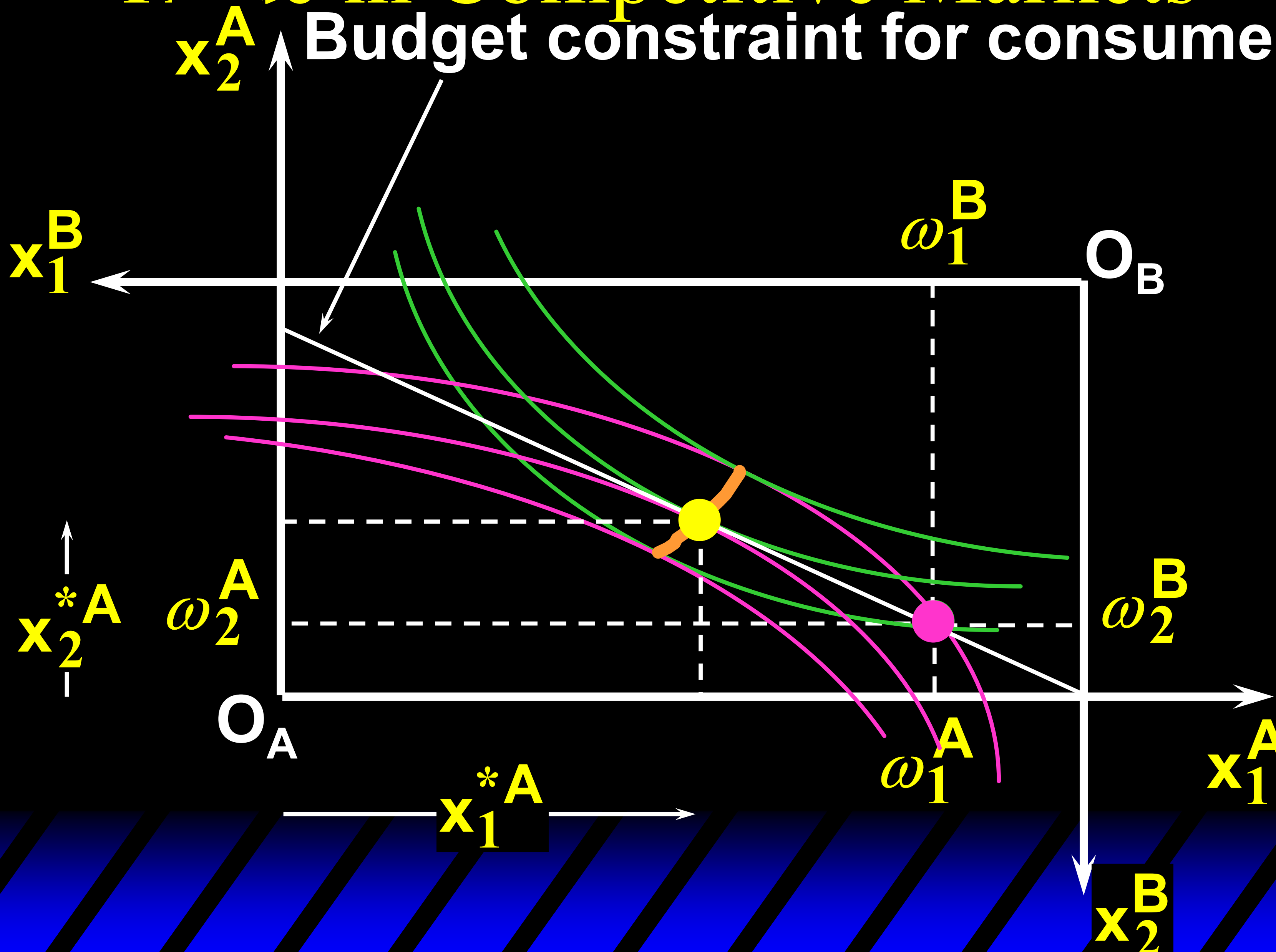


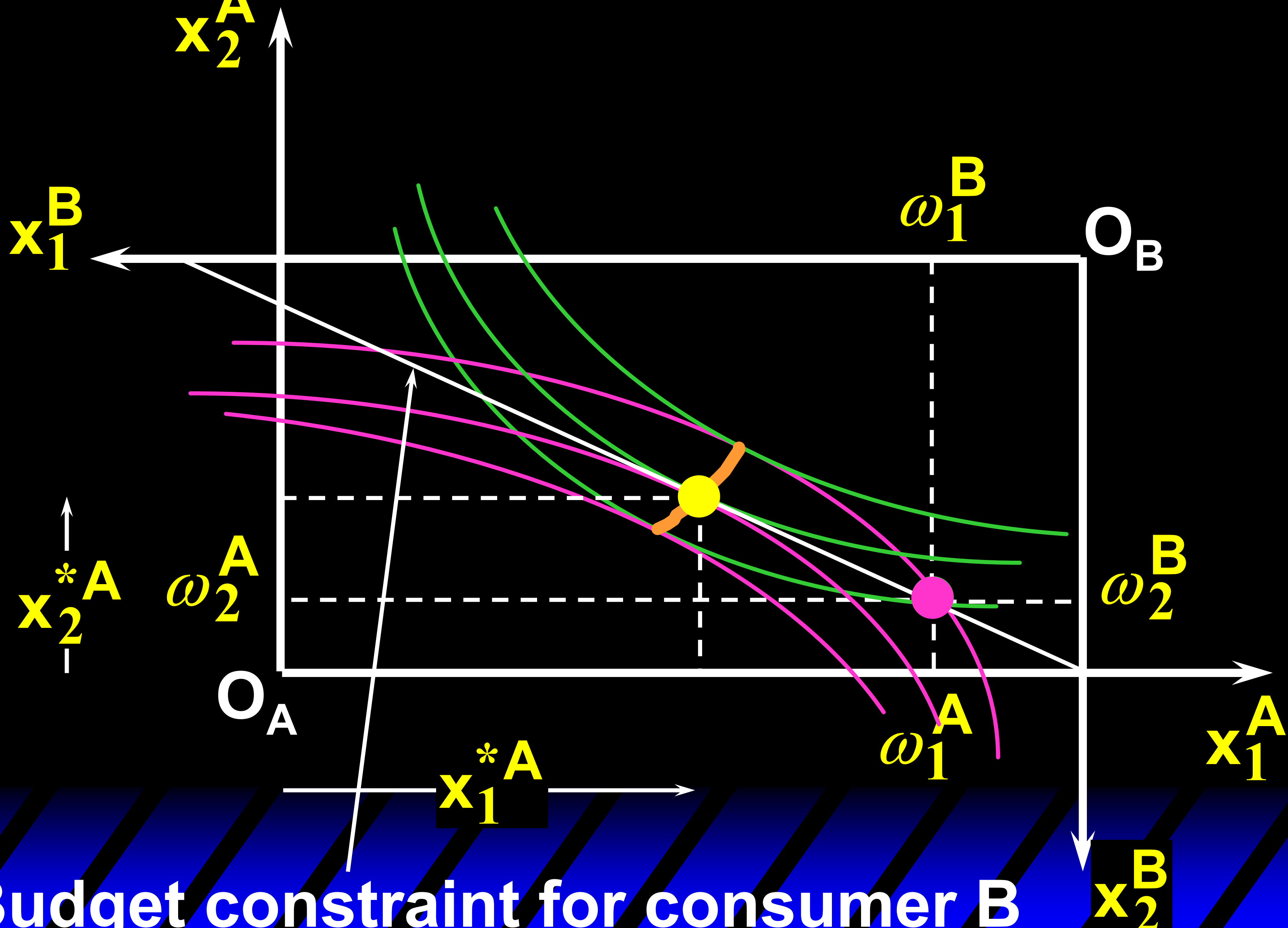
Trade in Competitive Markets



Trade in Competitive Markets

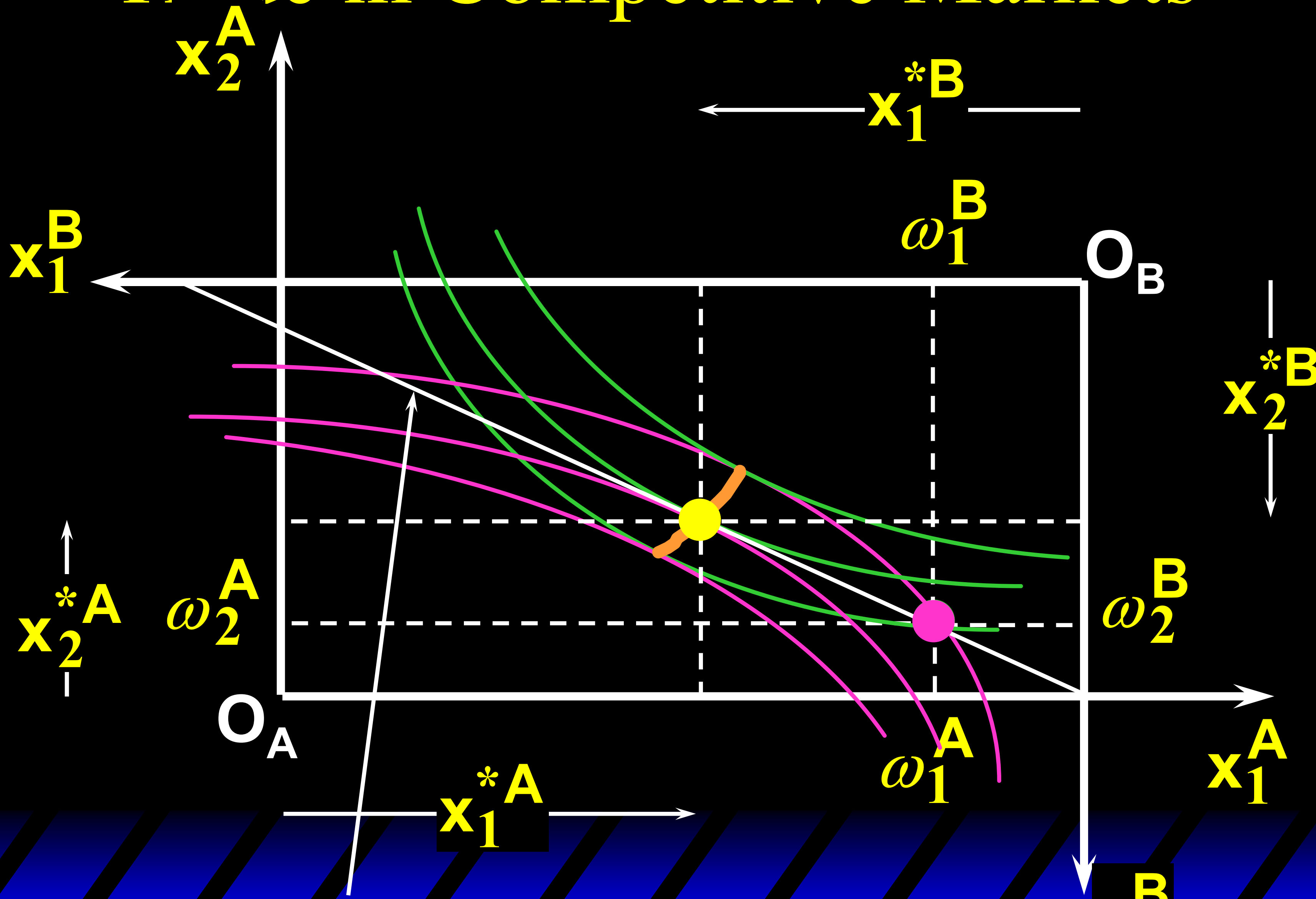


Trade in Competitive Markets



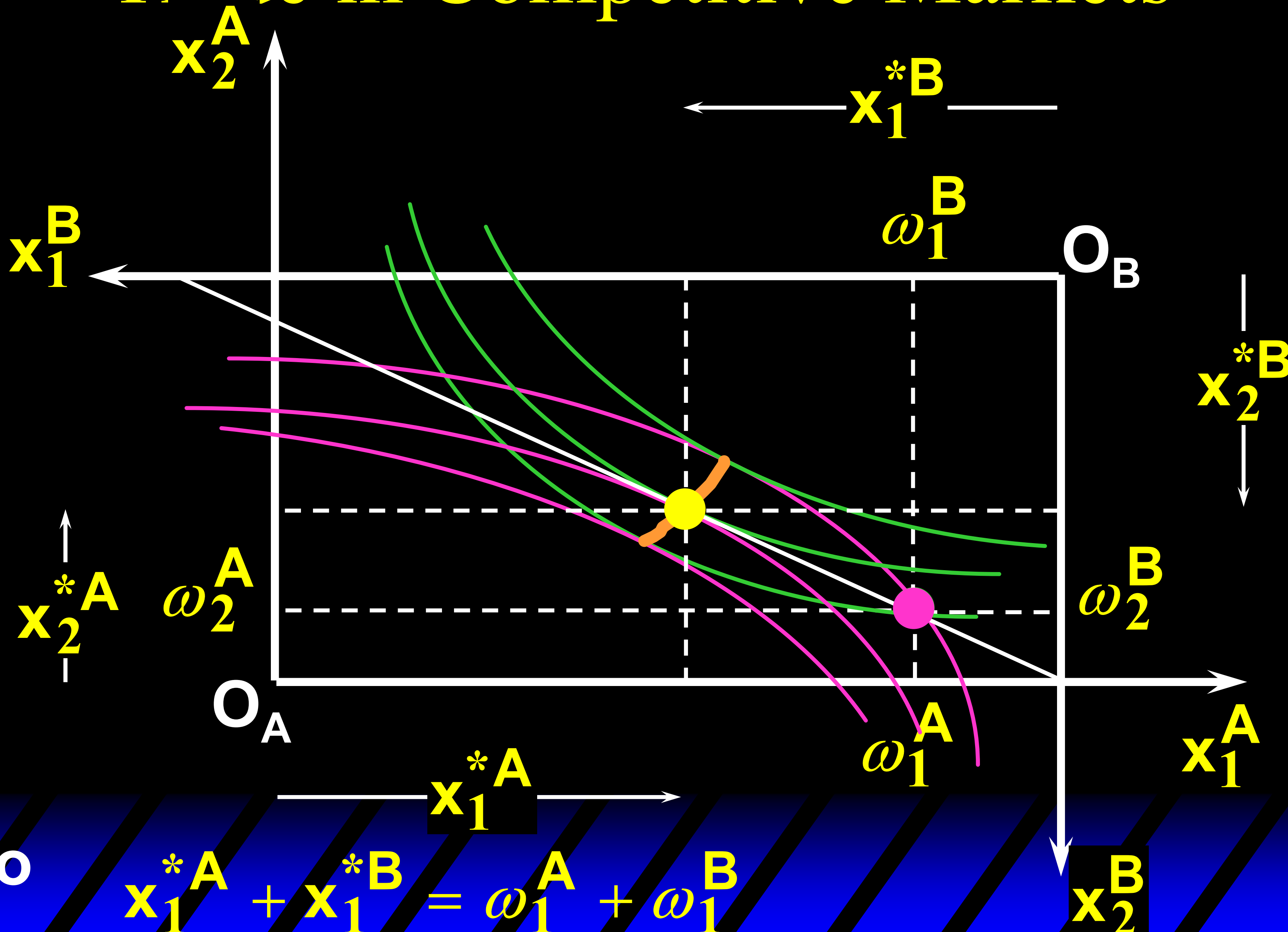
Budget constraint for consumer B

Trade in Competitive Markets



Budget constraint for consumer B

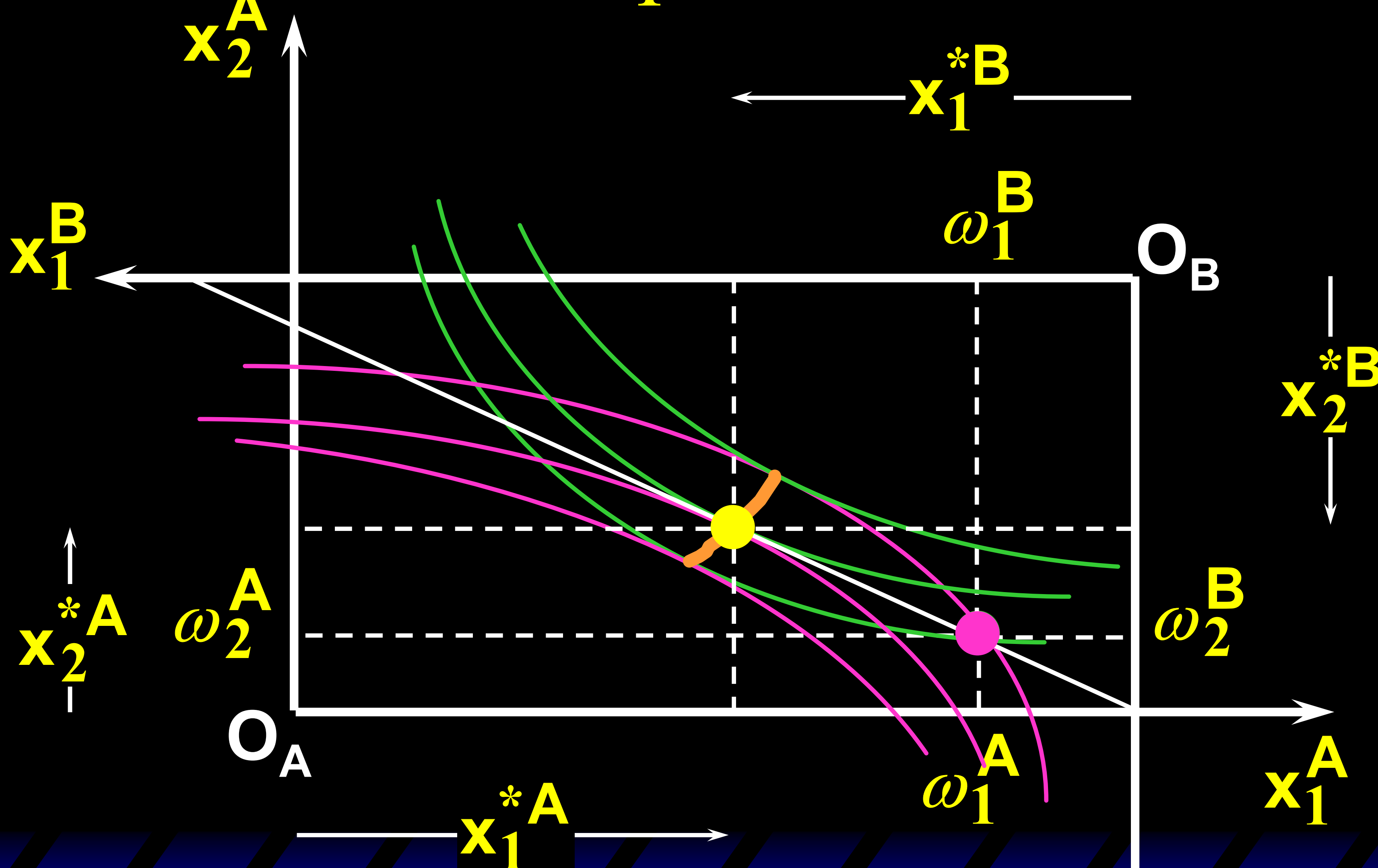
Trade in Competitive Markets



So

$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$$

Trade in Competitive Markets



and $x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B$

Trade in Competitive Markets

- At the new prices p_1 and p_2 both markets clear; there is a general equilibrium.
- Trading in competitive markets achieves a particular Pareto-optimal allocation of the endowments.
- This is an example of the **First Fundamental Theorem of Welfare Economics**.

First Fundamental Theorem of Welfare Economics

- Given that consumers’ preferences are well-behaved, an equilibrium in perfectly competitive markets yields a Pareto-optimal allocation of the economy’s endowment.

