Asymmetric Information and the Core: The Interim Stage

Rajiv Vohra

The Interim Stage without Incentive Constraints: Wilson's Coarse Core

At the ex ante stage, asymmetry of information plays a role only through incentive constraints.

Most economic problems of asymmetric information involve contracting problems at the interim stage. This is the essence of adverse selection.

But, for the interim stage, there isn't an obvious definition of the core.

What is the set of players: n, $\sum_i |T_i|$ or $|T|$?

What is the characteristic function?

What is the meaning of a coalitional improvement?

Should it require all types of all agents to gain? As Holmström and Myerson argue, this is indeed the correct way to define an improvement for the grand coalition. This may be the only way for an uninformed outsider to verify a Pareto improvement.

But this notion of domination should not be mimicked in defining objections in a coalition which is not the grand coalition.

For example, consider the coalition consisting of agent i alone.

Since i knows her type, say t_i, surely i will ‘object’ to a status-quo if she is better-off (an increase in $U_i(. \mid t_i)$) with her own endowment.

In other words, for an objection from a singleton coalition, $\{i\}$, it suffices that some type (not necessarily all types) of agent i can do better with her endowment - the standard notion of interim individual rationality; e.g. Myerson (1991).

The statement that all agents of all types can be made better-off turns out to be essentially equivalent to the statement that there is an informational event $E \subseteq T$ which is common knowledge to all i and all agents of all types in E can be made better-off (over E).

And the statement that agent i of type t_i is better-off means that there is an informational event known to i (common knowledge to i) over which she is better-off.

This idea, of an 'interim objection' by a coalition being common knowledge among members of the coalition, is the basis for the notion of the coarse core defined by Wilson (1978).

For an event $E \subseteq T$ let E_i denote the corresponding set of types for agent i, i.e., $E_i = \{t_i \mid t \in E\}$. An event $E \subseteq T$ is said to be common knowledge for S if

$$q(t'_{-i} \mid t_i) = 0 \text{ for all } i \in S,\ t_i \in E_i \text{ and } (t'_{-i}, t_i) \notin E.$$

	t_1	t_2	.	.	t_k	t_{k+1}	.	.	t_m
s_1						0	0	0	0
s_2			E			0	.	.	0
.						0	.	.	0
s_j						0	0	0	0
s_{j+1}	0	0	0	0	0				
.	0	.	.	.	0			F	
s_q	0	0	0	0	0				

E and F are common knowledge events for these two agents.

Based on the fact that all agents within a coalition can discern a common knowledge event, a coarse objection can be directed at any such event. An objection over event E requires all types in E_i to gain in interim utility.

Let $\mu \in \mathcal{F}$ be a feasible mechanism. S has a coarse objection to μ if there exists an event E which is common knowledge for S and a mechanism $\nu_S \in \mathcal{F}_S$ such that

$$U_i(\nu_S \mid t_i) > U_i(\mu \mid t_i) \qquad \forall i \in S, \forall t_i \in E_i.$$

(Note that with this notion of dominance, there is no loss of generality in restricting attention to common knowledge events for S which are of the form $E = \prod_{i \in S} E_i \times \prod_{j \notin S} T_j$.)

The coarse core is the set of all feasible mechanisms to which no coalition has a coarse objection.

Proposition 1 *(Wilson) The coarse core is non-empty.*

The Incentive Compatible Coarse Core

Suppose we accept Wilson's notion of the core but also require that the relevant mechanism satisfy incentive compatibility (as well as the measurability conditions). This leads to the notion of the core in Vohra (1999).

For an incentive compatible and feasible mechanism $\mu \in \mathcal{F}^*$, S has an incentive compatible coarse objection if there exists an event E which is common knowledge for S and a mechanism $\nu_S \in \mathcal{F}^*_S$ such that

$$U_i(\nu_S \mid t_i) > U_i(\mu \mid t_i) \qquad \forall i \in S, \forall t_i \in E_i.$$

The incentive compatible coarse core consists of all incentive compatible and feasible mechanisms to which no coalition has an incentive compatible coarse objection.

