Efficiency in an Economy with Fixed Costs*

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Abstract

It is by now well known that in an economy with increasing returns, first-best efficiency may be impossible to attain through an equilibrium concept based on market prices, even if firms are regulated to follow marginal cost pricing. We examine the efficiency issue in a special but important class of economies in which the only source of non-convexities is the presence of fixed costs. Even in this context, it is possible that none of the equilibria based on marginal cost pricing are efficient (unless additional, strong assumptions are made). We argue that available results on the existence of an efficient two-part tariff equilibrium rely on very strong assumptions, and provide a positive result using a weak surplus condition. Our approach can also be used to establish the existence of an efficient marginal cost pricing equilibrium with endogenously chosen lump-sum taxes if the initial endowment is efficient in the economy without the production technology.

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1 Introduction

The recent general equilibrium literature on increasing returns has made it clear that the optimal (first-best) regulation of a natural monopoly is an issue that is far more subtle than is suggested by the standard partial equilibrium analysis. Consider an economy in which consumers have exogenously given private endowments. Suppose, for simplicity, that there is a single firm and this firm faces increasing returns to scale in production. If, as in the standard Arrow-Debreu model, consumers have exogenously given shares in the ownership of the firm one may consider the usual notion of a Walrasian equilibrium as a possible way of achieving efficiency. The problem, of course, is that the existence of a Walrasian equilibrium is not to be expected in the presence of increasing returns. In order to achieve efficiency one may then consider regulating the firm, i.e., departing from the assumption of profit maximizing behavior. Indeed, the traditional solution to this problem, going back at least to Hotelling (1939), is to regulate the firm to follow marginal cost pricing. Several versions of a generalized second welfare theorem are now available which show that, under very general conditions, efficiency requires marginal cost pricing, with some appropriate, general notion of marginal costs.[1]

Clearly, marginal cost pricing, a necessary, first order-condition for efficiency, need not be sufficient in the presence of non-convexities; some allocations based on marginal cost pricing may be efficient while others may be inefficient. Unfortunately, the inefficiency problem runs even deeper. Consider first, the standard approach in which consumers have exogenously given endowments as well as shares in all the firms. It is possible that (in an economy with at least two consumers) *none* of the marginal cost pricing equilibria attain Pareto efficiency; see Guesnerie (1975), Brown and Heal (1979), Beato and Mas-Colell (1985) and Vohra (1990).[2]

Since marginal cost pricing may result in losses for a non-convex firm, consumers' shares in such a firm specify an exogenously given rule for collecting

[1]See, for example, Brown (1991), Cornet (1986), Guesnerie (1975), Khan (1999), Khan and Vohra (1987), Quinzii (1992) and Yun (1985).

[2]It follows from the second welfare theorem that this inefficiency problem cannot arise in a single consumer economy. In fact, the problem can be traced to that of aggregating preferences, as explained in some of the references cited above; see also Chichilnisky and Heal (1991). It is possible to avoid the problem by making suitable assumptions which relate Scitovsky indifference curves to the technology. For results in this direction see Quinzii (1988) and Dierker (1986).

these losses. In other words, lump-sum taxes for financing the regulated firm are exogenously given. And for this reason one may question the significance of the above mentioned results on the inefficiency of marginal cost pricing equilibria; in many cases, inefficiency may be avoided by appropriately changing the shares of consumers in the regulated firm. In any event, as a matter of public policy for optimal regulation it seems more reasonable to allow the policy not only to regulate the firm (to follow marginal cost pricing, for example) but also to determine endogenously the tax burden on each consumer. We shall refer to the corresponding equilibrium concept as a Marginal Cost Pricing Equilibrium with Variable Shares (MCPEVS). While a MCPEVS allows for some additional flexibility in income re-distribution compared to an equilibrium with fixed shares, this too may be insufficient for obtaining efficiency; see Vohra (1990, 1994). Another possibility, closely related to a MCPEVS, is to cover the losses by appropriately charging fixed amount to those consumers who choose to consume the produced good, i.e., to use two-part tariffs. In general, this again may not suffice to restore efficiency; see Vohra (1990), Quinzii (1992), and Brown Heller and Starr (1992).

Our aim in this paper is to explore the possibility of obtaining a first best equilibrium with two-part tariffs or a MCPEVS. In order to make some progress on this important problem we shall consider, throughout this paper, a simple but economically important class of increasing returns economies. We shall assume that the only source of non-convexities is the presence of fixed costs. In this context, it is instructive to consider the well studied partial equilibrium analysis of two-part tariffs; see for example, Brown and Sibley (1986) and Coase (1946). Suppose there are two goods, one of which is produced by incurring a fixed cost and constant marginal cost. Efficiency may imply that the technology is not worth using or that there is a way of constructing discriminatory two-part tariffs to efficiently supply the produced good. In either case, there exists an efficient two-part tariff equilibrium in which the variable part is simply the marginal cost. Vohra (1990) shows that this argument can be extended to a general equilibrium model in which marginal cost is non-decreasing but the positive result need not be valid if there are more firms or more commodities. Edlin and Epelbaum (1995) and Moriguchi (1996) provide a positive result without restricting the number of commodities but by using a surplus condition which was introduced by Brown, Heller and Starr (1992). For a given vector of market prices define the surplus of a consumer as the compensating variation of income associated with the ability to buy the produced good at the market prices. The surplus condition requires that for any production plan and a corresponding vector of marginal cost prices if the firm makes a loss, then this loss is strictly less than the aggregate surplus. We shall argue that this condition is too strong. In fact, as we will show (in Proposition 2), it is *inconsistent* with marginal cost pricing and the assumption that fixed costs are the only source of non-convexities. Edlin, Epelbaum and Heller (1998) have shown that in a fixed cost economy a weaker form of the surplus condition is sufficient to establish the existence of a perfectly discriminating monopoly equilibrium. Our main result shows that an even weaker version of the surplus condition is enough to obtain a positive result with two-part tariffs; we require that for production plans in which the technology is used, the loss of the firm be no more than the surplus. Our extension of Moriguchi's (1996) result is therefore based on a similar (but stronger) condition used by Edlin, Epelbaum and Heller (1998) to prove the existence of a perfectly discriminating monopoly equilibrium. As such, it adds further support to the idea that there exist two-part tariffs which have efficiency properties similar to those of a perfectly discriminating monopoly.