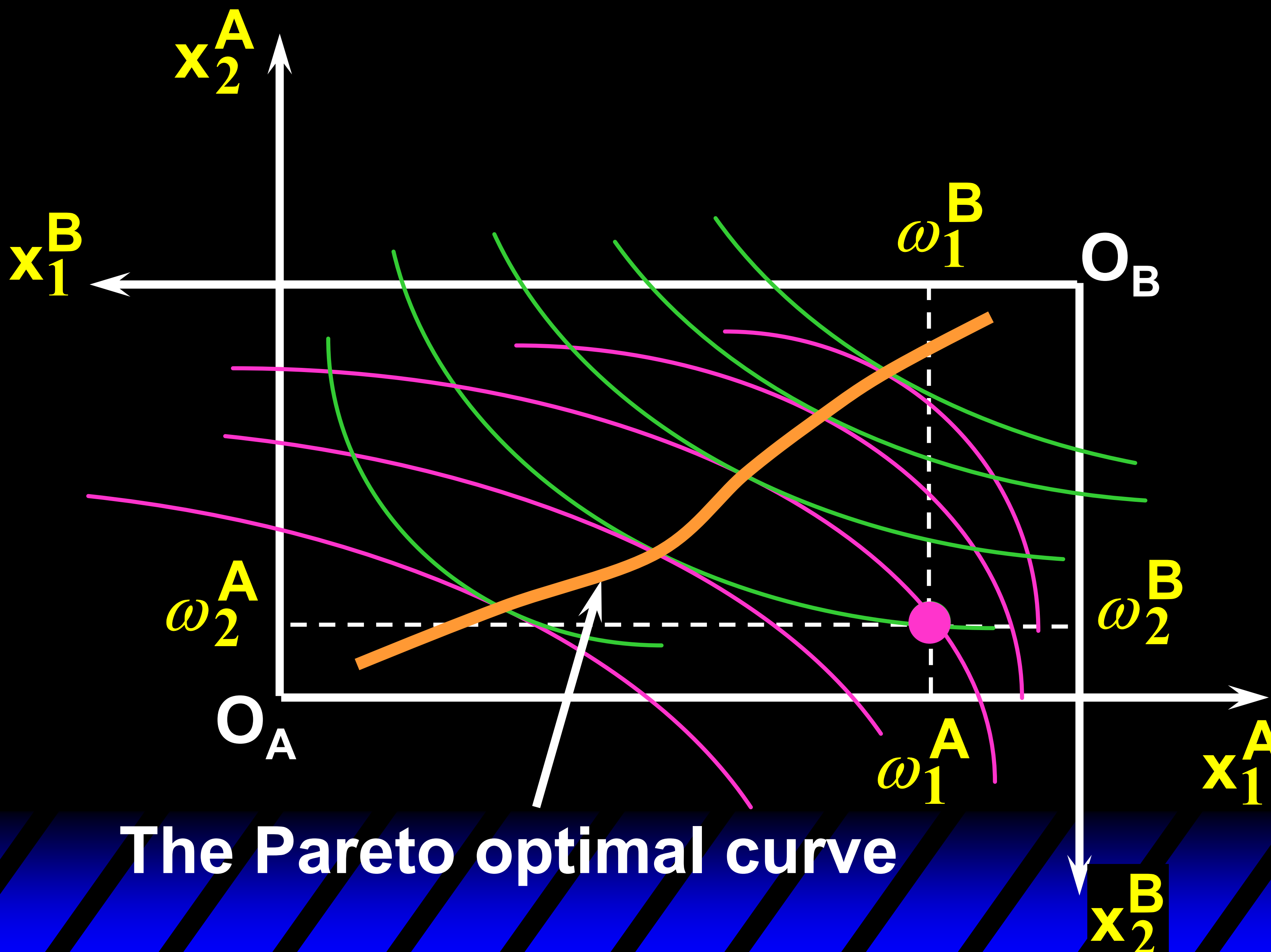
Second Fundamental Theorem of Welfare Economics

- **The First Theorem is followed by a second that states that any Pareto-optimal allocation can be achieved as an equilibrium allocation in competitive markets *provided* that endowments are first appropriately redistributed among the consumers.**

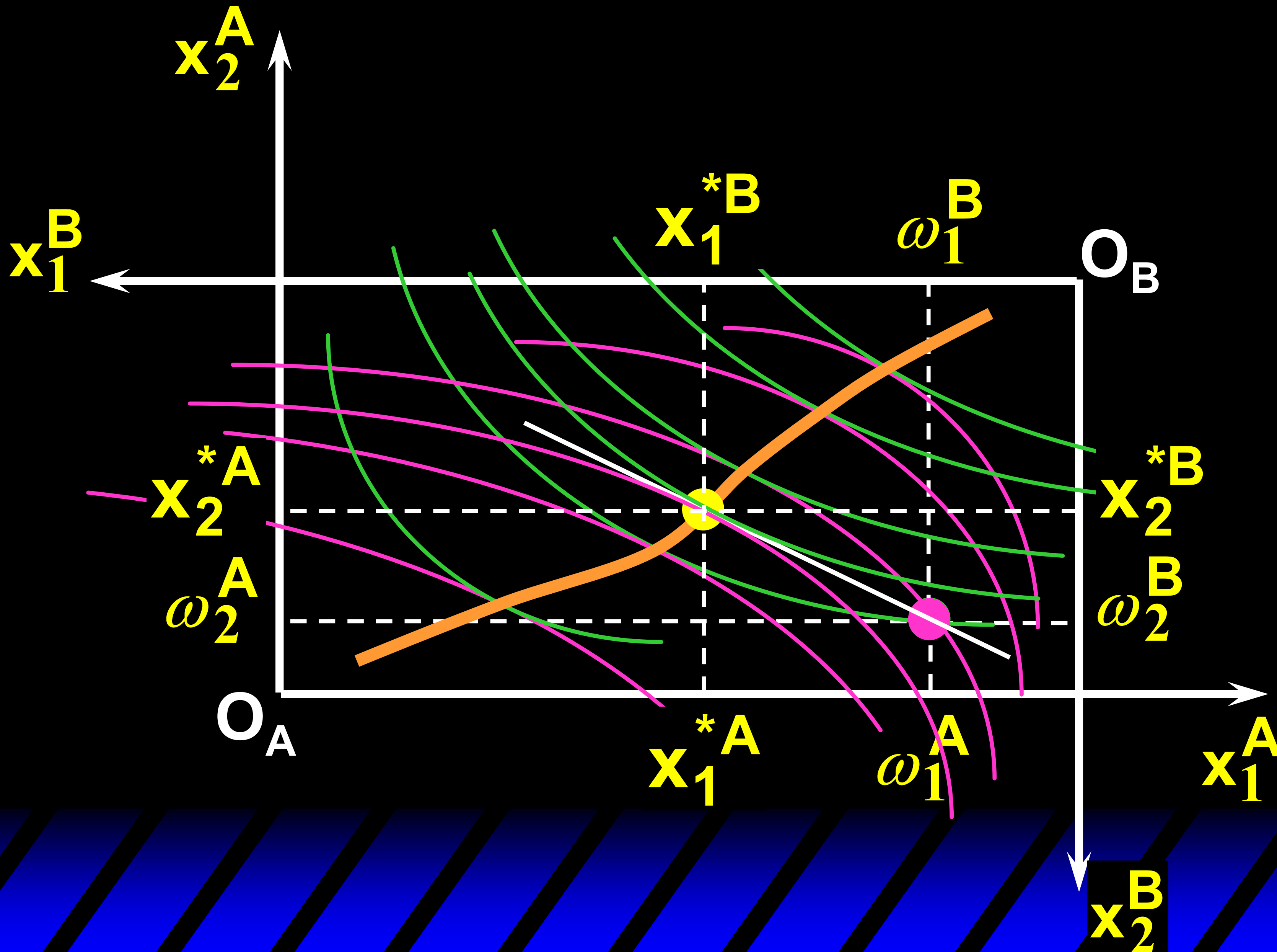
Second Fundamental Theorem of Welfare Economics

- **Given that consumers' preferences are well-behaved, for any Pareto-optimal allocation there are prices and an allocation of the total endowment that makes the Pareto-optimal allocation an equilibrium outcome in competitive markets.**