Information is non-exclusive if for every $i \in N$ and $t \in T^*$,

$$\text{for every } t \in T^*,\ q(t_{-i}, s_i) = 0 \text{ for all } i \in N \text{ and } s_i \neq t_i,$$

or

$$\forall t \in T^*, \quad \forall i \in N, \quad q(t_i \mid t_{-i}) = 1.$$

Example: Suppose $N = \{1, 2, 3\}$, $T_i = \{s_i, s_i')$ for all $i \in N$ and

$$T^* = \{(s_1, s_2', s_3'), (s_i', s_2, s_3'), (s_1', s_2', s_3)\}.$$

	s_2	s_2'
s_1	0	0
s_1'	0	

s_3

	s_2	s_2'
s_1	0	
s_1'		0

s_3'

Proposition 2 *(Vohra (1999)) If information is non-exclusive, then the incentive compatible coarse core is non-empty.*

Proof: Consider x in Wilson's coarse core, and let

$$\tilde{x}(t) = \begin{cases} x(t) & \text{if } t \in T^* \\ e(t) & \text{otherwise} \end{cases}$$

Since $u_i(\tilde{x}_i, t) = u_i(x, t)$ for all $t \in T^*$, it follows that

$$U_i(\tilde{x}_i|t_i) = U_i(x_i|t_i) \text{ for all } i \in N \text{ and } t_i \in T_i$$

Since there is no coarse objection to x there cannot be one against $\tilde{x}$. In particular, there is no incentive compatible coarse objection to $\tilde{x}$.

To complete the proof we show that $\tilde{x}$ is incentive compatible. Consider $s_i \in T_i$ such that $s_i \neq t_i$.

$$\begin{aligned} U_i(\tilde{x}_i, s_i \mid t_i) &= \textstyle\sum_{t_{-i}\in T_{-i}} q_i(t_{-i}|t_i)u_i(\tilde{x}_i(t_{-i}, s_i), t) \\ &= \textstyle\sum_{\{t_{-i}\in T_{-i}|(t_{-i},t_i)\in T^*\}} q_i(t_{-i}|t_i)u_i(\tilde{x}_i(t_{-i}, s_i), t) \end{aligned}$$

For every $(t_{-i}, t_i) \in T^*$ we know that $(t_{-i}, s_i) \notin T^*$ (by non-exclusivity) and therefore $\tilde{x}_i(t_{-i}, s_i) = e_i$.

Thus

$$\begin{aligned} U_i(\tilde{x}_i, s_i \mid t_i) &= \textstyle\sum_{\{t_{-i}\in T_{-i}|(t_{-i},t_i)\in T^*\}} q_i(t_{-i}|t_i)u_i(e_i(t_i), t) \\ &\leq U_i(x_i \mid t_i) = U_i(\tilde{x}_i \mid t_i), \end{aligned}$$

where the inequality follows from interim individual rationality of x. ∎

Non-emptiness is not guaranteed when incentive constraints are imposed.

The example in Forges-Mertens-Vohra also has an empty incentive compatible coarse core.

Unfortunately, the mechanism design approach, which was so fruitful in the ex ante case, does not immediately extend to the interim case.

That approach proceeded as follows:

Pick an allocation the core of the Arrow-Debreu economy.

Apply the transfers that yield incentive compatibility.

Another round of transfers (independent of state) to restore ex ante utilities to their original levels.

Now, this last step doesn't necessarily work.

While monetary transfers make it possible to transfer ex ante utility across consumers without affecting incentive constraints, the same need not be true in terms of transfers of interim utility (across types).

Restrictions on interim utility (such as interim individual rationality) may be too demanding if one insists on first-best efficiency even in the case of independent, private values, e.g. the Myerson and Satterthwaite theorem.

While that approach may not longer be fruitful, the possibility remains that the incentive compatible coarse core is non-empty under the conditions such as independent private values. This is an open question.

Endogenous Information Sharing

The theory should make endogenous the amount of private information that is shared within a coalition.

There may be situations in which some members of a coalition could credibly convince others in the coalition of an event which is not common knowledge.

Restricting attention to efficiency, Holmström and Myerson (1983) study this issue by considering a proposal for the grand coalition which is tested with a voting procedure and formalize the notion of durable decision rules.

A durable decision rule is one to which there does not exist a threat from a proposal which is in some sense a credible objection to the status-quo even though it may not be an improvement over a common knowledge event. In a voting game where an alternative mechanism is proposed, there should be a (perfect) equilibrium in which the status-quo is not rejected in favor of the alternative.

Similar ideas can be applied to a notion of the core in which coalitions are permitted to carry out objections over events finer than a common knowledge event.