Unfortunately, it is not possible to assert the existence of an efficient equilibrium if the weak surplus condition fails or if there are more than two non-produced commodities. In section 3 we show (without using the surplus condition) that an efficient MCPEVS exists if the initial endowments are efficient in the economy without the production technology. The proof is an adaptation of the technique used in section 2 for proving the existence of an efficient two-part tariff equilibrium.

2 Efficient Two-Part Tariffs

The economy consists of n consumers and one public sector firm. For simplicity we shall assume that the firm has one potential output which is indexed as commodity q. The set of other commodities in the economy is denoted L. Thus, letting $|L| = l$, there are $l+1$ commodities in the economy. For $x \in R^{l+1}$, we denote by x_L and x_q the restriction of x to the coordinates corresponding to L and q respectively. The consumption sets, utility functions and endowments of the consumers are denoted X_i, u_i and $\omega_i \in X_i$ respectively. The aggregate endowment is denoted $\omega = \sum_i \omega_i$. We make the standard assumptions on consumers' characteristics:

(C) For all i, $X_i = R^{l+1}_+$, u_i is continuous, strictly monotonic and quasi concave.

The production set of the firm is denoted $Y \subset R^{l+1}$. The only source of non-convexity in the technology is the presence of a fixed cost. This means that there exists a set of inputs $L' \subseteq L$ and $f_{L'} \ll 0$ such that positive production requires inputs more than $f_{L'}$. Formally, we assume:

(F)
$$Y = C \cup R_{-}^{l+1},$$
where $C \subseteq R_{-}^{l} \times R$ is a closed, convex set satisfying free disposal and there exists a set $L' \subseteq L$ and $f_{L'} \ll 0$ such that $(f_{L'}, 0) \in C$ and $y_q > 0$ implies that $y_{L'} \ll f_{L'}$.

For the usual reasons, we will also need a survival assumption to guarantee that quasi equilibria are equilibria. Since we have already assumed preferences to be monotonic, it will be enough to make the following assumption.

(S) $\omega_L \gg 0$ and $\omega_{L'} \gg -f_{L'}$.

Assumption (S) can be interpreted simply as the requirement that every commodity is either available as an endowment or the technology allows for it to be produced. It should be clear that without such an assumption it is generally impossible to show the existence of an equilibrium rather than a quasi-equilibrium.

A private ownership economy is defined as $\mathcal{E} = \{(X_i, u_i, \omega_i), Y\}$. Our interest lies in achieving Pareto efficiency through a public policy which involves regulating the firm in the context of a market economy. More precisely, we study equilibrium concepts in which consumers behave competitively, i.e., maximize utility taking market prices and incomes as given. The main issue then is to determine how the firm should be regulated in order to achieve efficiency. One important property of efficient regulation is that it must involve setting market prices equal to marginal costs. For our purposes, it will be adequate to consider the Clarke normal cone as a formalization of marginal cost prices. The Clarke normal cone, at the production plan $y \in Y$, is denoted $N(Y, y)$. Let S denote the unit simplex in R_{+}^{l+1}.

An *equilibrium with transfers* (using marginal cost pricing) is defined as $((\bar{x}_i), \bar{y}, \bar{p}) \in \prod_i X_i \times Y \times S$ such that:

(i) for all i, $u_i(x_i) \geq u_i(\bar{x}_i)$ implies that $\bar{p} \cdot x_i \geq \bar{p} \cdot \bar{x}_i$.

(ii) $\bar{p} \in N(Y, \bar{y})$.

(iii) $\sum_i \bar{x}_i = \bar{y} + \omega$.

Efficiency requires marginal cost pricing. This is the essential message of a generalized second welfare theorem, the following version of which will suffice for our purposes.[3]

Proposition 1 *(Generalized Second Welfare Theorem) Suppose (C) holds and Y satisfies free disposal. If $((\bar{x}_i), \bar{y})$ is Pareto efficient, then there exists $\bar{p} \in S$, such that $((\bar{x}_i), \bar{y}, \bar{p})$ is an equilibrium with transfers.*

In the present context, given assumption (F), we will not need to consider the general definition of the Clarke normal cone. It will be enough for us to rely on the following (partial) characterization of the Clarke normal cone, which also makes it clear that we can identify 'marginal cost prices' as points in the normal cone.

Lemma 1 *If (F) holds then*

$$N(Y, y) \cap S = \begin{cases} S & \text{if } y = 0 \\ \{p \in S \mid p \cdot y \geq p \cdot y' \text{ for all } y' \in C\} & \text{if } y_q > 0 \end{cases}$$

Thus, an efficient, regulated market equilibrium requires that the market prices are set equal to marginal cost prices for the firm.[4] It means that in searching for an efficient equilibrium there is essentially no loss of generality in concentrating on regulation which involves marginal cost pricing.