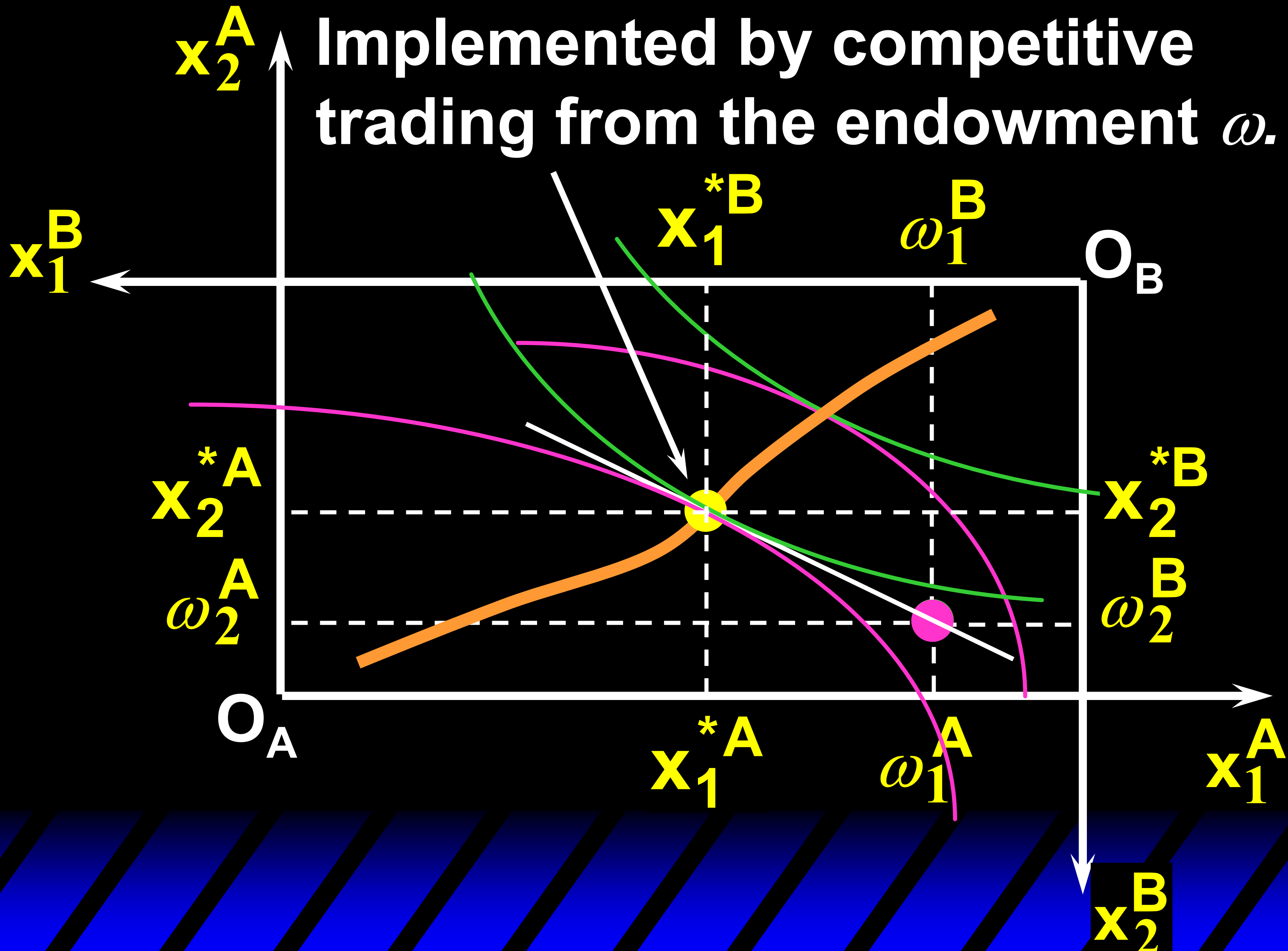
Second Fundamental Theorem



Second Fundamental Theorem

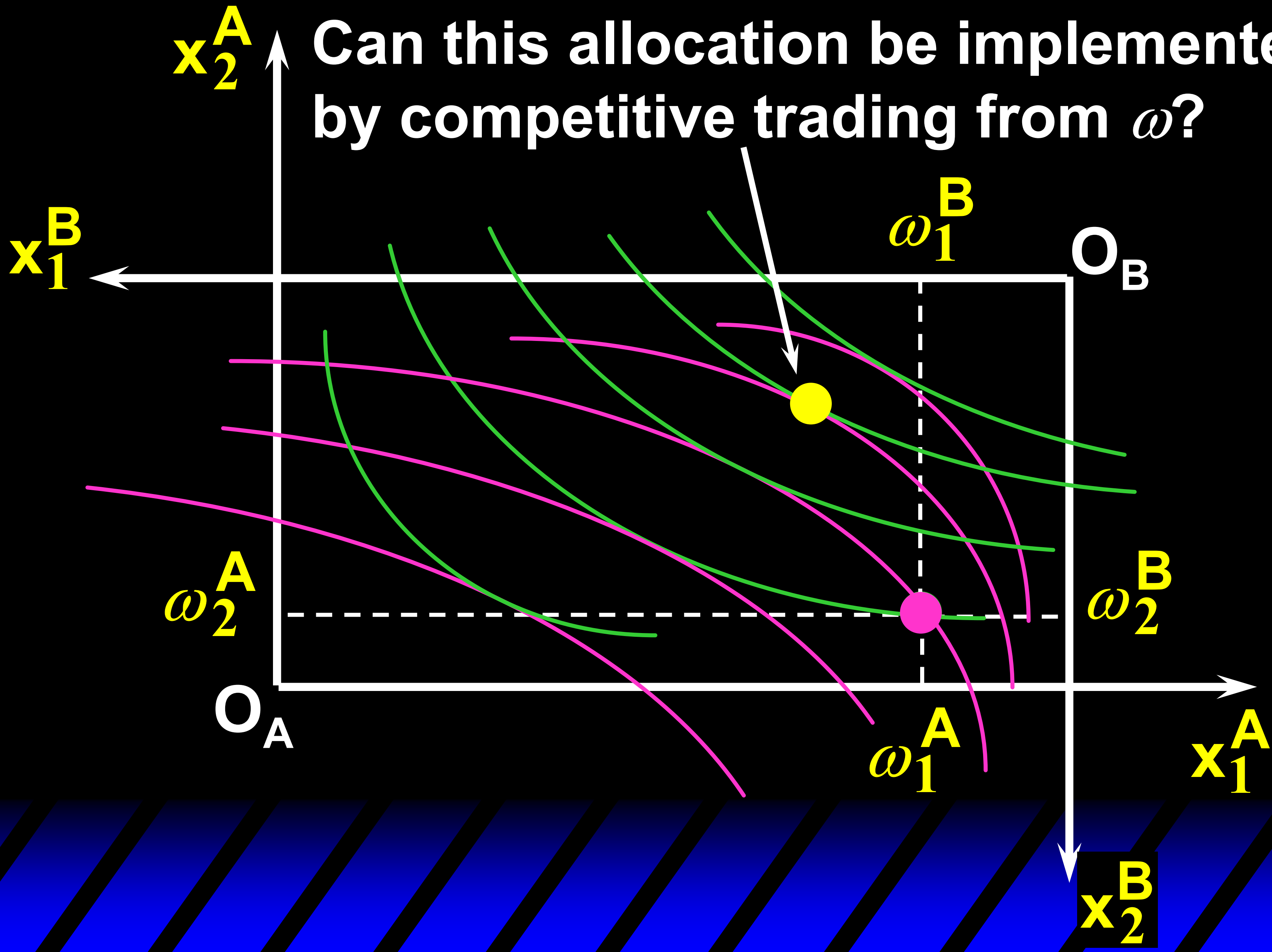


Second Fundamental Theorem



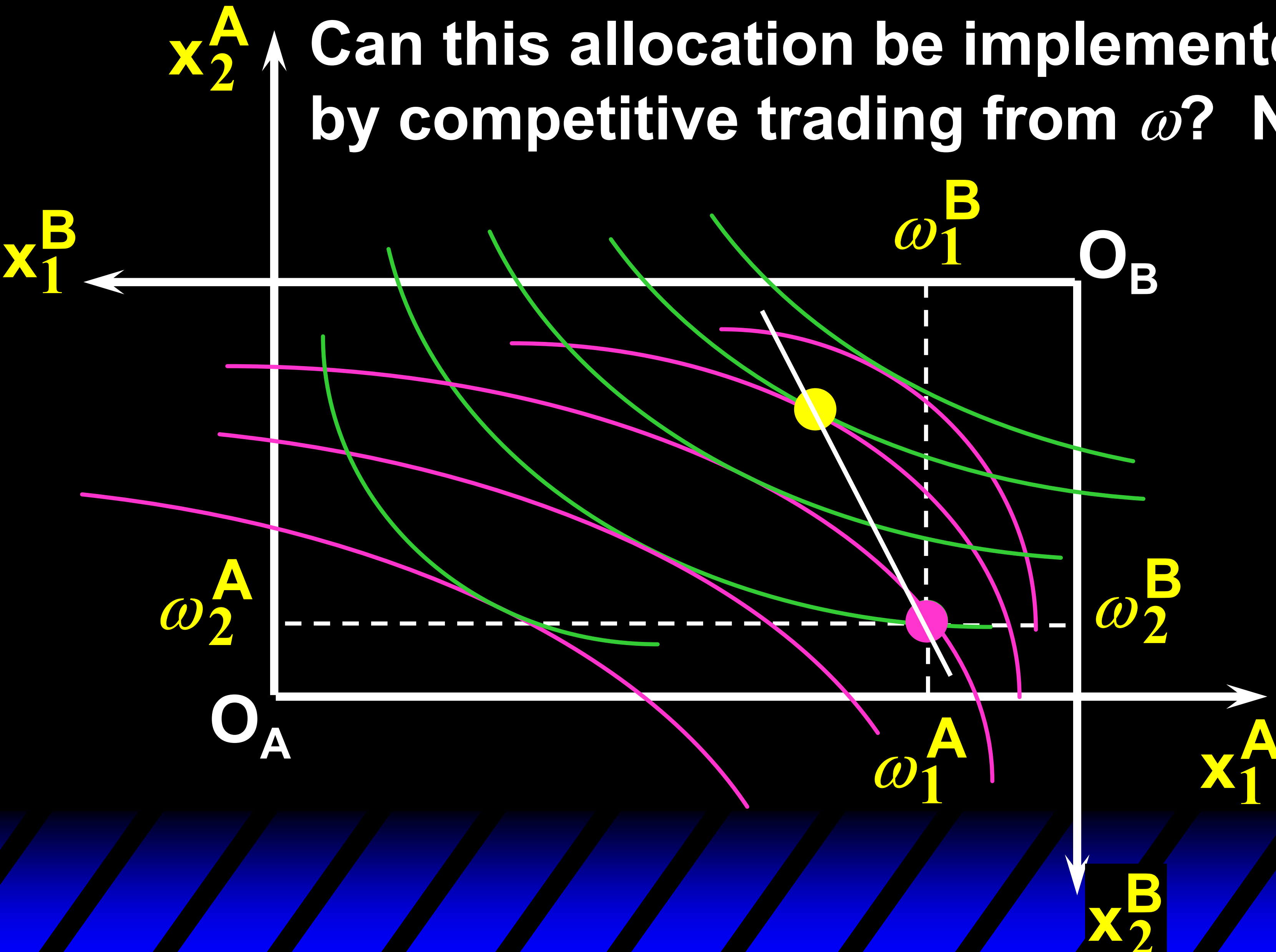
Second Fundamental Theorem

Can this allocation be implemented by competitive trading from ω?



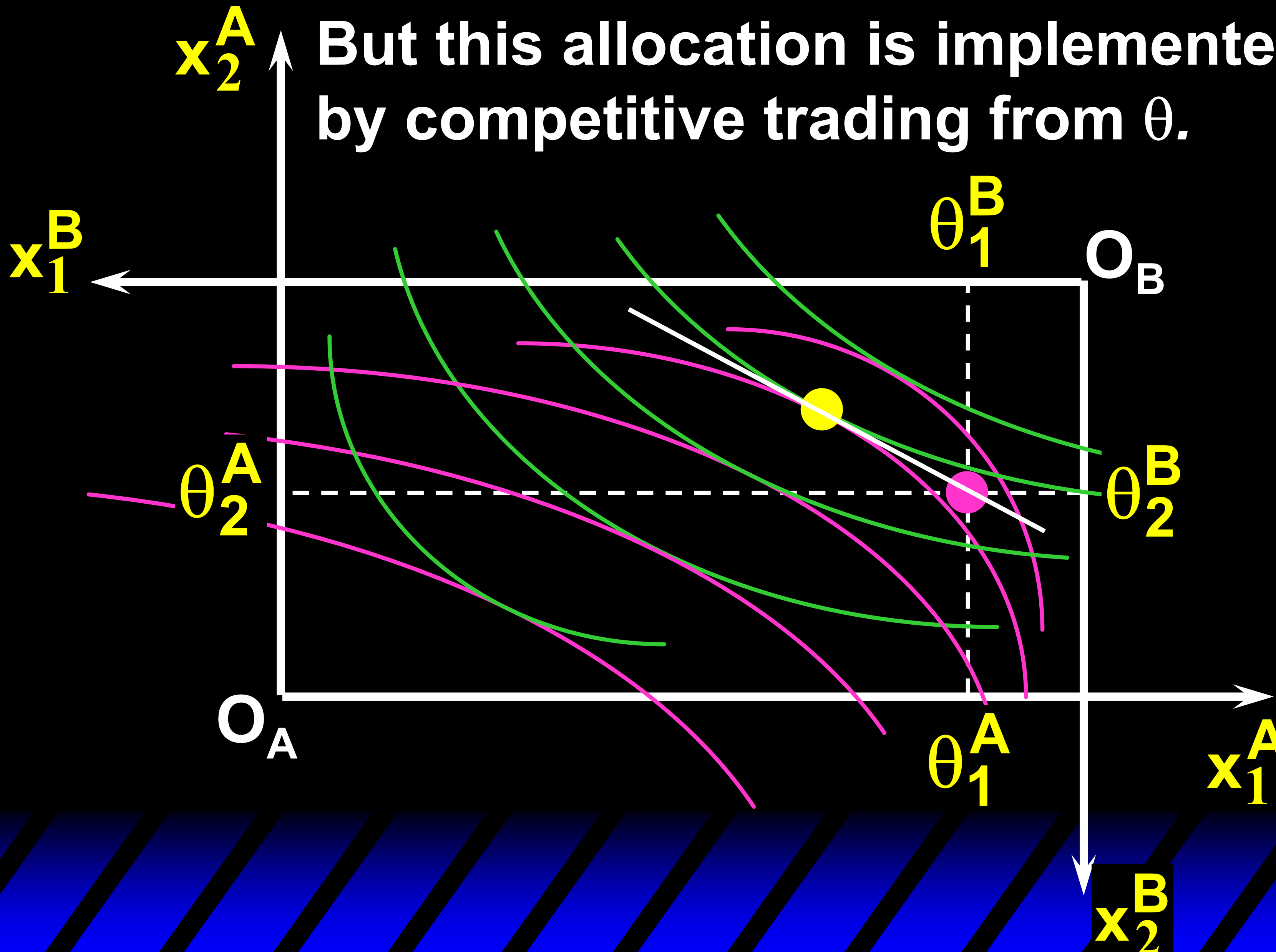
Second Fundamental Theorem

Can this allocation be implemented by competitive trading from ω? No.

