EC 206 Microeconomic Theory II Rajiv Vohra
March 6, 2003 2 hours 20 min.

1. State Roy's identity and prove it. Suppose the indirect utility function is $v(p, w) = \gamma + \ln w - \alpha \ln p_1 - (1-\alpha) \ln p_2$. Derive the demand functions for the two commodities.

   [15 points]

2. Suppose $u(x_1, x_2) = x_1 + 4\sqrt{x_2}$. The consumption set is R^2_+. Compute the demand function taking into account possible corner solutions. (The demand function is specified as one functional form in case a certain condition holds, and another if this condition does not hold.) Is the demand function continuous at $p = (1, 1)$ and $w = 1$?

   [15 points]

   Now suppose that commodity 1 is perfectly divisible but commodity 2 can only be consumed in integer amounts. The consumption set is therefore $X = R_+ \times K$, where K is the set of non-negative integers. The utility function is the same as before, but of course, it is only to be used to compare points in X. Show that in this case the demand function is not continuous at $p = (1, 1)$ and $w = 1$. Hint: Draw a figure with some budget lines close to the one with $p = (1, 1)$ and $w = 1$; identify $x((1, 1), 1)$; and construct a sequence $p^k \to p$ and $w^k \to w$ such that $x(p^k, w^k)$ does not converge (in R^2) to $x((1, 1), 1)$.

   [10 points]

3. Suppose the consumption set is R^L_+ and for all $p \gg 0$ and $w > 0$, the consumer has a unique demand, i.e., there exists a well-defined demand function $x(p, w)$. Assume that preferences are continuous in the sense that all better-than sets are open. Prove that for $p \gg 0$ and $w > 0$ the demand function is continuous in p. In other words, letting $x^k = x(p^k, w)$ for all k, prove that if $p^k \to p$, $w > 0$, $p \gg 0$, and $x^k \to x^*$, then $x^* = x(p, w)$. [Hint: First prove that x^* is affordable at (p, w), and then (using a proof by contradiction) show that there does not exist $x' \succ x^*$ and affordable at (p, w).]

   [30 points]

4. Suppose there are I consumers, and x_i denotes the demand of consumer i at prices p and wealth w_i (obtained by utility maximization). Let $\bar{x} = \sum_{i \in I} x_i$ be the aggregate demand. All consumers face the same prices p, although their wealths and preferences could differ. (There are L commodities, so $\bar{x} \in R^L$). A benevolent planner has the power to re-allocate $\bar{x}$ among the consumers but does not have additional resources. In other words, he can change the original allocation, $(x_i)_{i \in I}$, to some other allocation $(x'_i)_{i \in I}$, provided $\sum_i x'_i = \bar{x}$. His aim is to find a feasible re-allocation, (x'_i), such that $\sum_{i \in I} x'_i = \bar{x}$ and $u_i(x'_i) > u_i(x_i)$ for all $i \in I$. (This kind of change is known as a weak Pareto improvement). Prove that there does not exist a weak Pareto improvement.

[15 points]

A feasible re-allocation, (x'_i), (with $\sum_i x'_i = \bar{x}$) is said to be a Pareto improvement if $u_i(x'_i) \geq u_i(x_i)$ for all $i \in I$ and $u_j(x'_j) > u_j(x_j)$ for at least one $j \in I$. Prove that there does not exist a Pareto improvement over $(x_i)_{i \in I}$. (As before x_i is consumer i's demand at prices p and wealth w_i). You will need to make an assumption on the preferences to prove this. State this assumption clearly, and provide an example in which this assumption is not satisfied, and the conclusion does not hold. A two-consumer example with two commodities, illustrated with a couple of figures should suffice. [15 points]