Example 3 (Dutta and Vohra (2001))

There are three consumers in an economy with two commodities. Each consumer i can be of two possible types. Let $T_i = \{a_i, b_i\}$. Of the eight information states, only three arise with positive probability. These states are denoted

$$t^1 = (a_1, b_2, b_3), \quad t^2 = (b_1, a_2, b_3) \quad t^3 = (b_1, b_2, a_3)$$

All consumers have identical priors q, where

$$q(t) = \begin{cases} 1/3 & \text{if } t \in T^* = \{t^1, t^2, t^3\} \\ 0 & \text{otherwise} \end{cases}$$

In each state with positive probability there is exactly one consumer who is fully informed; consumer i in state t^i.

$$\omega_i(t_i) = \begin{cases} (1, 0) & \text{if } t_i = b_i \\ (0.5, 0.5) & \text{if } t_i = a_i \end{cases}$$

The utility functions are as follows.

$$u_1(x,t) = \begin{cases} 1.5(x_1 + x_2) & \text{if } t = t^3 \\ x_1 + x_2 & \text{otherwise} \end{cases}$$

$$u_2(x,t) = \begin{cases} 1.5(x_1 + x_2) & \text{if } t = t^1 \\ x_1 + x_2 & \text{otherwise} \end{cases}$$

$$u_3(x,t) = \begin{cases} 1.5(x_1 + x_2) & \text{if } t = t^2 \\ x_1 + x_2 & \text{otherwise} \end{cases}$$

The incentive compatible, coarse core contains $\bar{x}$, where

$$\begin{aligned} \bar{x}(t^1) &= ((0.5, 0.5), (2, 0), (0, 0)) \\ \bar{x}(t^2) &= ((0, 0), (0.5, 0.5), (2, 0)) \\ \bar{x}(t^3) &= ((2, 0), (0, 0), (0.5, 0.5)) \end{aligned}$$

$\bar{x}$ is not in the incentive compatible fine core. Consumers 1 and 3 have a fine objection over the event $\{t^1\}$. If private information can be shared, as is implicit in the notion of the fine core, then clearly $\bar{x}$ is not viable in state t^1.

But more can be said to justify the claim that $\bar{x}$ is not stable.

Suppose the state is t^1, which consumer 1 knows. Consumer 3 knows that the true state is either t^1 or t^2. Consider an offer from consumer 1 to consumer 3 of the contract $\tilde{x}(t)$, where

$$(\tilde{x}_1(t), \tilde{x}_3(t)) = \begin{cases} ((1.1, 0), (0.4, 0.5)) & \text{if } t = t^1 \\ (\omega_1(t), \omega_3(t)) & \text{otherwise} \end{cases}$$

In state t^1, the corresponding net-trades is $z_1(t^1) = (0.6, -0.5)$. Note that t^1 is the only state in which her endowments permit her to make this trade.

While 3 does not know whether the true state is t^1 or t^2, she does know that the informed agent would be better off with this contract only if the true state is t^1; if the state is actually t^2, the net-trade $z_1 = (0.6, -0.5)$ is infeasible for agent 1.

The informed agent's claim, that the state is t^1, is credible and should, therefore, be accepted by agent 3. Acceptance requires only that agent 3 infer (correctly) from the contract that the state is t^1 – not that 1's private information becomes explicitly available to agent 3.

Agents should be able to coordinate on an event that can be inferred simply by the fact that all members of the coalition are willing to sign a contract that is to their benefit only on the given event.

In the present example, this makes it hard to justify the coarse core as the appropriate core notion. The fine core seems more appropriate.

But in other cases, the coarse core seems more reasonable than the fine core. If in the previous example there is only one good agent 1 will not be able to signal to agent 3 with a contract such as $\bar{x}$.

Suppose each i in coalition S claims, independently, not to be of any type $\widehat{t}_i \notin E_i$. This type, $\widehat{t}_i$,cannot be ruled out by agent $j \in S$, with her private information, if

$$\text{for some } t \in E,\ \widehat{t}_i \notin E_i,\ q_j(t_{-i}, \widehat{t}_i) > 0 \qquad (1)$$

For each $i \in S$ let $V_i(E) \subseteq T_i \setminus E_i$ denote the set of all $\widehat{t}_i$ satisfying (1). Of course, if the event E is not a common knowledge event, $V_i(E) \neq \emptyset$ for some $i \in S$.

Given an admissible event E for coalition S define for each $i \in S$ and $\widehat{t}_i \in V_i(E)$,

$$q_i(t_{-i} \mid \widehat{t}_i, E) = \frac{q_i(t_{-i}, \widehat{t}_i)}{\sum_{t'_{-i} \in E_{-i}} q_i(t'_{-i}, \widehat{t}_i)}.$$

For an event admissible for coalition S, we can now define for each $i \in S$ and a type $\widehat{t}_i \in V_i(E)$, the conditional expected utility (conditional on E), of a contract x as

$$U_i(x \mid \widehat{t}_i, E) = \sum_{t'_{-i} \in E_{-i}} q_i(t'_{-i} \mid \widehat{t}_i, E) u_i(x(t'_{-i}, \widehat{t}_i), (t'_{-i}, \widehat{t}_i))$$