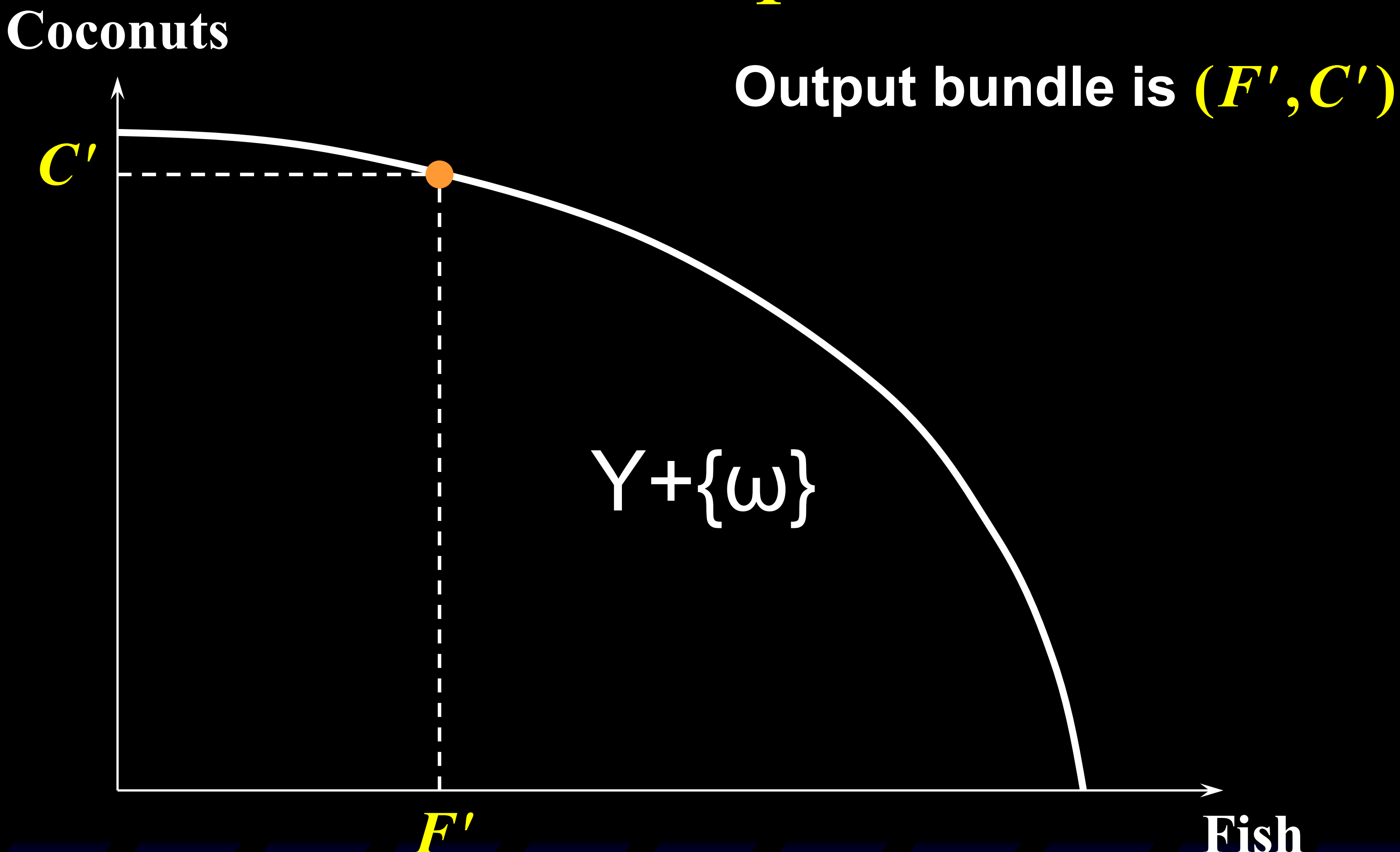


Second Fundamental Theorem

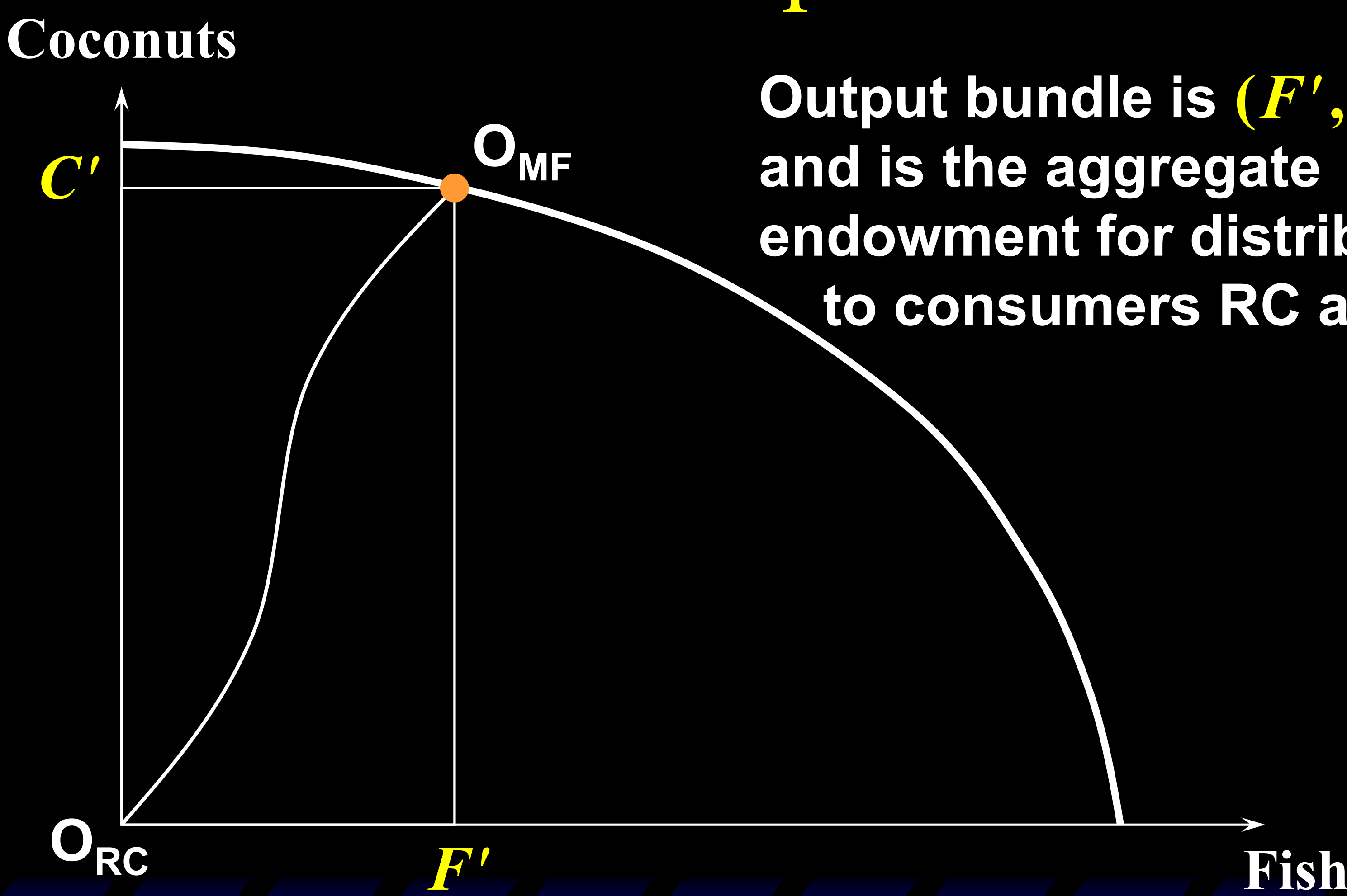
But this allocation is implemented by competitive trading from θ.