Of course, marginal cost pricing in the presence of increasing returns implies that the positive production may lead to a loss in the firm. An equilibrium concept based on marginal cost pricing must, therefore, involve a rule for covering such losses. A two-part tariff achieves this by setting the fixed part of the tariff such that the sum of these fixed charges, across all consumers who choose to consume the produced good, equals the loss of the

[3]The references cited in footnote 1 include alternatives to Clarke's normal cone as formulation of marginal costs. Guesnerie (1975) provided an early version of this theorem using the Dubovickii-Miljutin cone and assuming the tangent cone to be convex. Khan (1999) has shown that an even sharper result can be obtained by using the Mordukhovich cone. For more recent developments on this topic see Malcolm and Mordukhovich, (2000).

[4]Strictly speaking, this implication also requires a smoothness assumption on preferences, as explained in Remark 4, Vohra (1994).

firm. A consumer can but the q-th commodity at marginal cost after paying a fixed (hook-up) charge. The fixed charge (and the marginal cost) can be completely avoided by not buying commodity q.

To define a two-part tariff equilibrium it is also necessary to assume that there is no endowment of this commodity, i.e., $\omega_q = 0$. Given market prices p, where p_q refers to the variable part of the two-part tariff, and given a fixed part t_i, the income of consumer i can now be defined as

$$\hat{m}_i(p, t_i, x_i) = \begin{cases} p \cdot \omega_i & \text{if } x_{i_q} = 0 \text{ and } t_i \geq 0 \\ p \cdot \omega_i - t_i & \text{otherwise} \end{cases}$$

and the budget set is $\gamma_i(p, t_i) = \{x_i \in X_i \mid p \cdot x_i \leq \hat{m}_i(p, t_i, x_i)\}$.

A *Two Part Tariff Equilibrium*, TPTE, is defined as $((\bar{x}_i), \bar{y}, (\bar{t}_i), \bar{p})$ such that

(i) For all i, $\bar{x}_i \in \gamma_i(\bar{p}, \bar{t}_i)$ and $u_i(\bar{x}_i) \geq u_i(x_i)$ for all $x_i \in \gamma_i(\bar{p}, \bar{t}_i)$.

(ii) If $\bar{p} \cdot \bar{y} \geq 0$, then $\sum_i \bar{t}_i = -\bar{p} \cdot \bar{y}$ and $\bar{t}_i \geq 0$ for all i. If $\bar{p} \cdot \bar{y} < 0$, then $\sum_{\{i \mid x_{iq} > 0\}} \bar{t}_i = -\bar{p} \cdot \bar{y}$ and $\bar{t}_i \geq 0$ for all i.

(iii) $\bar{p} \in N(Y, \bar{y})$.

(iv) $\sum_{i=1}^n \bar{x}_i = \bar{y} + \sum_i \omega_i$.

It should be noted that condition (ii) above is slightly different from the corresponding condition used in some other papers. For instance, Brown, Heller and Starr (1992) rely on a fixed set of shares in the firm to distribute profits whenever the profits are positive. This is not an essential difference; our result can easily be adapted to their definition a TPTE. (It is, however, important that the sign of $\bar{t}_i$ be the same for all consumers).

Recall that for simplicity, we refrain from considering a more general model with many firms. While such an extension is important, and introduces new issues regarding the interpretation of the 'surplus', the basic approach in that direction has already been well studied by Edlin and Epelbaum (1993, 1995), Moriguchi (1996) and Clay (1994).

From our previous discussion it should not be surprising that the existence of an efficient two-part tariff equilibrium cannot generally be established; see Vohra (1990), Quinzii (1992) and Brown, Heller and Starr (1992).

However, it has been shown that in a pure fixed cost economy, an efficient two-part tariff equilibrium exists under certain conditions. Vohra (1990) provides such a result assuming that $l = 1$ and $q = 1$ while Edlin and Epelbaum (1995), and Moriguchi (1996) do so using a surplus condition based on a similar condition introduced by Brown, Heller and Starr (1992). We shall argue that the surplus condition is unduly strong, and prove a positive result based on a weaker version of this condition.

In the present context, given a single output and a single firm, the surplus condition can be defined exactly as in Brown, Heller and Starr (1992).[5]

Let K be a compact cube which contains all attainable sets in its interior and define $\hat{X}_i = X_i \cap K$ for all i. Let

$$v_i(p) = \max\{u_i(x_i) \mid x_i \in \hat{X}_i, x_q = 0, p \cdot x_i \leq p \cdot \omega_i\},$$

$$e_i(p) = \min\{p \cdot x_i \mid u_i(x_i) \geq v_i(p), x_i \in \hat{X}_i\},$$

and

$$r_i(p) = p \cdot \omega_i - e_i(p).$$

Given prices p, $v_i(p)$ is the indirect utility when the consumer is not allowed to consume commodity q and $e_i(p)$ is the minimum expenditure necessary to achieve this level of utility when trade in commodity q is permitted. (Note that $v_i(p)$ is well defined since utility maximization is restricted to the compact set $\hat{X}_i$.) Thus, at market prices p, $r_i(p)$ is a measure of the surplus (or compensating variation) associated with the ability to purchase commodity q. Note that $r_i(p) \geq 0$ for all $p \in S$. The surplus condition can now be stated as follows:

(SR) for all $y \in \partial(Y)$ and $p \in N(Y, y)$, $\sum_i r_i(p) > \max(-p \cdot y, 0)$.

This assumption is extremely strong. It rules out the possibility of the aggregate surplus being 0 (for $y \in \partial(Y)$ and $p \in N(Y, y)$). Since, in the presence of fixed costs, the normal cone imposes no restriction on the marginal cost prices at the production plan 0, it is easy to construct prices such that the aggregate surplus is 0. In fact, as the next proposition shows, this assumption is inconsistent with the assumption that non-convexity of Y is due only to the presence of a fixed cost.