Similarly, define the conditional expected utility of x to $\widehat{t}_i \in V_i(E)$ if $\widehat{t}_i$ pretends to be of type $s_i \in T_i$ as

$$U_i(x, s_i \mid \widehat{t}_i, E) \equiv \sum_{t'_{-i} \in T_{-i} \mid (t'_{-i}, \widehat{t}_i) \in E} q_i(t'_{-i} \mid \widehat{t}_i, E) u_i(x_i(t'_{-i}, s_i \mid \widehat{t}_i), (t'_{-i}, \widehat{t}_i))$$

Suppose $x \in \mathcal{F}^*$, $y \in \mathcal{F}_S$ and E is an admissible event for coalition S. A contract y is said to satisfy self-selection with respect to x over E if

$$U_i(y, s_i \mid \hat{t}_i, E) \leq U_i(x \mid \hat{t}_i, E) \text{ for all } \hat{t}_i \in V_i(E),\ s_i \in E_i \text{ for all } i \in S \tag{SS}$$

This constraint can be seen as an extension of incentive compatibility to those types who are not supposed to be part of the objecting coalition.

Coalition S is said to have an incentive compatible, credible objection to an incentive compatible contract $x \in \mathcal{F}^*$ if there exists $y \in \mathcal{F}_S^*$ and an admissible event E such that all agents in S gain over the event E and (SS) is satisfied.

The incentive compatible, credible core consists of all incentive compatible allocations to which there does not exist an incentive compatible, credible objection.

Notice that if E is a common knowledge event for S then $V_i(E) = \emptyset$ for all $i \in S$, and (SS) is, therefore, trivially satisfied. Thus the credible core is a subset of the coarse core.

Non-emptiness cannot be guaranteed in general.

One approach to non-emptiness:

Consider a TU model (monetary transfers are available).

Suppose there exists a mechanism such that the outcome is in the core state-by-state (in the 'ex post core'). Suppose further that the mechanism is incentive compatible.

This mechanism belongs to the ex ante incentive compatible core, the incentive compatible coarse core, as well as the credible core.

The condition that there exist a selection from the state-by-state core that is incentive compatible seems very stringent.

But it does hold in the case of an auction (of a single commodity); private values, or interdependent values satisfying single crossing (generalized VCG mechanism).

The set of core allocations can be much larger.

Suppose there are 2 buyers with the following valuations:

$$(t_1^h, t_1^L) = (10, 5), \qquad (t_2^h, t_2^L) = (6, 1).$$

The seller has 0 as reservation value. Types are equally likely.

Consider following an efficient allocation with the following utility profile.

	t_2^H	t_2^L
t_1^H	$(4, 1, 5)$	$(6, 0, 4)$
t_1^L	$(0, 1, 5)$	$(0, 0, 5)$

This is in the credible core, but not in the fine core. When t_1^H, 1's expected payoff is 5. So, to make an objection, he has to offer a price less than 5, (but greater than 4.5). But, then buyer 1 of type t_1^L would also make this offer.

	t_2^H	t_2^L
t_1^H	$(4, 1, 5)$	$(5, 0, 5)$
t_1^L	$(0, 0, 6)$	$(0, 0, 5)$

A credible objection is a price offer of 5.1 by buyer 1. Only type t_1^H would make this offer. So this is not in the credible core. But it does belong to the incentive compatible coarse core.

Credibility and Bayesian Nash Equilibria of Voting Games

Consider an incentive compatible allocation x which is the status-quo.

Suppose coalition S is free to choose a new mechanism y to be played by members of S.

y is only 'implemented' if all members of S vote to accept it.

If all players in S accept y, they play a direct revelation game in which the strategy set for each $i \in S$ is T_i, and the outcome is $y(t)$ for $t \in T$.

In case of a rejection, the outcome function used in the second stage is $x(t)$.

Players vote confidentially, and only the outcome of the vote is revealed publicly. Thus a voting game is defined as $\Gamma(S, y, x)$, in which the players are the members of S, $y \in A_S$ and $x \in A_N$.

A truthful Bayesian Nash equilibrium of the voting game is one in which each player reports truthfully in stage 2 (regardless of whether or not the proposal is accepted).

Proposition 3 *Suppose* $x \in A_N$ *is incentive compatible and* $\Gamma(S, y, x)$ *is a voting game for coalition* S*. If there exists a truthful Bayesian Nash equilibrium, and an admissible event* E*, such that* i *accepts* y *if and only* $t_i \in E_i$*, and all players in* S *gain compared to the status-quo whenever* y *is accepted, then* (S, y) *is an incentive compatible credible objection to* x *over the event* E*. The converse also holds if* x *is uniformly incentive compatible.*