Coordinating Production & Consumption

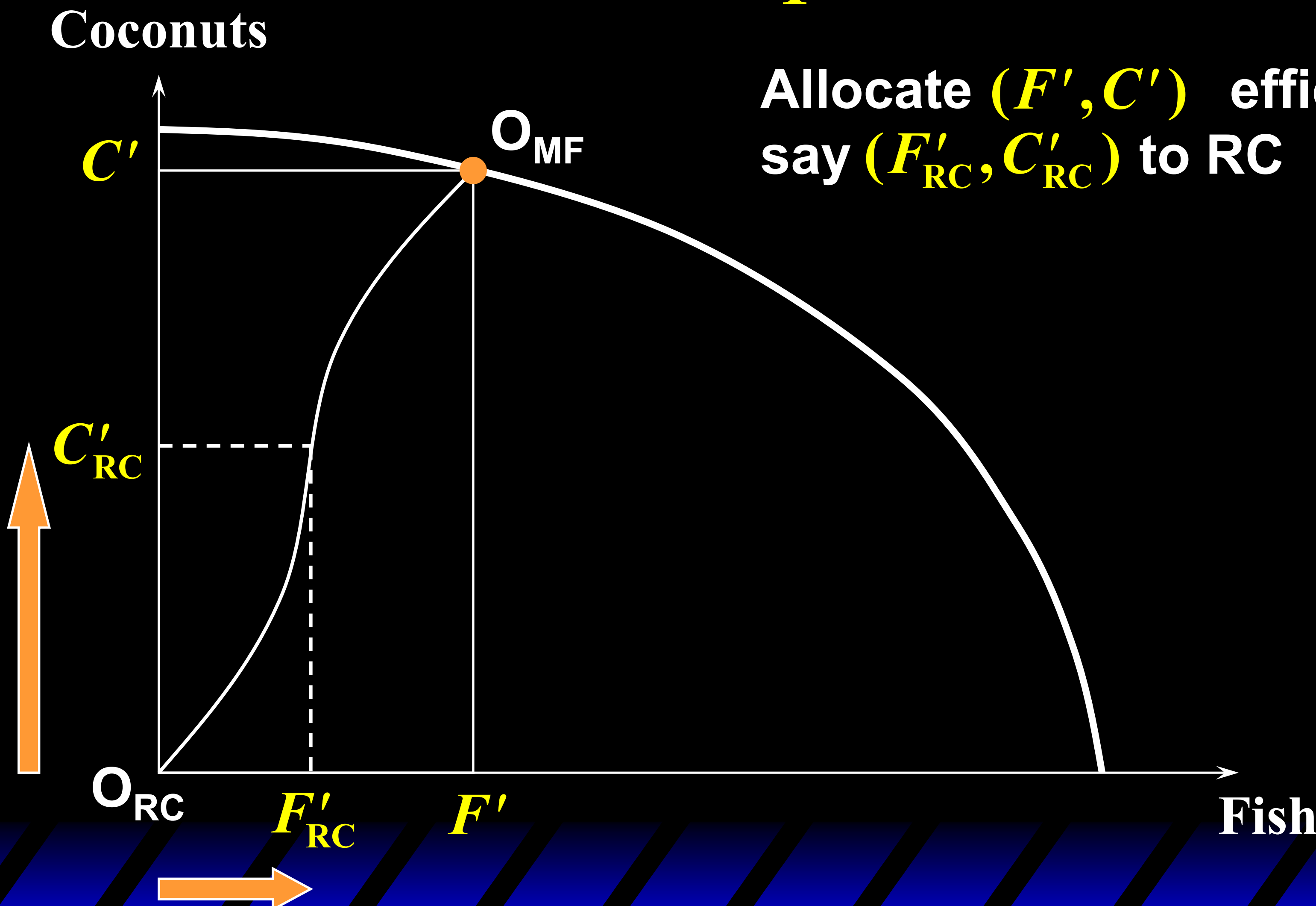


Coordinating Production & Consumption



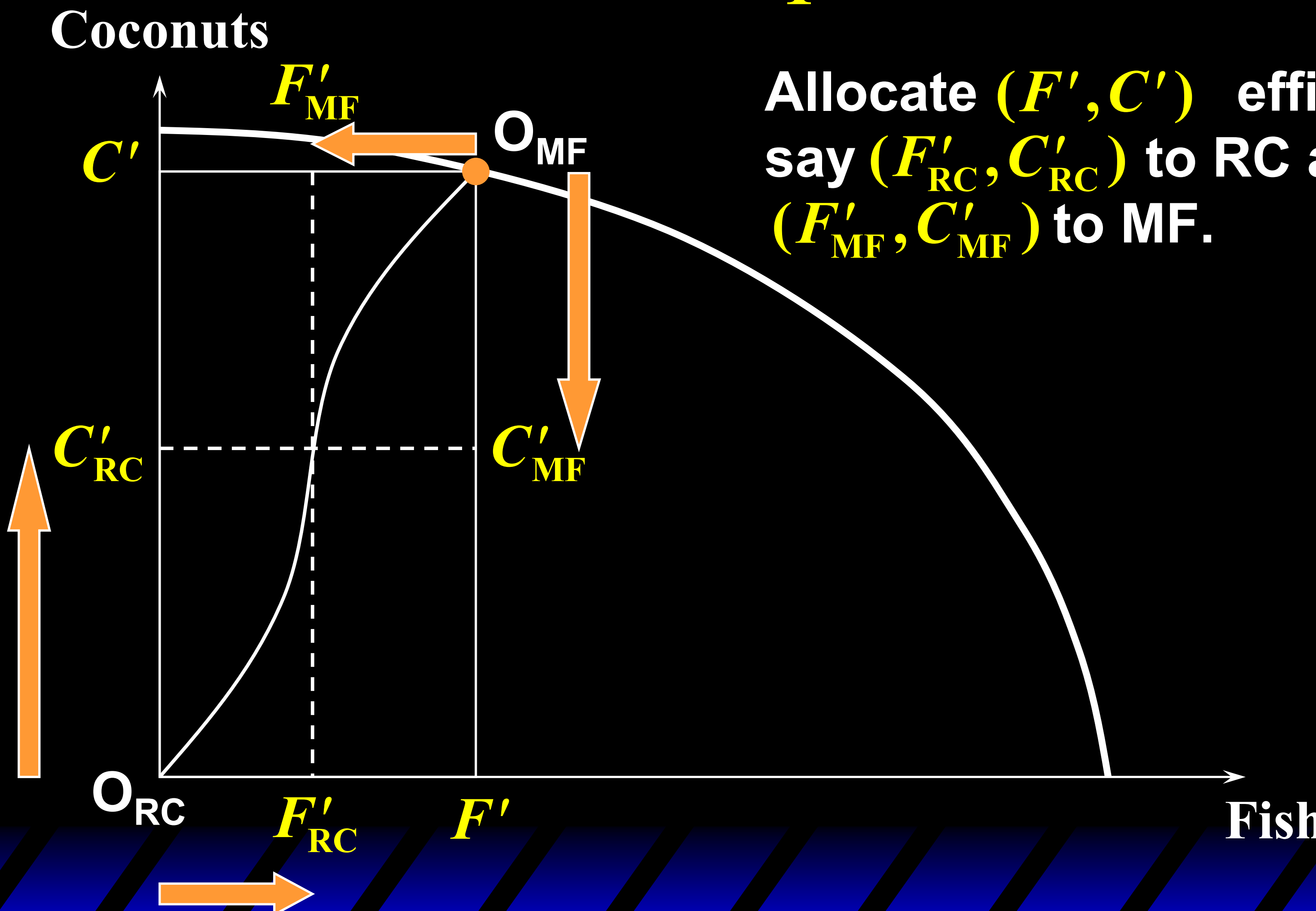
Output bundle is (F', C') and is the aggregate endowment for distribution to consumers RC and MF.

Coordinating Production & Consumption



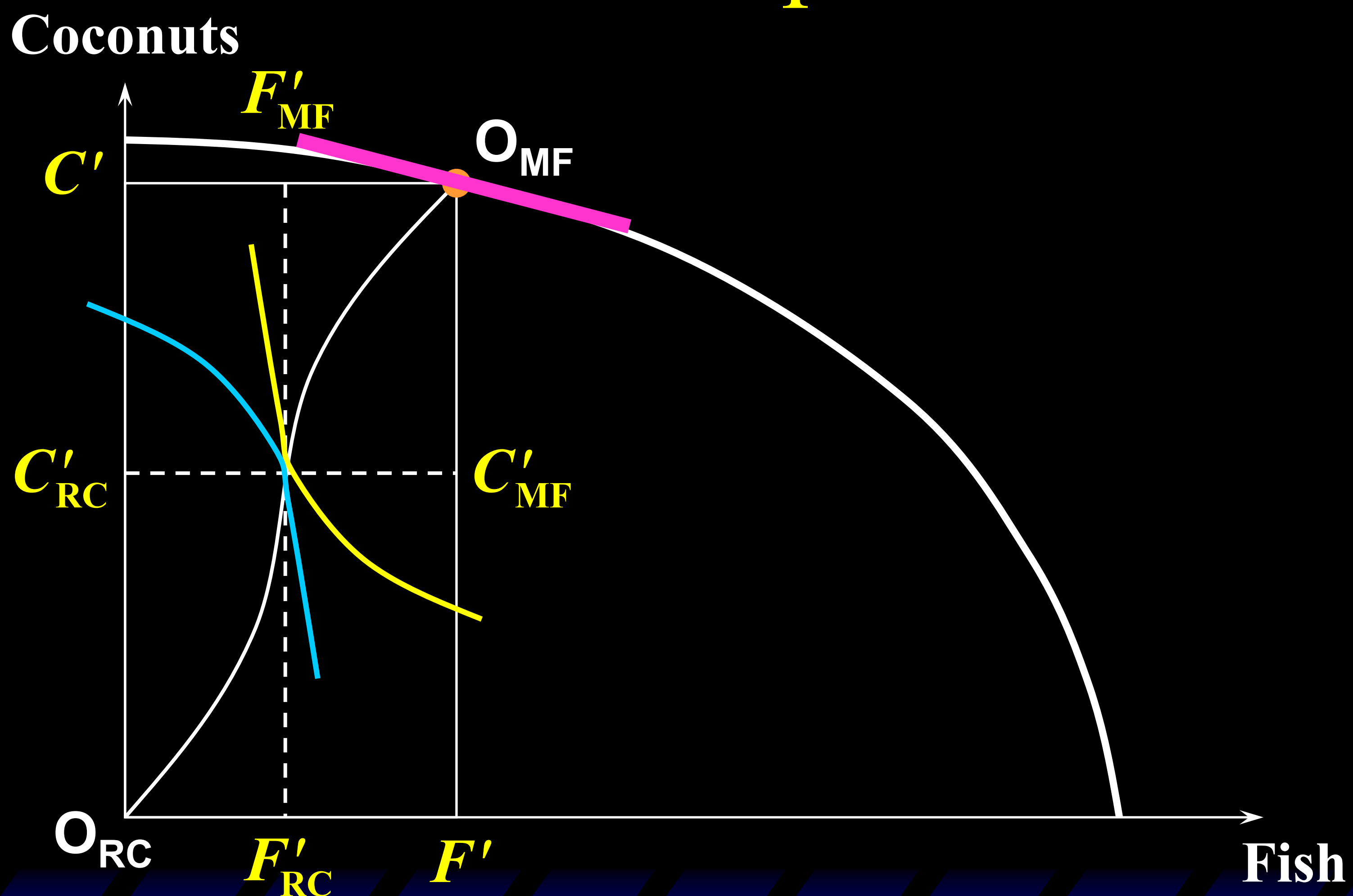
Allocate (F', C') **efficiently; say** (F'_{RC}, C'_{RC}) **to RC**