Proposition 2 *Condition (SR) cannot hold in an economy satisfying (F).*

[5] It is somewhat different from Moriguchi's but our comments will apply to that as well.

Proof: Consider $(p^*, y^*) \in S \times Y$, where $p^*_L = 0$ and $p^*_q = 1$, and $y^* = 0$. Of course, $p^* \in N(Y, y^*)$ (by lemma 1) and $p^* \cdot y^* = 0$. However, the surplus for each consumer is 0 since $e_i(p^*) = 0$ and $p^* \cdot \omega_i = 0$. Thus (SR) cannot hold for (p^*, y^*). ■

Remark 1. Note that argument in Proposition 2 applies, unchanged, even to the case where the firm produces several commodities (by choosing $p^*_L = 0$). The crucial assumption in Proposition 2 is (F). Thus, the result does not apply to more general non-convex technologies, as were considered by Brown, Heller and Starr (1992). In the more general case, our argument shows that given any technology, it is possible to construct well behaved preferences such that (SR) cannot hold at $y = 0$. The difficulty of applying (SR) to a pure fixed cost economy was recognized in Edlin and Epelbaum (1993, p. 917) and in Moriguchi (1996, section 3.2).[6] Proposition 2 can be seen as a more formal and direct expression of the problem.

Fortunately, as we shall show, a weaker version of the surplus condition is enough to establish the existence of an efficient equilibrium.[7] With Proposition 2 in mind, we shall assume the surplus condition as a weak inequality, and assume it only for those allocations and prices which are equilibria with transfers for the (convex) economy with production set C. The latter property makes our assumption similar to one used by Edlin, Epelbaum and Heller (1998) in proving the existence of a perfectly discriminating monopolist equilibrium. (Theorem 1 is a direct complement to their Proposition 4).

Define the (convex) economy with production set C as $\mathcal{E}^c = \{(X_i, u_i, \omega_i), C\}$. Notice that by assumptions (F) and (S), the set of feasible allocations in this economy is non-empty even though 0 production is not feasible. Recall also that $C \subset Y$ and $y \in Y$ such that $y_q > 0$ implies that $y \in C$.

(WSR) Suppose $((x_i), y, p)$ is an equilibrium with transfers for $\mathcal{E}^c$ (in particular, $y \in C$). Then $\sum_i r_i(p) \geq \max(-p \cdot y, 0)$.

We can now state our main result.

[6] In both cases the emphasis was on the case in which the price vector is not strictly positive but it should be clear that the problem remains even if prices are strictly positive.

[7] The solution we propose to the problem identified in Proposition 2 is different from the approach taken in Moriguchi (1996) in that we shall only need to apply a weaker condition to production plans which belong to C.

Theorem 1 *Suppose an economy $\mathcal{E}$ satisfies (C), (F), (S) and (WSR). Then there exists an efficient TPTE.*

To prove Theorem 1, we shall find it useful to rely on the following lemma.

Lemma 2 *Suppose $((\bar{x}_i), \bar{y}, \bar{p})$ is an equilibrium with transfers for economy $\mathcal{E}^c$. If assumptions (C), (F) and (S) hold, then:*

(i) $\bar{p} \cdot (\bar{y} + \omega) > 0$,

(ii) $\bar{p} \gg 0$,

(iii) for all i, $u_i(x_i) > u_i(\bar{x}_i)$ implies that $\bar{p} \cdot \bar{x}_i > \bar{p} \cdot \bar{x}_i$.

Proof: Suppose $((\bar{x}_i), \bar{y})$ and $\bar{p}$ satisfy the conditions of lemma 2.

We shall first show that condition (i) of lemma 2 is satisfied, i.e., $\bar{p} \cdot (\bar{y} + \omega) > 0$. Suppose $\bar{p}_L = 0$ (and $\bar{p}_q > 0$). By assumption (S), there exists $y \in C$, $y_q > 0$ such that $y + \omega \geq 0$. Of course, $\bar{p}_q = 0$ and $\bar{p}_L = 0$ implies that $\bar{p} \cdot y > 0$. Since $\bar{p} \in N(C, \bar{y})$, $\bar{p} \cdot \bar{y} \geq \bar{p} \cdot y > 0$, implying that $\bar{p} \cdot (\bar{y} + \omega) > 0$. Next, suppose $\bar{p}_L > 0$, and $\bar{y}_q = 0$. By assumption (S), this implies that $\bar{y}_L + \omega_L > 0$ and, $\bar{p} \cdot (\bar{y} + \omega) > 0$. Finally, suppose $\bar{p}_L > 0$ and $\bar{y}_q > 0$. Since $\bar{y}_q > 0$, $\bar{y}_{L'} \ll f_{L'}$. Using the fact that $\bar{y} + \omega \geq 0$, we can claim that

$$\omega_{L'} + f_{L'} \gg 0.$$

Since $\bar{p} \cdot \bar{y} \geq \bar{p} \cdot (f_{L'}, 0)$,

$$\bar{p} \cdot (\bar{y} + \omega) \geq \bar{p} \cdot ((f_{L'}, 0) + \omega) > 0,$$

where the last inequality uses the fact that $\omega_{L'} + f_{L'} \gg 0$ and $\bar{p}_L > 0$.

Since $\bar{p} \cdot (\bar{y} + \omega) > 0$, there must exist a consumer i such that $\bar{p} \cdot \bar{x}_i > 0$. By the usual argument then, condition (i) of Theorem 1 implies that

$$u_i(x_i) > u_i(\bar{x}_i) \text{ implies that } \bar{p} \cdot x_i > \bar{p} \cdot \bar{x}_i.$$

Monotonicity of preferences then implies that $\bar{p} \gg 0$, i.e., condition (ii) of the lemma holds. Of course, this must mean that condition (iii) of lemma 2 also holds for all i. ■

Proof of Theorem 1: Consider the (exchange) economy, $\mathcal{E}'$, in which the set of commodities is L, the consumption set is $\bar{X}_i = R^l_+$ for each consumer and the utility functions are restricted to R^l. Let S' be the unit simplex in R^l_+. Given assumption (C), there exists $(\hat{x}_i, \hat{p})$, a Walrasian equilibrium of $\mathcal{E}'$. We can now extend this to a TPTE of $\mathcal{E}$ in which the production is 0 and the fixed part of the tariff is chosen to be so high that no consumer chooses to buy commodity q. More precisely, let $p \in S$ such that $p_L = \alpha \hat{p}$ for some $\alpha > 0$. Let $y' = 0$, $x'_i = (\hat{x}_i, 0)$ and let $t'_i = p \cdot \omega_i$ for all i. It is easy to see that $((x'_i), y', p, (t'_i))$ is a TPTE of $\mathcal{E}$.

If $((x'_i), y')$ is Pareto efficient for $\mathcal{E}$, the proof would be complete. Suppose, therefore, that $((x'_i), y') \notin \mathcal{P}$, where $\mathcal{P}$ is the of Pareto efficient allocations of $\mathcal{E}$. Notice that any allocation in $\mathcal{P}$ which Pareto dominates $((x'_i), y')$ must involve positive production; recall that $((\hat{x}_i))$ was a Walrasian, and therefore efficient, allocation of $\mathcal{E}'$.

Consider the (convex) economy $\mathcal{E}^c = \{(X_i, u_i, \omega_i, C\}$. We begin by showing that there is an 'equilibrium' in this economy, and then argue that this is a TPTE for the original economy $\mathcal{E}$. Normalize the utility functions such that $u_i(\omega_i) = 0$ for all i and define:

$$U = \{u \in R^n | \; \exists \; ((x_i), y) \in \textstyle\prod_i X_i \times C, \; \sum_i x_i \leq y + \omega \text{ and } u_i \leq u_i(x_i) \text{ for all } i\}.$$

Let $\bar{U}$ be the boundary of U in R^n_+. By hypothesis, there exist an allocation which Pareto dominates $((x'_i), 0)$, and any such allocation must involve positive production. Thus, there exists $u \in \bar{U}$, $u \gg 0$. Since the set of feasible allocations is compact and the utility functions are monotonic, it can be shown that there is a homeomorphism between S^n, the unit simplex in R^n, and $\bar{U}$; see Arrow and Hahn (1971, lemma 5.3) and Vohra (1990, Theorem 5.1). In particular, there exists a continuous function $v : S^n \mapsto \bar{U}$ such that $v(s) = \lambda(s)s$ for some $\lambda(s) > 0$. The set of efficient allocations of $\mathcal{E}^c$ is denoted $\mathcal{P}^c$.

Let $\psi : S^n \mapsto \mathcal{P}^c$ be defined as:

$$\psi(s) = \{((x_i), y) \in \mathcal{P}^c \mid v_i(s) = u_i(x_i) \text{ for all } i\}$$

and let $\pi : \mathcal{P}^c \mapsto S$ be defined as:

$$\pi((x_i), y) = \{p \in S \mid p \in N(C, y) \text{ and } p \cdot x'_i \geq p \cdot x_i \text{ if } u_i(x'_i) \geq u_i(x_i)\}.$$

It is easy to see that π is non-empty (by Proposition 1), convex-valued and upper hemicontinuous. Since C is convex, $p \in N(C, y)$ implies that

$$p \cdot y \geq p \cdot y' \text{ for all } y' \in C. \tag{1}$$

By a standard argument (see, for example, Vohra (1990)), we can now claim that

$$\text{If } ((x_i), y) \in \psi(s) \text{ and } ((x'_i), y') \in \psi(s), \text{ then } \pi((x_i), y) = \pi((x'_i), y') \text{ and, for any } p \in \pi((x_i), y),\ p \cdot x_i = p \cdot x'_i \text{ for all } i \text{ and } p \cdot y = p \cdot y' \quad (2)$$

Let $\alpha : S \mapsto S^n$, where

$$\alpha(p) = \begin{cases} \frac{r_i(p)}{\sum_i r_i(p)} & \text{if } \sum_i r_i(p) > 0 \\ S^n & \text{otherwise} \end{cases}$$

(A similar construction for determining the income distribution was used in Vohra (1992).)

Let $\delta : S^n \times S^n \mapsto R^n$, where

$$\delta(s, p, a) = \{(p \cdot \omega_i + a_i p \cdot y - p \cdot x_i \mid ((x_i), y) \in \psi(s) \text{ and } p \in \pi((x_i), y)\}.$$

Notice that by (2), $p \cdot x_i$ and $p \cdot y$ in the construction of $\delta(s)$ are unique. Thus $\delta : S^n \mapsto D$ (where $D = Co(\delta(S^n))$ is a well defined continuous function.

Let $h : S^n \times D \mapsto S^n$ be defined as

$$h_i(s, d) = \frac{s_i + \max(d_i, 0)}{1 + \sum_{k=1}^n \max(d_k, 0)} \qquad i = 1, \ldots, n.$$

Certainly, h is a well defined, continuous function.

Finally, consider $\beta : S^n \times S \times S^n \times D \mapsto S^n \times S \times S^n \times D$, where $\beta(s, p, a, d) = h(s, d) \times \pi(s) \times \alpha(p) \times \delta(s, p, a)$.