Coordinating Production & Consumption

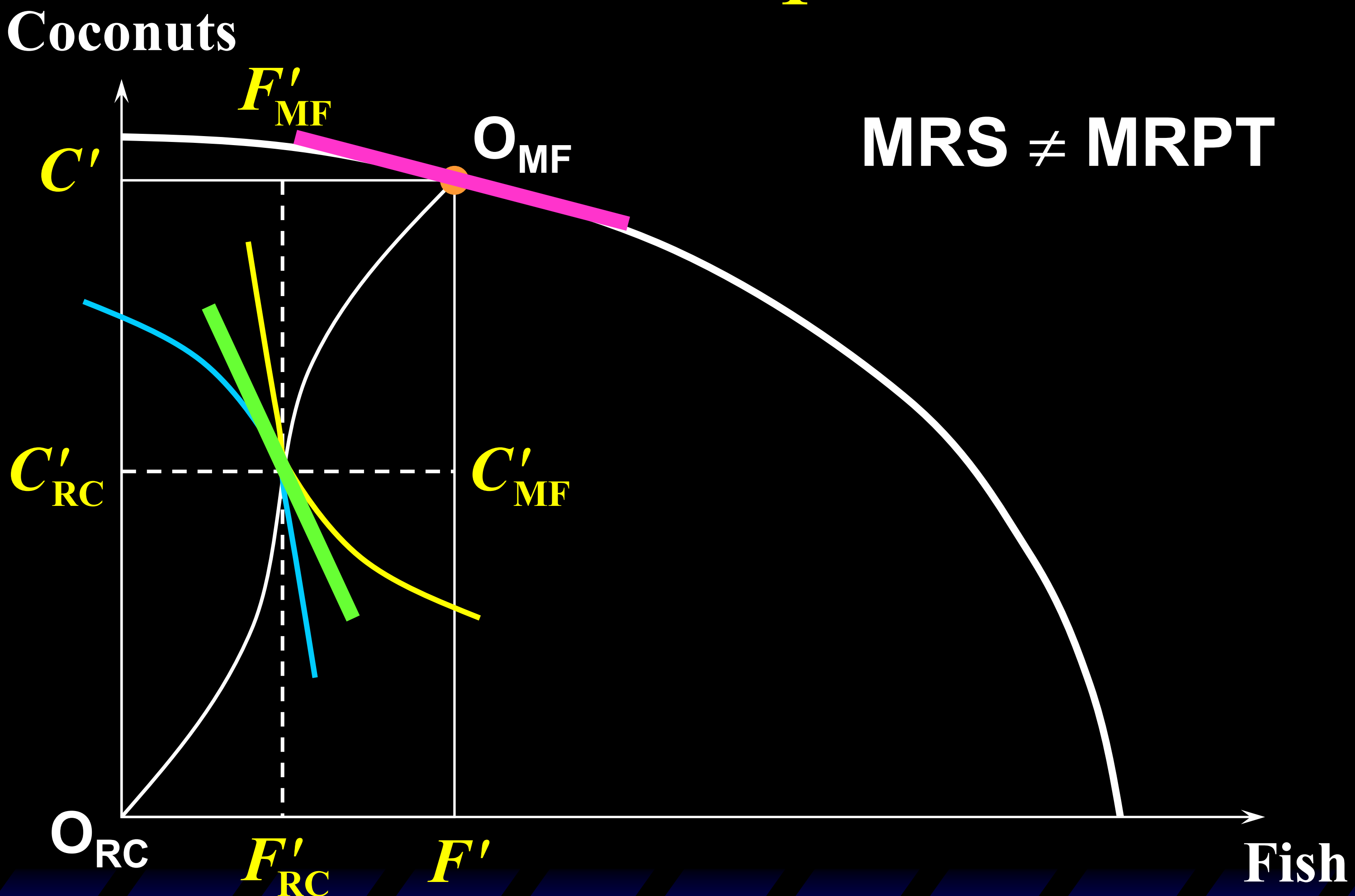


Allocate (F', C') efficiently; say (F'_{RC}, C'_{RC}) to RC and (F'_{MF}, C'_{MF}) to MF.

Coordinating Production & Consumption

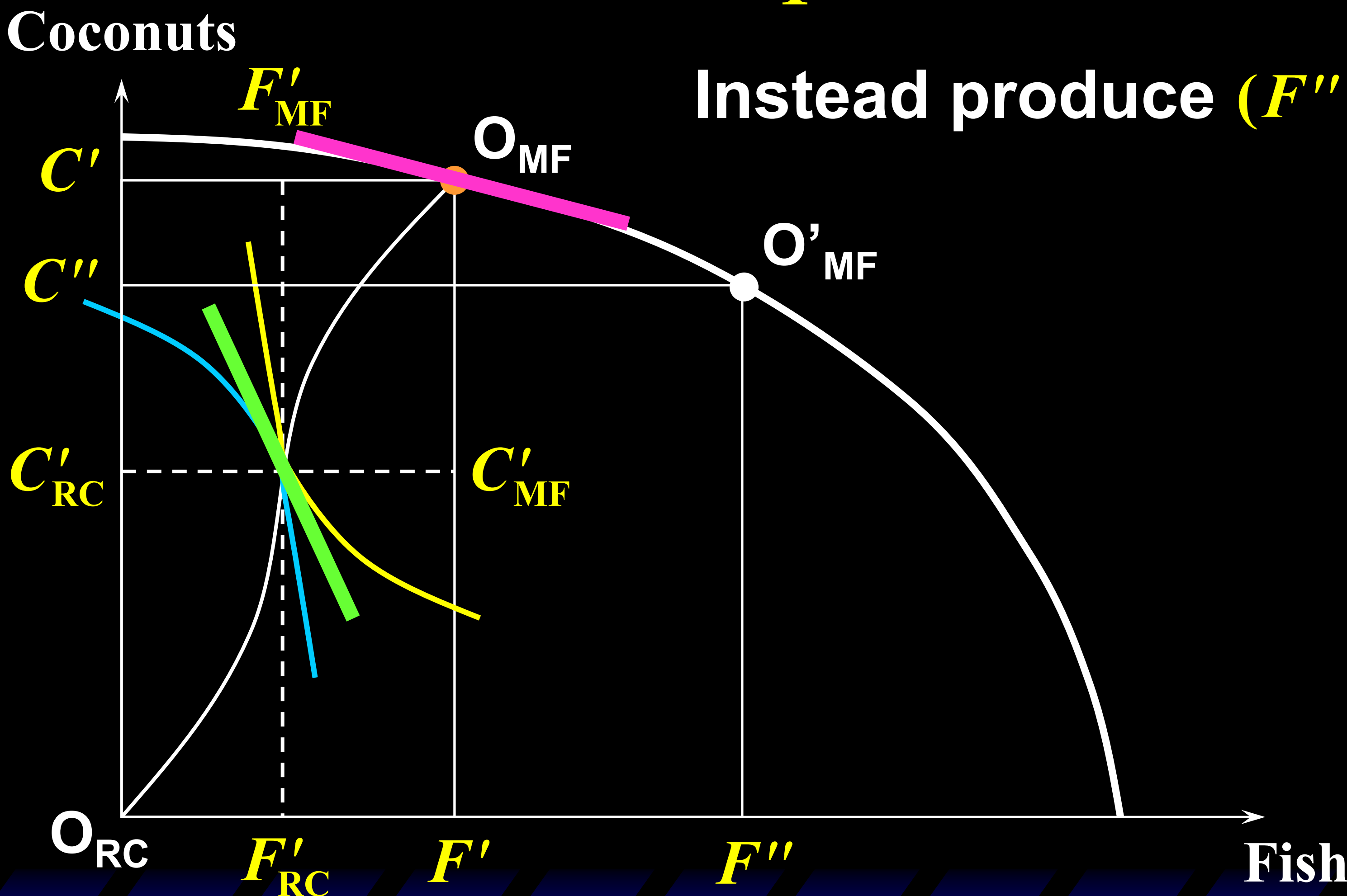


Coordinating Production & Consumption

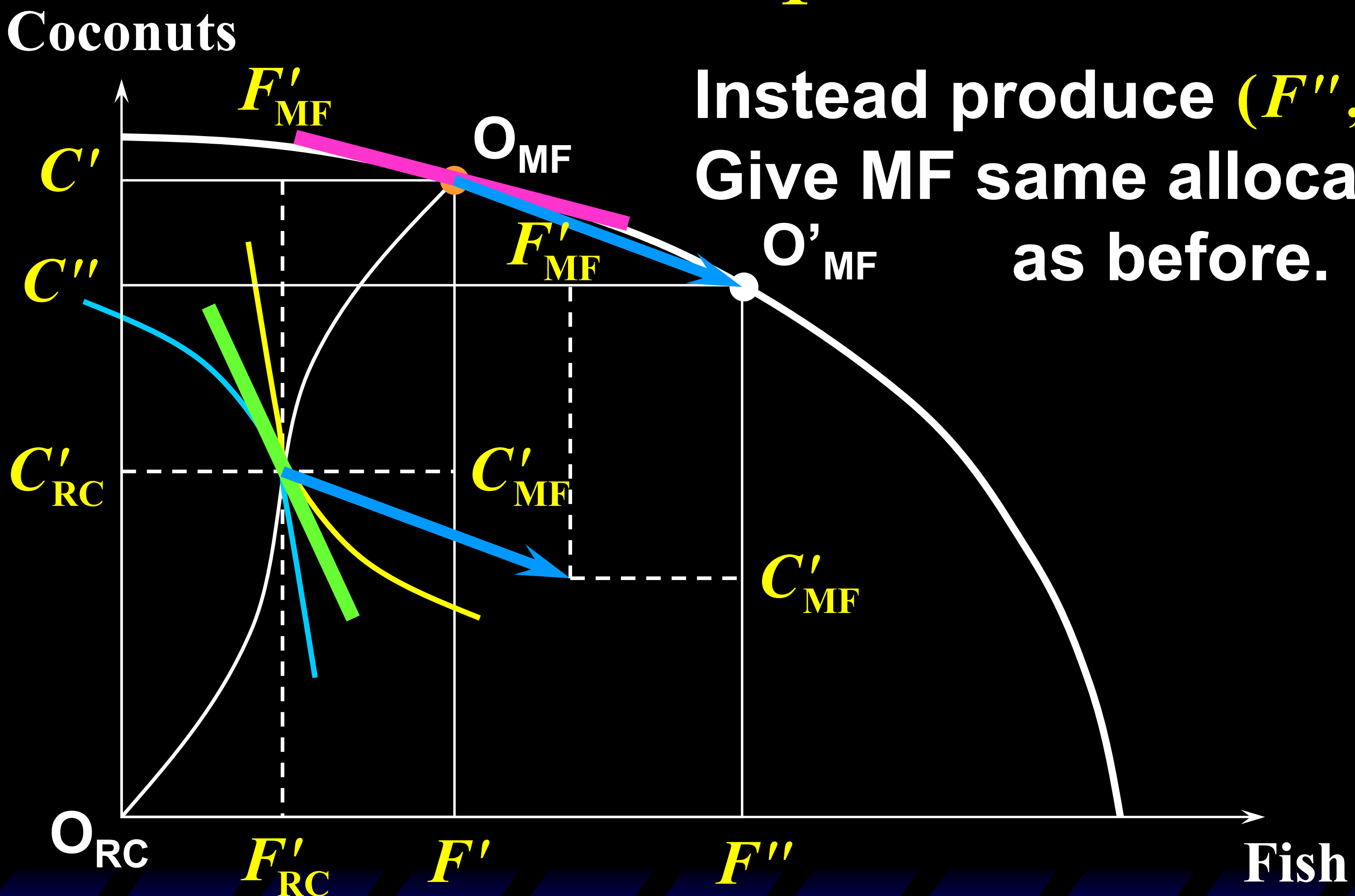


Coordinating Production & Consumption

Instead produce (F'', C'').