By Kakutani's fixed point theorem, this mapping has a fixed point: $(\bar{s}, \bar{p}, \bar{a}, \bar{d})$, where

$$\bar{s} \in h(\bar{s}, \bar{d}), \bar{p} \in \pi(\bar{s}), \bar{a} \in \alpha(\bar{p}), \bar{d} = \delta(\bar{s}, \bar{p}, \bar{a}).$$

In particular,

$$\bar{s}_i(\textstyle\sum_{k=1}^n \max(\bar{d}_k, 0)) = \max(\bar{d}_i, 0) \qquad \text{for all } i. \quad (3)$$

Let $((\bar{x}_i), \bar{y}) \in \psi(\bar{s})$ be an allocation (in $\mathcal{P}^c$). We will prove that $((\bar{x}_i), \bar{y}, \bar{p}, (\bar{t}_i))$ is a TPTE for $\mathcal{E}$, where

$$\bar{t}_i = \bar{a}_i \bar{p} \cdot \bar{y}.$$

We begin by showing that

$$\bar{d}_k = 0 \text{ for all } k = 1, \dots n. \quad (4)$$

Clearly, $\sum_{k=1}^{n} \bar{d}_k = 0$ (recall the construction of δ), which implies that we only need to prove that $\bar{d}_k$ has the same sign for all k.

Let $I = \{i \mid \bar{s}_i > 0\}$ and $J = \{i \mid \bar{s}_i = 0\}$. Notice that for any $j \in J$, $u_j(\omega_j) = u_j(\bar{x}_j)$ (because $\bar{s}_j = 0$). Moreover, $\bar{p} \cdot \omega_i - r_i(\bar{p}) = e_i(\bar{p})$, which can yield utility $v_i(\bar{p}) \geq u_i(\omega_i)$. Since $u_i(x'_i) \geq u_i(\bar{x}_i)$ implies $\bar{p} \cdot x'_i \geq \bar{p} \cdot \bar{x}_i$ (because $\bar{p} \in \pi(\bar{s})$), we can assert that

$$\bar{p} \cdot \omega_j - r_i(\bar{p}) - \bar{p} \cdot \bar{x}_j \geq 0 \text{ for all } j \in J. \quad (5)$$

We shall consider three cases, as follows:

(a) Suppose $\bar{d}_i \leq 0$ for some $i \in I$. From (3), this means that $\sum_{k=1}^{n} \max(\bar{d}_k, 0) \leq 0$, and since $\bar{d}_k = \bar{s}_k \sum_{k=1}^{n} \max(\bar{d}_k, 0)$, this implies that $\bar{d}_k \leq 0$ for all $k = 1, \dots n$. Thus $\bar{d}_k \leq 0$ for all k, and the proof of (4) is complete.

(b) Suppose $\bar{d}_i > 0$ for all $i \in I$ and $\bar{p} \cdot \bar{y} > 0$. Since $\bar{p} \cdot y \geq 0$, it follows from (5) that $\bar{d}_j \geq 0$ for all $j \in J$. By hypothesis, $\bar{d}_i > 0$ for all $i \in I$, and the proof of (4) is complete.

(c) Suppose $\bar{d}_i > 0$ for all $i \in I$ and $\bar{p} \cdot \bar{y} < 0$. From (WSR) it follows that $\sum_i r_i(\bar{p}) \geq -\bar{p} \cdot \bar{y} > 0$. Given the definition of α, this means that $\bar{a}_k = r_k(\bar{p}) / \sum_i r_i(\bar{p})$ and $\bar{d}_k \geq \bar{p} \cdot \omega_k - r_k(\bar{p}) - \bar{p} \cdot x_k$ for all $k = 1, \dots n$. By (5), this means that $\bar{d}_j = \bar{p}(\omega_j - \bar{x}_j) \geq 0$ for all $j \in J$. Thus $\bar{d}_k \geq 0$ for all $k = 1, \dots n$, as required.

From lemma 2 we know that $\bar{p} \cdot (\bar{y} + \omega) > 0$, $\bar{p} \gg 0$, and for all i, $u_i(x'_i) > u_i(\bar{x}_i)$ implies that $\bar{p} \cdot x'_i > \bar{p} \cdot \bar{x}_i$. From (4) it follows that $\bar{p} \cdot \bar{x}_i = \bar{p} \cdot \omega_i - \bar{t}_i$ for all $i = 1, \dots n$. Thus, each consumer is maximizing utility given prices $\bar{p}$ subject to the constraint that expenditure does not exceed $\bar{p} \cdot \omega_i - \bar{t}_i$.

If $\bar{p} \cdot \bar{y} \geq 0$, this clearly implies that $\bar{y}_q > 0$, and $\bar{p} \in N(C, \bar{y})$ means that $\bar{p} \in N(Y, \bar{y})$. It then follows that $((\bar{x}_i), \bar{y}, \bar{p}, (\bar{t}_i))$ is a TPTE for $\mathcal{E}$.

If $\bar{p} \cdot \bar{y} < 0$, (WSR) implies that $r(\bar{p}) \geq -\bar{p} \cdot \bar{y} > 0$. Thus $\bar{a}_i = r_i(\bar{p}) / \sum_i r_i(\bar{p})$, and $\bar{t}_i \leq r_i(\bar{p})$ for all i. Since each consumers pays a fixed part which does not exceed her surplus, it follows that no consumer can gain by avoiding the fixed part and not consuming commodity q. Obviously, this also means that $\bar{y}_q > 0$ and $\bar{p} \in N(Y, \bar{y})$, and $((\bar{x}_i), \bar{y}, \bar{p}, (\bar{t}_i))$ is a TPTE for $\mathcal{E}$.

To complete the proof of Theorem 1 it remains to be shown that $((\bar{x}_i), \bar{y}) \in \mathcal{P}$. Of course, this allocation cannot be Pareto dominated by an allocation in the (convex) economy $\mathcal{E}^c$. Because the equilibrium is a TPTE, it cannot be dominated by 0 production in economy $\mathcal{E}$. The argument is as follows.[8] Suppose there exists an allocation $((x_i'), 0)$ in $\mathcal{E}$ such that $u_i(x_i') > u_i(\bar{x}_i)$ for all i. Since $((x_i'), 0)$ involves 0 production, $x_{iq}' = 0$ for all i. Since $((\bar{x}_i), \bar{y}, \bar{p}, (\bar{t}_i))$ is a TPTE, $\bar{p} \cdot x_i' > \bar{p} \cdot \omega_i$ for all i, which contradicts the hypothesis that $\sum_i x_i' = \omega$. ∎

3 Efficiency and Variable Shares

It is important to realize that while we have shown the existence of an efficient TPTE, involving positive production, under the weak surplus condition, one cannot draw the conclusion that if the weak surplus condition does not hold then there exists an efficient equilibrium with 0 production; see the example in Vohra (1990). In this section we consider the possibility of restoring efficiency without assuming the surplus condition. To do so we shall consider an equilibrium notion which is closely related to that of two-part tariffs. The fixed part of a two-part tariff cannot generally be viewed as a lump-sum tax since it can be avoided by not consuming the produced good. However, the equilibrium we constructed in proving Theorem 1 can be interpreted as one with lum-sum taxes because the fixed part for each consumer was chosen to be no more than the surplus.

More generally, we can consider an equilibrium notion involving marginal cost pricing and a rule for distributing the profits (or losses) of the regulated firm among the consumers. In the interest of achieving efficiency, we shall allow for as much flexibility in the income distribution rules as is legitimate for studying equilibrium theory. We shall also assume that the profits or losses of the firm are distributed to the consumers in a lump-sum manner. For $p \in S$, the unit simplex in R^{l+q}, and a lump-sum tax t_i, the income of consumer i is defined as

$$m_i(p, t_i) = p \cdot \omega_i - t_i.$$

A general equilibrium concept which encompasses various feasible public policies can now be defined as follows:

[8] See Theorem 1 in Moriguchi (1996) for a more general version of this result.

A *Marginal Cost Pricing Equilibrium with Variable Shares* (MCPEVS) consists of $((\bar{x}_i,), \bar{y}, \bar{p}, (\bar{t}_i)) \in \prod_i X_i \times Y \times S \times R^n$ such that

(i) For all i, $\bar{p} \cdot \bar{x}_i \leq m_i(\bar{p}, \bar{t}_i)$ and $u_i(x_i) > u_i(\bar{x}_i)$ implies that $\bar{p} \cdot x_i > m_i(\bar{p}, \bar{t}_i)$.

(ii) $\sum_i \bar{t}_i = -\bar{p} \cdot \bar{y}$ and either, $\bar{t}_i \geq 0$ for all i or $\bar{t}_i \leq 0$ for all i.

(iii) $\bar{p} \in N(Y, \bar{y})$.

(iv) $\sum_{i=1}^n \bar{x}_i = \bar{y} + \omega$.

The requirement that $t_i \geq 0$ for all i or $t_i \leq 0$ for all i in condition (ii) is important. Allowing some consumers to get a subsidy while others are taxed would allow arbitrary redistribution and efficiency in that case is easy to obtain (as the second welfare theorem implies).[9] On the other hand, a MCPEVS loosens the income distribution rules compared to the notion of a marginal cost pricing equilibrium in which consumers are assigned exogenously given shares in the firm, θ_i, where $\theta_i \geq 0$ for all i.[10]

Fixed shares in the firm imply that $t_i = \theta_i p \cdot y$. Examples have been provided in Guesnerie (1975), Brown and Heal (1979) and Beato and Mas-Colell (1985) to show that none of the equilibria (for a given profile of shares) may be efficient. Clearly, a MCPEVS, while not allowing arbitrary redistribution, is more conducive to the search for an efficient equilibrium. Unfortunately, it is not enough; Vohra (1990, 1994) provides examples, satisfying (F), in which none of the MCPEVS are efficient.

The inefficiency problem can be resolved by assuming that there is a single input and a single output (Vohra (1990)) or by imposing a surplus condition as we did in the previous section. The approach we used in section 2 can be applied to the more general case of many inputs and outputs, as we shall now show.[11]

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[9] See Vohra (1990) for further discussion.

[10] The lump-sum taxes/subsidies in condition (ii) can be interpreted as (endogenously) constructed shares in the firm; hence the term marginal cost pricing equilibrium with variable shares.

[11] In order to retain comparability with the previous section we continue to assume that there is a single produced good, but it should be clear that this is only a simplifying assumption.

To consider the existence of an efficient MCPEVS without assuming (WSR), we shall assume that the initial endowment is efficient for the exchange economy, i.e., it is not possible to make a Pareto improvement over the initial endowment without undertaking production. More precisely:

(E) There does not exist a feasible allocation $((x'_i), 0)$ in $\mathcal{E}$ which Pareto dominates (ω_i).

Of course, (E) is satisfied if $l = 1$. Our next result, therefore, explains the role played by the assumption that $l = 1$ in Vohra (1990, Theorem 5.1).

The following result is an application of the approach used in proving Theorem 1.