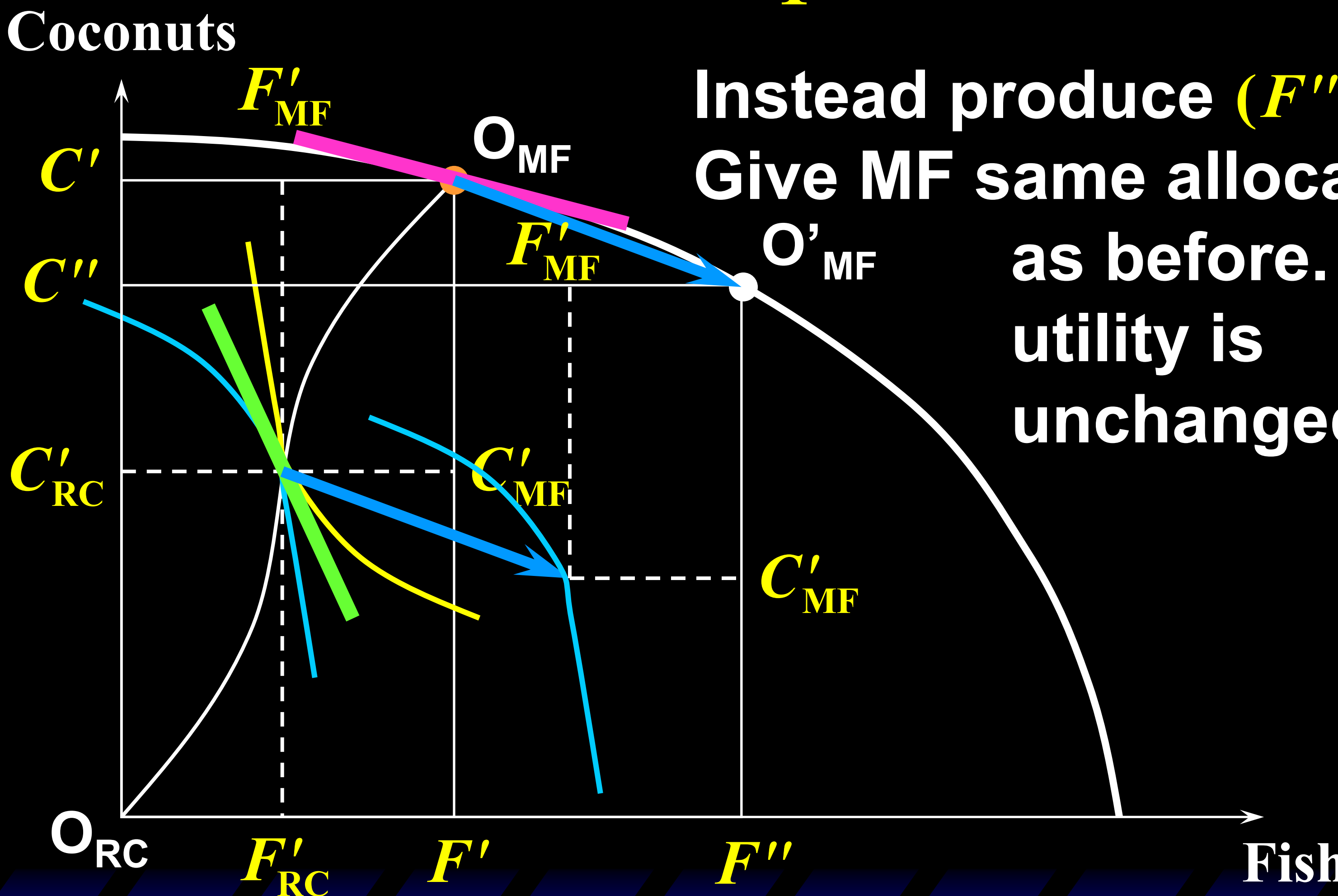


Coordinating Production & Consumption



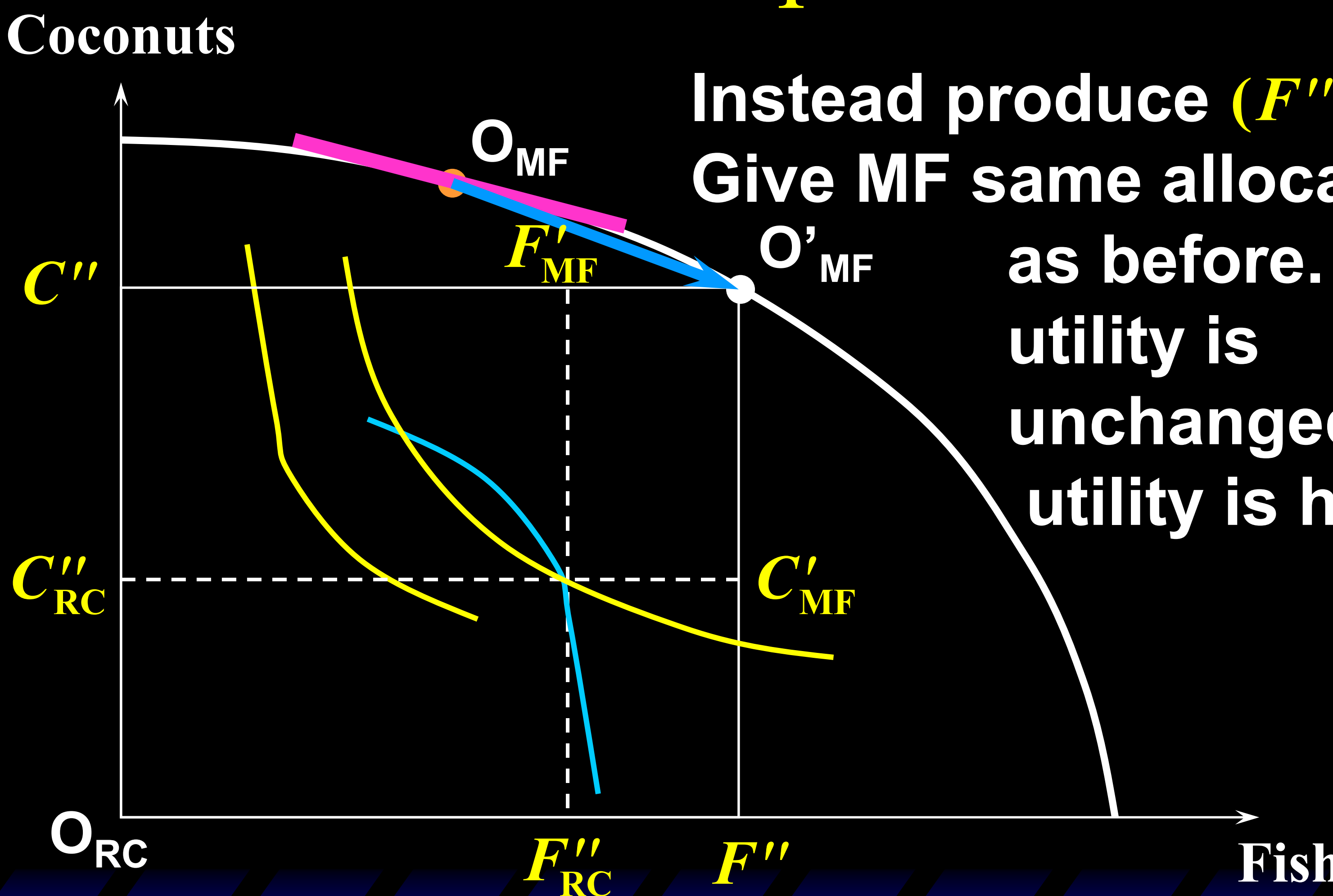
Instead produce (F'', C''). Give MF same allocation as before.

Coordinating Production & Consumption



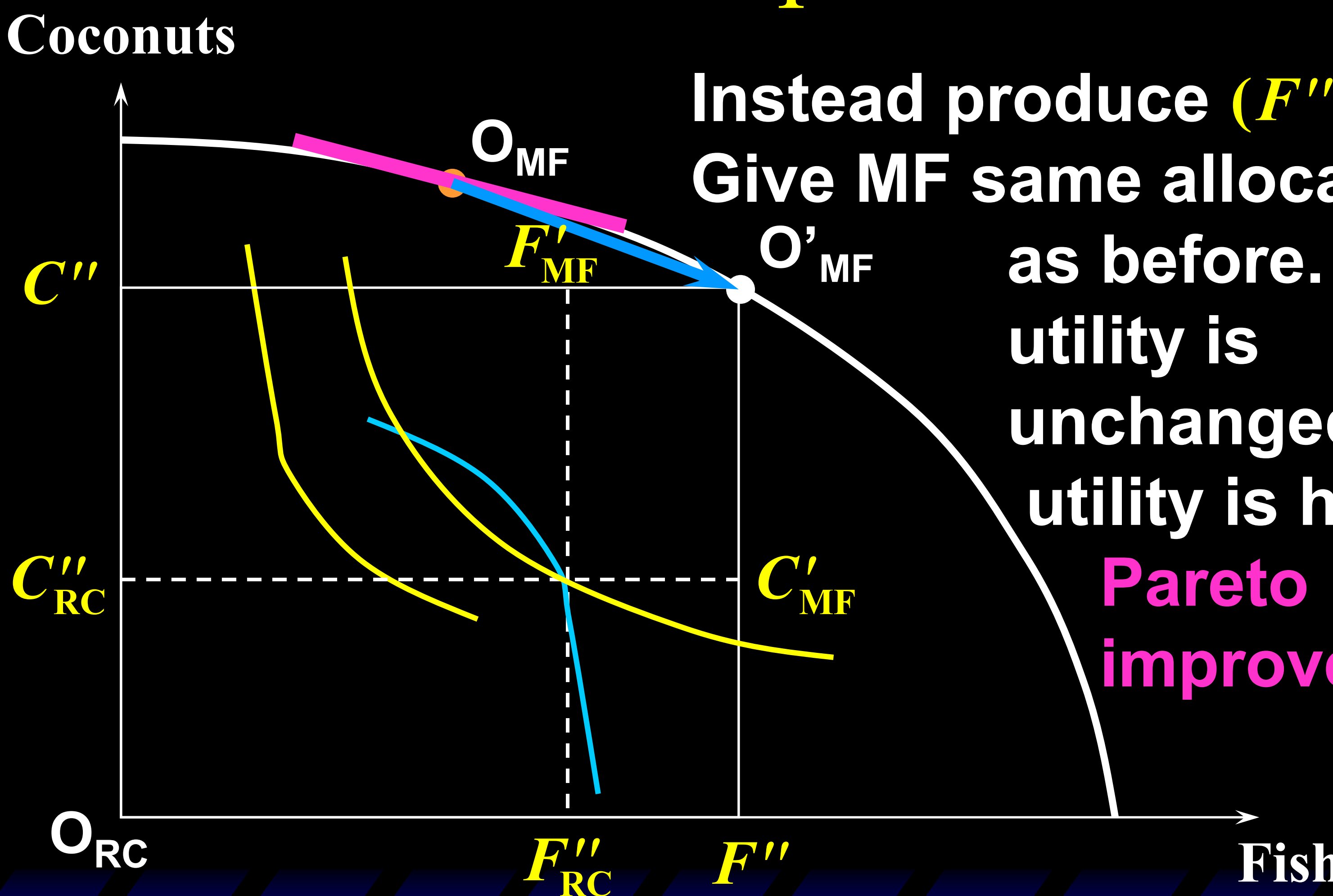
Instead produce (F'', C''). Give MF same allocation as before. MF's utility is unchanged.

Coordinating Production & Consumption



Instead produce (F'', C''). Give MF same allocation as before. MF's utility is unchanged, RC's utility is higher

Coordinating Production & Consumption



Instead produce (F'', C''). Give MF same allocation as before. MF's utility is unchanged, RC's utility is higher; Pareto improvement.

Coordinating Production & Consumption

- **MRS $\neq$ MRPT $\Rightarrow$ inefficient coordination of production and consumption.**
- **Hence, MRS = MRPT is necessary for a Pareto optimal economic state.**

Coordinating Production & Consumption