Theorem 2 *Suppose an economy $\mathcal{E}$ satisfies (C), (F), (S) and (E). Then there exists an efficient MCPEVS.*

Proof: As in the proof of Theorem 1, consider $\mathcal{E}'$, the exchange economy with l commodities. Assumption (E) implies that (ω_i) is a Walrasian allocation of $\mathcal{E}'$. Let $\hat{p} \in S'$ be corresponding Walrasian prices. To interpret $(\omega_i), \hat{p})$ as an MCPEVS in $\mathcal{E}$ we only need to augment $\hat{p}$ by introducing a price for the produced good. (Recall that $N(Y, 0) = R_+^{l+1}$). It suffices to set $\hat{p}_q = \infty$; a similar approach is taken in Edlin, Epelbaum and Heller (1998]. Allowing such prices then allows us to claim that this is an MCPEVS. Alternatively, if we assume that marginal rates of substitution for all consumers are bounded it will be possible to augment $\hat{p}$ to obtain prices in S. Thus, if $((\omega_i), 0) \in \mathcal{P}$ the proof would now be complete. Suppose, therefore, that $((\omega_i), 0) \notin \mathcal{P}$. Normalizing utilities such that $u_i(\omega_i) = 0$ for all i, let U be the utility possibility set of the economy $\mathcal{E}^c$. Let $\bar{U}$ be the boundary of U in R_+^n. By hypothesis, there exist an allocation which Pareto dominates $((\omega_i), 0)$, and any such allocation must involve positive production. Thus, there exists $u \in \bar{U}$, $u \gg 0$. Moreover, any allocation which Pareto dominates $((\omega_i), 0)$ must involve positive production. Proceeding as in the proof of Theorem 1, and using exactly the same fixed point argument, we obtain a fixed point $(\bar{s}, \bar{p}, \bar{a}, \bar{d})$, where

$$\bar{s} \in h(\bar{s}, \bar{d}), \bar{p} \in \pi(\bar{s}), \bar{a} \in \alpha(\bar{p}), \bar{d} = \delta(\bar{s}, \bar{p}, \bar{a}).$$

Let $((\bar{x}_i), \bar{y}) \in \psi(\bar{s})$ be an allocation (in $\mathcal{P}^c$). By construction, this allocation Pareto dominates $((\omega_i), 0)$. Assumption (E) now implies that $((\bar{x}_i), \bar{y})$ cannot be dominated by an allocation with 0 production. Thus $((\bar{x}_i), \bar{y}) \in \mathcal{P}$ and $\bar{y}_q > 0$. (Notice that this is the only step where we make crucial use of assumption (E)). Clearly then, $\bar{p} \in N(Y, \bar{y})$.

Let

$$\bar{t}_i = \bar{p} \cdot \omega_i - \bar{p} \cdot \bar{x}_i \text{ for all } i.$$

We shall now argue that $((\bar{x}_i), \bar{y}, \bar{p}, (\bar{t}_i))$ is an efficient MCPEVS. The only conditions in the definition of a MCPEVS that remain to be checked are (i) and (ii).

We begin by showing that $\bar{t}_i$ has the same sign for all i. As in the proof of Theorem 1, consider the three cases, (a), (b) and (c). Recall that in cases (a) and (b), we showed that $\bar{d}_i = 0$ for all i without using (WSR). Thus, in those case, $\bar{t}_i = \bar{a}_i \bar{p} \cdot y$ for all i. Now consider case (c), i.e., the case in which $\bar{d}_i > 0$ for $i \in I$ and $\bar{p} \cdot \bar{y} < 0$. Of course, $\bar{d}_i > 0$ for all $i \in I$ and $\bar{p} \cdot \bar{y} > 0$ means that

$$\bar{t}_i = \bar{p} \cdot \omega_i - \bar{p} \cdot \bar{x}_i = \bar{d}_i - \bar{a}_i \bar{p} \cdot \bar{y} > 0 \text{ for all } i \in I.$$

For $j \in J$, $\bar{s}_j = 0$ implies that $u_i(\omega_i) \geq u_i(\bar{x}_i)$ and

$$\bar{t}_j = \bar{p} \cdot \omega_i - \bar{p} \cdot \bar{x}_i \geq 0.$$

Thus, the sign of $\bar{t}_i$ is the same for all i. Of course, $\sum_i \bar{t}_i = \bar{p} \cdot y$.

Finally, the proof that aggregate income is positive and that condition (i) of equilibrium holds follows from lemma 2 exactly as in the proof of Theorem 1. ■

Remark 2. Assumption (E) can also be used to strengthen the conclusion of Proposition 2. Recall that in proving Proposition 2 we constructed prices which were not necessarily related to the marginal rates of substitution. This raises the possibility that the inconsistency between (SR) and the presence of fixed costs may be avoided by restricting attention to prices which correspond to marginal rates of substitution (in addition to being in the appropriate normal cone). But this is not possible if (E) holds (or $l = 1$). If the marginal rates of substitution are bounded, the argument used in the beginning of the proof of Theorem 2 shows that (given (E)), there exists $p \in S$ such that $((\omega_i), 0, p)$ is an equilibrium with transfers for $\mathcal{E}$. Since the prices reflect marginal rates of substitution, the surplus is 0 for each consumer and (SR) cannot hold.

Remark 3. While assumption (E) is quite restrictive, Theorem 2 can be re-interpreted in the following way. Suppose resources are allocated sequentially. In the first stage, the l non-produced commodities are first traded without using the technology. This leads to a 'Walrasian equilibrium' in the exchange economy. By the first welfare theorem, the corresponding allocation satisfies assumption (E). The firm is then activated in the second stage, using marginal cost pricing and variable shares. Theorem 2 shows that efficiency can be achieved without assuming (WSR), and with this re-interpretation, without relying on assumption (E).

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